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CONCEPT OF SECTIONAL ANALYTICITY IN SEQUENCE SPACES

Kunwar Pal Singh, Associate Professor

Department of Mathematics CCR (PG) College, Muzaffarnagar- 251001

Abstract

This article deals with a special dual called Analytical dual, apart from the general Kothe-Toeplitz duals. In this article, the concepts Λ-dual and Λ-perfect are introduced. The Λ-dual and Λ-perfect space of X with $\Gamma \subseteq X \subseteq \Lambda$ are investigated. Also the concept of sectional analyticity is introduced and the relation between f-dual and Λ-dual has been established. In this article, the definition of Λ-dual and Λ-perfect are given. The Λ-duals of some known sequence spaces are determined. Also, Sectional analiticity in sequence spaces is introduced.

Keywords: sequences, theory, fourier, power, series

1. INTRODUCTION

In several branches of analysis, the study of sequence spaces occupies a very prominent position. The theory of sequence spaces is a part of functional analysis, motivated by problems in Fourier series, power series and systems of equations with infinitely many variables [1]. Apart from this, it is a powerful tool for obtaining positive results concerning Schauder bases and their associated types. Also it has made remarkable advances in recent times in enveloping summability theory via unified techniques effecting transformations from one sequence space into another [2]. Therefore this study is made on this subject matter.

2. ANALYTICAL DUAL OF A SEQUENCE SPACE X

Here we define Analytical dual and give some properties and illustrations [3].

Definition 2.1. Let X be an FK space.

Let$\Lambda = \{\mathrm{x}: \mathrm{x} \in \omega, \sup|x_n|^{\frac{1}{n}} \text{ exists}\}$

The Λ-dual of X (denoted by X^{Λ}) and may be called analytic dual of X, is defined as

$X^{\Lambda} = \{x \in \omega: x_u \in \Lambda$ for every $u \in X\}$.

Definition 2.2. An FK space X is called perfect if $X^{\Lambda\Lambda} = X$.

Remark 2.1. The above definitions also hold when X is singleton or a sequence space instead of an FK space [4].

Lemma 2.1. The Λ-dual of a sequence space has the following properties.

(i) X^{Λ} is a linear subspace of ω for $X \subset \omega$.

(ii) $X \subset Y$ implies $X^{\Lambda} \supset Y^{\Lambda}$ for every $X, Y \subset \omega$.

(iii) $X^{\Lambda\Lambda} = (X^{\Lambda})^{\Lambda} \supset X$ for every $X \subset \omega$.

Theorem 2.1. If $\Gamma \subseteq X \subseteq \Lambda$ then $X^{\Lambda} = \Lambda$ [5].

Proof. Step (i): We first claim that $\Gamma^{\Lambda} = \Lambda$.

If $x \in \Lambda$ then $(|x_k|^{\frac{1}{k}})$ is bounded.

For any $u \in \Gamma$ and $x \in \Lambda$, we have $ux \in \Lambda$. Therefore $x \in \Gamma^{\Lambda}$.

On the other hand, suppose that $x \notin \Lambda$, [6].

Then there exists an increasing sequence of positive integers $n_1 < n_2 < \ldots < n_k < \ldots$ such that $|x_k|^{\frac{1}{n_k}} > p^{n_k}$, where $p > 1$ is an integer

Construct a sequence $u = (u_n)$ as follows [7].

$$u_n = \begin{cases} \dfrac{k^n}{p^{n_k}} & \text{if } n = n_k \text{ for } k = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Obviously u ∈ Γ. But$|x_{n_k} u_{n_k}|^{\frac{1}{n_k}} > k$.

Hence ($|x_n u_n|^{1/n}$) is unbounded, which is a contradiction to the fact that x ∈ Γ^Λ . Thus $\Gamma^\Lambda = \Lambda$.

Step (ii): We claim that $\Lambda^\Lambda = \Lambda$.

Since Γ ⊂ Λ, we have $\Lambda^\Lambda \subset \Gamma^\Lambda = \Lambda$ (by Step (i)).

That is, $\Lambda^\Lambda \subset \Lambda$. Also we have $\Lambda \subset \Lambda^\Lambda$. Hence $\Lambda = \Lambda^\Lambda$.

Step (iii): We claim that $X^\Lambda = \Lambda$.

Since Γ ⊆ X ⊆ Λ, we have $X^\Lambda \subseteq \Gamma^\Lambda$.

Then by Step (i), $X^\Lambda \subseteq \Lambda$. Also X ⊆ Λ implies $\Lambda^\Lambda \subseteq X^\Lambda$.

Then by Step (ii), we have $\Lambda \subseteq X^\Lambda$. Thus $X^\Lambda = \Lambda$ [8].

Corollary 2.1. The only Λ-perfect space X with Γ ⊆ X ⊆ Λ is Λ.

Proof. Let X be such that $X^{\Lambda\Lambda} = X$.

