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FUNCTIONS RELATED WITH DEGREE OF ESTIMATE IN HOLDER METRIC

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Abstract: The authors evaluated the degree of estimate of meanings linked by the similar series in Holder metric by Borel's mean after establishing the Fourier atmosphere of series.
Keywords: Holder Metric, Vital

1. INTRODUCTION

The grade of calculation of a meaning f going to several courses by diverse Suability technique takes remained resolute by numerous Geometrician, Chandra (1982) bargain the degree of calculation of meaning thru Norlund alter. Well ahead continuously Chandra (1982) find the mark of guesstimate in Holder metric by matrix convert. I n sequel singh (2018) get the fault bound of intermittent utility in Holder metric over Mishra (2019) gave the oversimplification of consequence of Singh (2018). In this weekly we discovery the step of calculation of meaning in Holder metric thru (N, Pn) revenues.

2. DEFINITION

Contract f is a broken utility of retro 2π inferable in logic of Lévesque terminated [π, - π]. Lease Fourierseries of f assumed through

$$f(t) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad (1)$$

Let $c_{2\pi}$ mean Banach Space of altogether 2π - intermittent unceasing function distinct continuously $[\pi, -\pi]$ below sub-norm. Aimed at $0 \leq \beta \leq 1$ and certain constructive continual k the meaning space H_α is assumed through next

$$H\alpha = \{f \in c_{2\pi} : |(x) - f(y)| \leq k|x - y|^\alpha \} \quad .(2)$$

Space H_α stands a Banach space by the average f_α de fined through

$$f_\alpha = "f"_c + \sup_{x,y} [\Delta^\alpha (y, x)] \quad (3)$$

Wherever, $f_c = \text{Sup}_{-\pi \leq y \leq \pi} |f(y)|$ and

$$\Delta^\alpha f(y, x) = \frac{|f(x) - f(y)|}{|x-y|^\alpha} \; x \neq y$$

We will procedure the assembly that $\Delta^0 f(x, y) = 0$

Metric influenced through average in (3) off H_α is christened Holder metric. We transcribe finished the paper $\phi_y = (y + t) - 2f(y) + f(y - t)$ (4)

$$K_n(t) = \frac{1}{2\pi P_n} \sum_{k=0}^{n} \frac{p_n}{(1+q)^k} \left\{ \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)} \right\} (5)$$

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2. RESULTS

In 1982 Mahapatra and Chandra (1982) careful the E_n^q (f,x) the container incessant utility f to attain mistake boundaries in Holder norm. The subsequent

Theorem – Contract $0 \le \beta < \alpha \le 1$ plus let $f \in H_\alpha$ then on behalf of m > 1

$$\| f - E_n^q(f) \|_\beta = o\,[(m)^{\frac{-(\alpha-\beta)}{2}} (\log m)^{\frac{\beta}{\alpha}}]\ (5)$$

Upstairs proposition better by Chandra in 1984 and shown

Theorem – Contract $0 \le \beta \le 1$ plus let $f \in H_\alpha$ then on behalf of m > 1

$$\| f - E_n^q(f) \|_\beta = o\,[(n)^{\beta-\alpha} (\log n)^{\beta}]\ (6)$$

Singh and Mahajan (1992) well-known the subsequent proposition to fault sure of indication transient finished (C,1)(E,1)alter.

Theorem 1 – Lease (z) distinct (4) be s . t .

$$\int_t^{\pi} \frac{(v)}{v^2} dv = \{(z)\} \qquad H(z) \ge 0\ (5)$$

$$\int_0^t (v)t = o\{tH(z)\}$$

$$as\ t \to 0^+\ (6)$$

Then on behalf of $0 \le \alpha \le 1$ and $\in H_w$ we obligate

$$\| {}^{(CE)}_n{}^1 (S; f) - f(x) \|_{w^*} = o\,[((n+1)^{-1} H(\frac{\pi}{n+1}))^{1-\beta}]\ (7)$$

Theorem 2 – Consider w(t) definite (4) and on behalf of $0 \le \alpha \le 1$ and $f \in H_w$ we obligate

$$\| {}^{(CE)}_n{}^1 (f) - f(x) \|_{w^*} = o\,\{(w(\frac{\pi}{m+1}))^{1-\frac{\beta}{\alpha}} + (m+1)^{-1} \sum_{k=0}^{n} \{w(\frac{1}{k+1}))^{1-\frac{\beta}{\alpha}}\}\qquad (8)$$

In equal Mishra and Chari contributed the comprehensive outcome of upstairs theorem. They show the next.

