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A NEW CLASS OF MINIMAL AND MAXIMAL SETS VIA BĜ – CLOSED SET

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ABSTRACT

In this paper a new class of sets namely, bĝ–minimal and bĝ–maximal closed sets in topological spaces are introduced and characterized so as to determine their behaviour relative to subspaces.

KEYWORDS

bĝ–closed sets, bĝ–minimal closed set, bĝ–maximal closed set, bĝ–minimal open set and bĝ–maximal open set.

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1 INTRODUCTION

Levine[9] introduced generalized closed (briefly g–closed) sets and studied their basic properties. Veera Kumar [12] introduced ĝ–Closed sets in topological spaces. Andrijevic[1] introduced a new class of open sets called b–open sets. R.Subasree and M.Mariasingam[11] introduced a new class of sets called bĝ–closed sets.

Recently minimal open sets and maximal open sets in topological spaces were introduced and studied by F.Nakaoka and N.Oda [5]. In section 3, a new class of sets called bĝ–minimal and bĝ–maximal closed sets in topological spaces are introduced and characterized so as to determine their behaviour relative to subspaces. The purpose of this present paper is to study some fundamental properties related to generalized minimal closed sets. The complement of a generalized minimal closed set is said to be a generalized maximal open set.

2 PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents a non–empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. Let us recall the following definitions.

DEFINITION 2.1[5] : A subset A of a space (X,τ) is called a

(i) A minimal open (resp. minimal closed) set if any open (resp. closed) subset of X which is contained in A, is either A or Φ.

(ii) A maximal open (resp. maximal closed) set if any open (resp. closed) subset of X which contains A, is either A or X.

The following duality principle holds [5] for subset A of a topological space X:

(1) A is minimal closed if and only if $X - A$ is maximal open.

(2) A is maximal closed if and only if $X - A$ is minimal open.

DEFINITION 2.2 [5]**:** A topological space is said to be locally finite space if each of its elements is contained in a finite open set.

DEFINITION 2.3: A subset A of a space (X,τ) is called a

(i) b–open set[1] if A ⊆ cl[Int (A)] ∪ Int [cl(A)]

(ii) generalized closed (briefly g–closed) set[9] if cl(A)⊆U whenever A⊆U and U is open set in X.

(iii) ĝ–closed set[12] if cl(A) ⊆U whenever A⊆U and U is a semi–open set in (X,τ).

(iv) bĝ–closed set[11] if bcl(A) ⊆U whenever A⊆U and U is ĝ–open set in (X,τ).

The complement of a b–open set is called b–closed set and the complement of a g–closed(resp. ĝ–closed and bĝ–closed) set is called g–open (resp. ĝ–open and bĝ –open) set .

The intersection of all bĝ–closed sets of X containing A is called the bĝ–closure and is denoted by bĝ–cl(A). The family of all b–closed (resp. g–closed, ĝ–closed and bĝ–closed) subsets of a space X is denoted by b–C(X) (resp. g–C(X), ĝ–C(X) and bĝ– C(X)).

3 MINIMAL bĝ –OPEN SETS AND MAXIMAL bĝ –CLOSED SETS

In this section we introduce and study the properties of Minimal bĝ–open sets and Maximal bĝ–closed sets.

DEFINITION 3.1: A proper non-empty bĝ–open subset A of X is said to be a minimal bĝ–open if any bĝ–open set contained in A is Φ or A.

DEFINITION 3.2: A proper non-empty $b\hat{g}$–closed subset A of *X* is said to be a maximal $b\hat{g}$–closed if any $b\hat{g}$–closed set containing A is either *X* or A.

EXAMPLE 3.3: Let X = {a,b,c,d} with a topology τ = {X,Φ,{b},{b,d},{a,b,d}}

$b\hat{g}$–O(X) = {X, Φ, {a}, {b}, {d},{a,b},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d}}

$b\hat{g}$–C(X) = {X, Φ, {a}, {c}, {d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}

Here {a},{b},{d} are minimal $b\hat{g}$–open sets of X and {a,b,c},{a,c,d},{b,c,d} are maximal $b\hat{g}$–closed sets of X.