Since Γ ⊆ X, we have XΛ ⊆ Γ Λ = Λ (by Step (i) of theorem 2.1).

By applying Step (ii) of theorem 2.1, $\Lambda = \Lambda^\Lambda \subseteq X^{\Lambda\Lambda} = X$.

Also by hypothesis X ⊆ Λ.

3. SECTIONAL ANALYTICITY

Now here, we define Sectional Analyticity and prove some related theorems [9].

Definition 3.1. Let X be an FK space containing Φ. Then A+ is defined as

$A^+(X) = \{z \in \omega : (z_k f(\delta^k)) \in \Lambda \text{ for all } f \in X'\}$

and we put $A = A^+ \cap X$.

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Lemma 3.1. Let X and Y be FK spaces containing Φ [10].

Then $A^+(X) \subset A^+(Y)$ whenever $X \subset Y$.

Proof. Let z ∈ A+(X). Then $(z_n f(\delta^n)) \in \Lambda$ for all f ∈ X ′ .

From this fact and since g|X∈ X′ , $(z_n g(\delta^n)) \in \Lambda$ for all g ∈ Y ′ .

This shows that $z \in A^+(Y)$. Hence $A+(X) \subset A^+(Y)$.

Definition 3.2. Let X be an FK space containing Φ. Then X is said to have AA-Property (AbschnittsAnalytique) or sectional analyticity if and only if X = A [11].

Lemma 3.2. Let X be an FK space containing Φ and z ∈ ω.

Then $z \in A^+$ if and only if $z^{-1}X \supset \Gamma$.

Proof. Let f ∈ (z −1X) ′ . Then by Theorem.

f(x) = α.x + g(zx), where α ∈ Φ, g ∈ X ′ and$\alpha x = \sum_{k=1}^{\infty} \alpha_k x_k$.

Consequently, $f(\delta^k) = \alpha k + g(z\delta^k)$.

That is, $f(\delta^k) = \alpha k + z_k g(\delta^k)$.

Hence if $z \in A^+$ then $(z_k f(\delta^k)) \in A$ and so $(f(\delta^k)) \in \Lambda$ for all $f \in (z^{-1}X)'$. That is, $(z^{-1} .X) f \subset \Lambda$.

But $\Lambda = \Gamma^f$. Since Γ has AD, by Theorem, $\Gamma \subset z^{-1} .X$. The reverse implication follows similarly [12].

Theorem 3.1. Let X be an FK space containing Φ. Then $z \in X^{f\Lambda}$ if and only if $z^{-1}X \supset \Gamma$ [13].

Proof. Note that, by definition of A^+, $z \in A^+$ if and only if $z_u \in A$ for every $u \in X^f$. Hence A+ = $X^{f\Lambda}$.

By Lemma 3.3, $z \in A^+$ if and only if $z^{-1}X \supset \Gamma$.

Hence $z \in X^{f\Lambda}$ if and only if $z^{-1}X \supset \Gamma$.

Theorem 3.2. Let X be an FK space containing Φ. If X has AA, then $Xf \subset X\Lambda$.

Proof. Suppose that X has AA. Then X = A = A+ ∩ X and so X ⊂ A+ = XfΛ . Therefore XΛ ⊃XfΛΛ. Hence we have XΛ ⊃Xf .

Theorem 3.3. Let X be an FK space containing Φ. If X has AK then X has AA.

Proof. Suppose X has AK. Then $X_\beta = X_f$ and so $X \subset X_{\beta\beta} = X_{f\beta}$. Further, $X \subset X_{f\beta} \subset X_f\Lambda$. This means that X ⊂ A+. Consequently, A = X. Hence X has AA property [14].

Remark 3.1. Consider the space c, A+(c) = c fΛ = ℓ Λ = Λ. Now, A = A+ ∩ c = Λ∩c = c and hence c has AA. But c does not posses AK Property.

Theorem 3.4. A+ is monotone.

Proof. Let $(x_k) \in A^+$. Suppose $|y_k| \leq |x_k|$ for all $k \in N$.

Since $(x_k) \in A^+$, $(x_k f(\delta^k)) \in \Lambda$ and so $\sup |x_k f(\delta^k)|^{1/k}$ exists.

Thus $|x_k f(\delta^k)|^{1/k} < M < \infty$ for some $M > 0$.

From this, we get $|xk|^{1/k}\ |f(\delta^k)|^{1/k} < M$.

Now $|yk| \leq |xk|$ and so $|yk|^{1/k} \leq |xk|^{1/k}$.

This implies $|yk|^{1/k}\ |f(\delta\ k)|^{1/k} \leq |xk|^{1/k}\ |f(\delta\ k)|^{1/k} < M$.

Therefore $(y_k) \in A^+$ and so A^+ is solid. Hence it is monotone.

4. CONCLUSION

It is concluded that in this article, the Analytical and Entire duals of some known sequence spaces are obtained. The concept of sectional analyticity is introduced and the relation between f-dual and Λ-dual is obtained. Some important results discussed in this article.

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