Theorem 3 – Contract (t) definite (4) be s.t.

$$\int_t^{\pi} \frac{(v)}{v^2} dv = \{(z)\} \qquad H(z) \ge 0$$

$$\int_0^t (v)v = o\{zH(z)\} \qquad A(s)\ t \to 0^+$$

Let Np stand the Orland suability matrix spawned through non –negative {Pn} s.t. (n+1) pn = o(Pn) $\forall n \ge 0$.

Then on behalf of $f \in H_w$ $0 \le \beta \le 1$ we ought to

$$\| t_n^{-NE}(f) - f(x) \| w^* = o\,\{\frac{w(|y-x|)^{\beta}}{w^*(|y-x|)} (\log(m+1))^{\beta} (m+1)^{-1} H(\frac{\pi}{m+1}))^{1-\beta}\}\ (9)$$

Plus if w(t) mollifies (5) then on behalf of $f \in H_w$ $0 \le \beta < \alpha \le 1$ we obligate

$$\| t_n^{-NE}(f) - f(x) \| w^* = o\,\{\frac{w(|y-x|)^{\frac{\beta}{\alpha}}}{w^*(|y-x|)} (\log(m+1)\, w(\frac{\pi}{m+1}))^{1-\frac{\beta}{\alpha}} + ((\frac{1}{m+1}) \sum_{k=0}^{n} w(\frac{\pi}{m+1}))^{1-\frac{\beta}{\alpha}}\}$$

(10)

In this tabloid we verify the subsequent proposition

Theorem – Aimed at $0 \le \beta < a \le 1$ *and* $f \in H_a$ before let $f \in H_a$ on behalf of $n > 1$

$$\| t_n(f) - f \|_\beta = o\left(n^{\beta-a} \log\left(\right)\right) \quad (11)$$

LEMMA

Lemma 5(a) - If $\emptyset_y(t)$ definite in (5) then on behalf of $f \in H_a$, obligate

$$|\emptyset_y - \emptyset_x| = (|y - x|) \quad (12)$$

$$|\emptyset_y - \emptyset_x| = (|t|) \quad (13)$$

Lemma 5(b) - On behalf of $0 \le t \le \frac{\pi}{n}$ we need $\sin nt = n \sin t$

$$|K_n(t)| = (n) \quad (14)$$

Proof - For $0 \le t \le \frac{\pi}{m}$ plus $\sin mt = m \sin t$ before

$$| K_n(t) = \frac{1}{2\pi P_n} \sum_{k=0}^{n} \frac{p_n}{(1+q)^k} \left\{ \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)} \right\}$$

$$\le$$

$$\le \frac{1}{2\pi P_n} |\sum_{k=0}^{n} p_{n-k} (2k+1)|$$

$$= \frac{(2n+1)}{2\pi P_n} |\sum_{k=0}^{n} p_{n-k}|$$

$$= o(n)$$

Lemma 5(c) - $\frac{\pi}{n} \le t \le \pi$, $\sin\frac{t}{2} \ge \frac{t}{\pi}$ *and* $\sin nt \le 1$ we require

$$| K_n(t)| = o\left(\frac{1}{t}\right) \quad (15)$$

Proof - $\frac{\pi}{m} \le t \le \pi$ $\sin\frac{t}{2} \ge \frac{t}{\pi}$

and
$\sin nt \le 1$ —

$$|K_m(t)| = \left| \frac{1}{2\pi P_n} \sum_{k=0}^{n} \frac{p_m}{(1+q)^k} \left\{ \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)} \right\} \right|$$

$$\le \frac{1}{2\pi P_n} \sum_{k=0} \frac{p_{n-k}}{(1+q)} \left(\sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\pi}{t} \right) |$$

$$\le \frac{1}{2t P_n} |\sum_{k=0}^{n} p_{n-k}|$$

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$= o\left(\frac{1}{t}\right)$

RESILIENT OF PROPOSITION 4

Let $S_n(x)$ signifies the incomplete quantity of fourier sequence assumed in (1) we need

$$S_n(y) - f(y) = \frac{1}{2\pi}\int_0^{\pi} \emptyset(t)\,\frac{\sin\left(n+\frac{1}{2}\right)}{\sin\frac{t}{2}}\,dt \tag{16}$$

The (E,q) convert E^q of S_n is specified by

$$E_n^q - f(x) = \frac{1}{2\pi P_m}\sum_{k=0}^{n}\frac{P_{n-k}}{2\pi(1+q)^n}\int_0^{\pi}\theta(t)\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}\right\}dt$$