REMARK 3.4: Minimal open and minimal $b\hat{g}$–open sets are independent to each other.

EXAMPLE 3.5: Let X = {a,b,c} with a topology τ = {X,Φ,{a,c}}

$b\hat{g}$–O(X) = {X, Φ, {a},{c},{a,b},{a,c},{b,c}}

Here {a,c} is minimal open but not minimal $b\hat{g}$–open and {a},{c} are minimal $b\hat{g}$–open sets but not minimal open.

REMARK 3.6: Maximal closed and Maximal $b\hat{g}$–closed sets are independent to each other:

EXAMPLE 3.7: In Example (3.5), {b} is maximal closed but not maximal $b\hat{g}$–closed and {a,b}, {b,c} are maximal $b\hat{g}$–closed but not maximal closed.

DEFINITION 3.8: A proper non-empty $b\hat{g}$–closed subset A of *X* is said to be a minimal $b\hat{g}$–closed if any $b\hat{g}$–closed set contained in A is Φ or A.

DEFINITION 3.9: A proper non-empty $b\hat{g}$–open subset A of *X* is said to be a maximal $b\hat{g}$–open if any $b\hat{g}$–open set containing A is either *X* or A.

EXAMPLE 3.10: Let *X* = {a, b, c, d}, τ = {X,Φ,{a},{a,b},{a,d},{a,b,d}}

$b\hat{g}$–O(X) = {X, Φ, {a}, {b}, {d},{a,b},{a,c}, {a,d},{b,d},{a,b,c},{a,b,d},{a,c,d}}

$b\hat{g}$–C(X) = {X, Φ, {b}, {c}, {d},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,c,d},{b,c,d}}

Here {b},{c} and {d} are minimal $b\hat{g}$–closed sets and {a, b, c}, {a,b,d} and {a,c,d} are maximal $b\hat{g}$–open sets.

THEOREM 3.11:

(i) Let A be minimal $b\hat{g}$–open set and B be a $b\hat{g}$–open set. Then $A \cap B = \Phi$ or $A \subset B$.

(ii) Let A and B be minimal $b\hat{g}$–open sets. Then $A \cap B = \Phi$ or A = B

PROOF:

(i) Let A be a minimal $b\hat{g}$–open set and B be a $b\hat{g}$–open set. If $A \cap B = \Phi$, then there is nothing to prove. If $A \cap B \neq \Phi$. Then $A \cap B \subset A$. Since A is minimal $b\hat{g}$–open set, we have $A \cap B = A$. Therefore $A \subset B$.

(ii) Let A and B be minimal $b\hat{g}$–open sets. If $A \cap B \neq \Phi$, then $A \subset B$ and $B \subset A$ by (i). Hence A = B.

THEOREM 3.12: Let A be a minimal b$\hat{g}$–open set. If $x \in$ A, then A $\subset$ B for any open regular neighborhood of B of x.

PROOF: Let A be a minimal b$\hat{g}$–open set and x be an element of A. Suppose there exists a regular open neighborhood A of x such that A $\not\subset$ B. Since B is regular, A $\cap$ B is minimal b$\hat{g}$–open with A $\cap$ B $\subset$ A and A $\cap$ B $\neq \Phi$. Since A is a minimal b$\hat{g}$–open set, we have A $\cap$ B = A. That is A $\subset$ B, which is a contradiction for A $\not\subset$ B.Therefore A $\subset$ B for any open regular neighborhood of B of x.

THEOREM 3.13: Let A be a minimal b$\hat{g}$–open set. If $x \in$ A, then A $\subset$ B for some b$\hat{g}$–open set B containing x.

THEOREM 3.14: Let A be a minimal b$\hat{g}$–open set. If $x \in$ A, then A = $\cap$ {B:B is b$\hat{g}$–open set containing x} for any element x in A.

PROOF: By theorem (3.13) and A is b$\hat{g}$–open set containing x, we have A $\subset$ {B: B is b$\hat{g}$–open set containing x} $\subset$ A. Hence A= $\cap$ {B:B is b$\hat{g}$–open set containing x}

THEOREM 3.15: For any $x \in X$ and a subset A in X, $x \in$ b$\hat{g}$–Cl(A) if and only if U $\cap$ A $\neq \Phi$ for every b$\hat{g}$–open set U containing x.

PROOF: Let $x \in X$, A $\subset X$ and $x \in$ b$\hat{g}$–Cl(A). We prove the result by the method of contradiction. Suppose there exists a b$\hat{g}$–open set U containing x such that U $\cap$ A = Φ. Then A $\subset X -$ U and $X -$ U is b$\hat{g}$–closed. We have b$\hat{g}$–Cl(A) $\subset X -$U. This gives $x \notin$ b$\hat{g}$–Cl(A), which is contradiction. Hence U $\cap$ A $\neq \Phi$ for every b$\hat{g}$–open set U containing x.

Conversely, let U $\cap$ A $\neq \Phi$ for every b$\hat{g}$–open set U containing x. We prove the result by the method of contradiction. suppose $x \notin$ b$\hat{g}$–Cl(A). Then there exists a b$\hat{g}$–closed subset V containing A such that $x \notin$ V. Then $x \in X$ – V and $X -$ V is b$\hat{g}$–open. Also ($X -$ V) $\cap$ A = Φ, which is a contradiction. Hence $x \in$ b$\hat{g}$–Cl(A).

THEOREM 3.16: Let A be a non-empty b$\hat{g}$–open set. Then the following are equivalent:

(1) A is a minimal b$\hat{g}$–open set.

(2) A $\subset$ b$\hat{g}$–Cl(U), for any non-empty subset U of A.

(3) b$\hat{g}$–Cl(A) = b$\hat{g}$–Cl(U), for any non-empty subset U of A.

PROOF: (1) $\Rightarrow$ (2): Let A be minimal b$\hat{g}$–open set. Let $x \in$ A and U be a non-empty subset of A. By theorem (3.13), there is a b$\hat{g}$–open set B containing x such that A $\subset$ B. Then we have U $\subset$ A $\subset$ B which implies U $\subset$ B. Now U = U $\cap$ A $\subset$ U $\cap$ B. Since U is non-empty, we have U $\cap$ B $\neq \Phi$. Since B is any b$\hat{g}$–open set containing x, by above theorem (3.15), $x \in$ b$\hat{g}$–Cl(U). That is $x \in$ A implies $x \in$ b$\hat{g}$–Cl(U). Hence A $\subset$ b$\hat{g}$–Cl(U), for any non-empty subset U of A.

(2)$\Rightarrow$(3): Let U be a non-empty subset of A and A$\subset$bĝ–Cl(U). Then bĝ–Cl(U)$\subset$bĝ–Cl(A) and bĝ–Cl(A)$\subset$bĝ–Cl(U). Hence bĝ–Cl(A) = bĝ–Cl(U), for any non-empty subset U of A.

(3)$\Rightarrow$(1): Let bĝ–Cl(A) = bĝ–Cl(U), for any non-empty subset U of A. Suppose A is not a minimal bĝ–open set. Then there exists a non-empty bĝ–open set B such that B$\subset$A and B $\neq$A. Now there exists an element $x \in$ A such that $x \notin$ B, which implies $x \in X -$ B. That is bĝ–Cl($\{x\}$) $\subset$bĝ–Cl($X -$ B) = $X -$ B, as $X -$ B is bĝ–closed in X. It follows that bĝ–Cl($\{x\}$) $\neq$ bĝ–Cl(A). This is a contradiction to the fact that bĝ–Cl($\{x\}$) = bĝ–Cl(A), for any non-empty subset $\{x\}$ of A. Thus A is a minimal bĝ–open set.