The (N,Pm) (E,q) of $S_m(x)$ is specified by

$$t_n^{NE}(f) - f(x) = \frac{1}{2\pi P_n}\sum_{k=0}^{n}\frac{P_{n-k}}{2\pi(1+q)^n}\int_0^{\pi}\theta\left\{\sum_{v=0}^{k}\binom{k}{v}q^{k-v}\frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}\right\}dt$$

$$= \int_0^{\pi}\theta(t)\,K_{(t)}$$

$$= \left[\int_0^{\frac{\pi}{n}} \cdot \int_{\frac{\pi}{m}}^{\pi}\cdot\right]\theta K_{m(t)}$$

Now $E_m(x) = |t_n^{NE}(f) - (x)|$ and $E_m(y,x) = |E_m(y) - E_m(x)|$

$$E_m(y,x) = |E_m(y) - E_m(x)|$$

$$= \left[\int_0^{\frac{\pi}{n}} \cdot \int_{\frac{\pi}{n}}^{\pi}\cdot\right]|\emptyset_x - \emptyset_v||K_m(t)|d(t)$$

$$= I_1 + I_2 \tag{18}$$

Again $I_1 = \int_0^{\frac{\pi}{n}} |\emptyset_y - \emptyset_x||k_n|\,dt$

With lemma (2) we grow

$$= (m)\int_0^{\frac{\pi}{n}} t^{\alpha}\,dt$$

$$= (m)\left[\left(\frac{\pi}{n}\right)^{\alpha+1}\right]$$

$$= (m)^{-\alpha} \tag{19}$$

Now

$$L_2 = \left[\int_0^{\frac{\pi}{n}} \cdot \int_{\frac{\pi}{n}}^{\pi}\cdot\right]|\emptyset_x - \emptyset_v||K_m|d(t)$$

$$= (m)^{-\alpha}$$

Again $I_1 = \int_0^{\frac{\pi}{m}} |\emptyset - \emptyset_y||k_m| \, dt$

$= (|y - x|^{\alpha} m)$ (21)

$I_2 = \int_{\frac{\pi}{n}}^{\pi} |\emptyset - \emptyset||k_n(t)| \, dt$

$= o|x - y|^{\alpha} \int_{\frac{\pi}{n}}^{\pi} k_n(t)| \, dt$

$= o|y - x|^{\alpha} \int_{\frac{\pi}{n}}^{\pi} (\frac{1}{t}) \, dt$

$= (|y - x|^{\alpha} \log m)$ (22)

Currently $I_r \overset{1-\beta}{=} I_r \quad {}_{\alpha}^{\beta} \quad I_r =$

r = 1,2……….

Since (6) and (8)

we contract

$I_1 = o\,[\{(m)\}^{-\alpha}]^{1-\frac{\beta}{\alpha}} \{|y - x| \; (m)^{\alpha}\}]^{\frac{\beta}{\alpha}}$

$= o\,[(m)^{-\alpha} |y - x|^{\beta} (m)^{\frac{\beta}{\alpha}}]$

$= o\,[(m)^{\beta - \alpha + \frac{\beta}{\alpha}} |y - x|^{\beta}]$ (23)

Since (7) and (9)

we get

$I_2 = o\,[[(m)^{-\alpha}]^{1-\frac{\beta}{\alpha}} \{|y - x|^{\alpha} (\log m)\}]^{\frac{\beta}{\alpha}}$

$= o\,[(m)^{-\alpha} |y - x|^{\beta} (\log m)^{\frac{\beta}{\alpha}}]$ (24)

$= o\,[(m)-\alpha \, |y - x|\beta \, (\log m)\alpha\,]$

Currently

from (10) besides (11)

we change to

$|f(y) - f(x)| = o\,[(m)^{\beta - \alpha + \frac{\beta}{\alpha}} |y - x|^{\beta}] + o\,[(m)^{\beta-\alpha}|y - x|^{\beta} (\log m)^{\frac{\beta}{\alpha}}]$

$= o\,[(m)^{-\alpha} |y - x|^{\beta} (\log m)^{\frac{\beta}{\alpha}}]$

$= o\,[(m)^{-\alpha} |y - x|^{\beta} (\log m)_{\alpha}]$

$= o\,[(m)^{-\alpha} |y - x|^{\beta} (\log m)_{\alpha}]$

$(y \neq x)$

$$=o\,[(m)^{-\alpha}\ (\log m)^{\frac{\beta}{\alpha}}] \qquad (24)$$

Now $\|f\|_c = [(m)^{-\alpha}]$ (25)

Joining (12) and (13) we grow

$$\|t_n(f) - f\|_\beta = o\,[(m)^{-\alpha}\ (\log n)_\alpha^{\frac{\beta}{\ }}].$$

These ample the proof of proposition.

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