THEOREM 3.17: Let B be a non-empty finite bĝ–open set. Then there exists at least one (finite) minimal bĝ–open set A such that A$\subset$ B.

PROOF: Let B be a non-empty bĝ–open set. Then we have the following two cases:

(1) B is a minimal bĝ–open.

(2) B is not a minimal bĝ–open.

Case (1): If we choose B = A, then the theorem is proved.

Case (2): If B is not a minimal bĝ–open, then there exists a non-empty (finite) bĝ–open set B_1 such that $B_1 \subset$B. If B_1 is minimal bĝ–open, we take A = B_1. If B_1 is not a minimal bĝ–open set, then there exists a non-empty(finite) bĝ–open set B_2 such that $B_2 \subset B_1 \subset$B. We continue this process and have a sequence of bĝ–open sets ... $\subset$ Bn $\subset$........$B_2 \subset B_1 \subset$B. Since B is finite, this process will end at finite number of steps. That is, for any natural number k, we have a minimal bĝ–open set B_k such that B_k = A. Hence the proof.

COROLLARY 3.18: Let X be a locally finita space and B be a non-empty bĝ–open set. Then there exists at least one (finite) minimal bĝ–open set A such that A $\subset$ B.

PROOF: Let X be a locally finite space and B be non-empty bĝ–open set. Let $x \in$B. Since X is finite, we have a finite open set B_x such that $x \in B_x$. Then B$\cap B_x$ is a non-empty finite bĝ–open set. By theorem (3.14), there exists at least one (finite) minimal bĝ–open set A such that A $\subset$B$\cap B_x$. That is A $\subset$B. Hence there exists at least one (finite) minimal bĝ–open set A such that A$\subset$B.

COROLLARY 3.19: Let B be a finite minimal open set. Then there exists atleast one (finite) minimal bĝ–open set A such that A $\subset$ B.

PROOF: Let B be a finite minimal open set. Then B is a non-empty finite bĝ–open set. By theorem (3.17), there exists at least one (finite) minimal bĝ–open set A such that A $\subset$ B.

THEOREM 3.20: Let A and A_λ be minimal bĝ–open sets for any $\lambda \in \Gamma$. If $A \subset \bigcup_{\lambda \in \Gamma} A_\lambda$ then there exists an element $\lambda \in \Gamma$ such that $A = A_\lambda$.

PROOF: Let $A \subset \bigcup_{\lambda \in \Gamma} A_\lambda$. Then $A \cap \bigcup_{\lambda \in \Gamma} A_\lambda = A$. That is $\bigcup_{\lambda \in \Gamma} (A_\lambda \cap A) = A$. Also by theorem(3.11), $A \cap A_\lambda = \Phi$ or $A = A_\lambda$ for any $\lambda \in \Gamma$. Hence there exists an element $\lambda \in \Gamma$ such that A = A_λ.

THEOREM 3.21: Let A and A_λ be minimal bĝ–open sets for any $\lambda \in \Gamma$. If A $\neq A_\lambda$ for any $\lambda \in \Gamma$, then $\bigcup_{\lambda \in \Gamma} A_\lambda \cap$ A = Φ.

PROOF: Suppose $\bigcup_{\lambda \in \Gamma} A_\lambda \cap$ A $\neq$ Φ**.** That is $\bigcup_{\lambda \in \Gamma} (A_\lambda \cap A) \neq$ Φ. Then there exists an element $\lambda \in \Gamma$ such that $A \cap A_\lambda \neq$ Φ. By theorem(3.11), we have A= A_λ which is a contradiction to the fact that A≠ A_λ for any $\lambda \in \Gamma$. Hence $\bigcup_{\lambda \in \Gamma} A_\lambda \cap$ A = Φ.

THEOREM 3.22: A proper non-empty subset A of X is maximal bĝ–closed if and only if X − A is minimal bĝ–open.

PROOF: Let A be a maximal bĝ–closed set. Suppose X−A is not a minimal bĝ–open set. Then there exists a non-empty bĝ–open set B such that B ⊂ X −A. That is A ⊂ X − B and X − B is a bĝ–closed set. This is a contradiction to the fact that A is a maximal bĝ–closed set.

Conversely, Let X −A is a minimal bĝ–open set. Suppose A is not a maximal bĝ–closed set. Then there exists a bĝ–closed set B ≠ A such that A ⊂ B ≠ X. That is Φ ≠ X−B ⊂ X−A and X−B is a bĝ–open set. This contradicts the fact that X − A is a minimal bĝ–open set. Hence A is a maximal bĝ–closed set.

THEOREM 3.23:

(i) Let A be maximal bĝ–closed set and B be a bĝ–closed set. Then A ∪ B = X or B ⊂ A.

(ii) Let A and B be maximal bĝ–closed sets. Then A ∪ B = X or A = B

PROOF:

(i) Let A be a maximal bĝ–closed set and B be a bĝ–closed set. If A ∪ B = X, then there is nothing to prove. If A ∪ B ≠ X. Then A ⊂ A ∪ B and A ∪ B is bĝ–closed. We have A ∪ B = A, as A is maximal bĝ–closed set. Then we have B ⊂ A.

(ii) Let A and B be maximal bĝ–closed sets. If A ∪ B ≠ X, then A ⊂ B and B ⊂ A by (i). Therefore A = B.

THEOREM 3.24: Let A be a maximal bĝ–closed set. If $x \in$ A then for any bĝ–closed set B containing x, A $\cup$ B = X or B $\subset$ A.

PROOF: Let A be a maximal bĝ–closed set and $x \in$ A. Suppose there exists bĝ–closed set B containing x such that A $\cup$ B $\neq X$. Then A $\subset$ A $\cup$ B and A $\cup$ B is a bĝ–closed. Since A is a maximal bĝ–closed, we have A $\cup$ B = A. Hence B $\subset$ A.

THEOREM 3.25: Let $A_\alpha, A_\beta, A_\gamma$ be maximal bĝ–closed sets such that $A_\alpha \neq A_\beta$. If $(A_\alpha \cap A_\beta) \subset A_\gamma$, then either $A_\alpha = A_\gamma$ or $A_\beta = A_\gamma$.

PROOF: Given that $(A_\alpha \cap A_\beta) \subset A_\gamma$. If $A_\alpha = A_\gamma$, then there is nothing to prove.

But if $A_\alpha \neq A_\gamma$ then we have to prove $A_\beta = A_\gamma$. Now

$$
\begin{aligned}
A_\beta \cap A_\gamma &= A_\beta \cap (A_\gamma \cap X) \\
&= A_\beta \cap (A_\gamma \cap (A_\alpha \cup A_\beta)) \\
&= A_\beta \cap ((A_\gamma \cap A_\alpha) \cup (A_\gamma \cap A_\beta)) \\
&= (A_\beta \cap A_\gamma \cap A_\alpha) \cup (A_\beta \cap A_\gamma \cap A_\beta) \\
&= (A_\alpha \cap A_\beta) \cup (A_\gamma \cap A_\beta) \quad (\text{since } (A_\alpha \cap A_\beta) \subset A_\gamma) \\
&= (A_\alpha \cup A_\gamma) \cap A_\beta \\
&= X \cap A_\beta \quad (\text{by theorem } 3.23) \\
&= A_\beta
\end{aligned}
$$

$$\Rightarrow A_\beta \subset A_\gamma$$

Since A_β *and* A_γ are maximal bĝ–closed sets, we have $A_\beta = A_\gamma$.

THEOREM 3.26: Let A_α, A_β *and* A_γ be maximal bĝ–closed sets which are different from each other. Then $(A_\alpha \cap A_\beta) \not\subset (A_\alpha \cap A_\gamma)$

PROOF: Let $(A_\alpha \cap A_\beta) \subset (A_\alpha \cap A_\gamma)$. Then $(A_\alpha \cap A_\beta) \cup (A_\beta \cap A_\gamma) \subset (A_\alpha \cap A_\gamma) \cup (A_\beta \cap A_\gamma)$, which implies $(A_\alpha \cup A_\gamma) \cap A_\beta \subset A_\gamma \cap (A_\alpha \cup A_\beta)$. Since by theorem (3.23) $(A_\alpha \cup A_\gamma) = X$,and $(A_\alpha \cup A_\beta) = X$ which implies $X \cap A_\beta \subset A_\gamma \cap X$ which gives $A_\beta \subset A_\gamma$. From the definition of maximal bĝ–closed set it follows that $A_\beta = A_\gamma$,which contradicts that A_α, A_β *and* A_γ are different from each other. Hence $(A_\alpha \cap A_\beta) \not\subset (A_\alpha \cap A_\gamma)$.

THEOREM 3.27: Let A be a maximal bĝ–closed set and $x \in$ A. Then A = {B : B is a bĝ–closed set containing x such that A $\cup$ B $\neq X$}.

PROOF: By theorem (3.24) and the fact that A is a bĝ–closed set containing x, we have A $\subset$ $\cup${B: B is a bĝ–closed set containing x such that A $\cup$ B $\neq X$ } $\subset$ A. Hence the result.

THEOREM 3.28: Let A be a proper non-empty co-finite bĝ–closed set. Then there exists (co-finite) maximal bĝ–closed set B such that A $\subset$B.

PROOF: If A is maximal bĝ–closed, we may take B = A. If A is not a maximal bĝ–closed set, then there exists (co-finite) bĝ–closed set A_1 such that A $\subset A_1 \neq X$. If A_1 is a maximal bĝ–closed set, we may take B = A_1. If A_1 is not a maximal bĝ–closed set, then there exists (co-finite) bĝ–closed set A_2 such that A $\subset A_1 \subset A_2 \neq X$. Continuing this process, we have a sequence of bĝ–closed sets such that A $\subset A_1 \subset A_2 \subset \ldots \subset A_k \subset \ldots \neq X$. since A is a co-finite set, this process will end in a finite number of steps. Then finally we get a maximal bĝ–closed set B = B_n for some natural number n.

THEOREM 3.29: Let A be a maximal bĝ–closed set. If $x \in X -$ A then $X -$ A$\subset$ B for any bĝ–closed set B containing x.

PROOF: Let A be a maximal bĝ–closed set and $x \in X -$A. Let B be bĝ–closed set containing x. Then by theorem (3.20), either A$\cup$B = X or B$\subset$ A. But B$\not\subset$A, for any bĝ–closed set B containing x. Therefore $X -$ A $\subset$B.

THEOREM 3.30: Let A be a non-empty bĝ–closed set. Then the following are equivalent:

(1) A is a minimal bĝ–closed set.

(2) A $\subset$ bĝ–Int(U), for any non-empty subset U of A.

(3) bĝ–Int(A) = bĝ–Int(U), for any non-empty subset U of A.

PROOF: (1)$\Rightarrow$(2): Let A be minimal bĝ–closed set. Let $x \in$A and U be a non-empty subset of A. There exists a bĝ–closed set B containing x such that A $\subset$B. Then we have U $\subset$ A $\subset$B which implies U $\subset$B. Now U = U $\cap$ A $\subset$U $\cap$B. Since U is non empty, we have U $\cap$B $\neq \Phi$. Since B is any bĝ–closed set containing x, $x \in$ bĝ–Int(U). That is $x \in$ A implies $x \in$ bĝ–Int(U). Hence A $\subset$ bĝ–Int(U), for any non-empty subset U of A.

(2) $\Rightarrow$ (3): Let U be a non-empty subset of A and A $\subset$bĝ–Int(U). Then bĝ–Int(U) $\subset$ bĝ–Int(A) and bĝ–Int(A) $\subset$bĝ–Int(U). Hence bĝ–Int(A) = bĝ–Int(U), for any non-empty subset U of A.

(3) $\Rightarrow$ (1): Let bĝ–Int(A) = bĝ–Int(U), for any non-empty subset U of A. Suppose A is not a minimal bĝ–closed set. Then there exists a non-empty bĝ–closed set B such that B $\subset$A and B $\neq$ A. Now there exists an element $x \in$ A such that $x \notin$ B, which implies $x \in X -$ B. That is bĝ–Int({x}) $\subset$ bĝ–Int($X -$ B) = $X -$ B, as $X -$ B is bĝ–open in X. It follows that bĝ–Int({x}) $\neq$ bĝ–Int(A). This is a contradiction to the fact that bĝ–Int({x}) = bĝ–Int(A), for any non-empty subset {x} of A. Thus A is a minimal bĝ–closed set.

Similarly we prove all the above properties for Minimal bĝ–closed and Maximal-open sets.

4 bĝ–SEMI-MAXIMAL OPEN SETS AND bĝ–SEMI-MINIMAL CLOSED SETS

In this section we introduce and study the properties of bĝ–semi-maximal open sets and bĝ–semi-minimal closed sets.

DEFINITION 4.1: A set A in X is said to be bĝ–semi-maximal open if there exists a maximal bĝ–open set U such that U $\subset$ A $\subset$ Cl(U). A set A of X is bĝ–semi-maximal open if and only if $X - A$ is bĝ–semi-minimal closed sets. That is the complement of bĝ–semi-maximal open sets is called as bĝ–semi-minimal closed sets.

EXAMPLE 4.2: Let X = {a, b, c}, τ = {X, Φ, {a}}.

bĝ–O(X) = {X, Φ, {b},{c},{a,b},{a,c},{b,c}}. Then {a, b}, {a, c} are bĝ–semi-maximal open and {b}, {c} are bĝ–semi-minimal closed.

REMARK 4.3: Every maximal bĝ–open (resp. minimal bĝ–closed) set is bĝ–semi-maximal open (resp.bĝ–semi-minimal closed) set.

THEOREM 4.4: Let A be a bĝ–semi-maximal open set of X and A $\subset$B $\subset$Cl(A), then B is a bĝ–semi-maximal open set of X.

PROOF: Since A is a bĝ–semi-maximal open set of X, then there exists a maximal bĝ–open set U of X such that U $\subset$ A $\subset$Cl(U). Then U $\subset$A$\subset$ B $\subset$Cl(A) $\subset$Cl(U). That is U $\subset$B $\subset$Cl(U). Thus B is a bĝ–semi-maximal open set of X.

THEOREM 4.5: Let A be a bĝ–semi-minimal closed set of X if and only if there exists a minimal bĝ–closed set B in X such that Int(B) $\subset$ A $\subset$ B.

PROOF: Suppose A is bĝ–semi-minimal closed set of X. By definition X – A is bĝ–semi-maximal open set of X. Then there exists a maximal bĝ–open set U such that U $\subset X - A \subset$ Cl(U). That is Int($X - U$) = X − Cl(U) $\subset$ A $\subset X - U$. Let B = $X - U$, so that B is a minimal bĝ–closed set in X such that Int(B) $\subset$ A $\subset$ B.

Conversely, Suppose that there exists a minimal bĝ–closed set B in X such that Int(B) $\subset$A $\subset$ B. Hence X−B $\subset X$−A $\subset X$−Int(B) = Cl(X−B). That is there exists a maximal bĝ–open set U = X−B such that U $\subset X$−A $\subset$ Cl(U). This implies $X - A$ is bĝ–semi-maximal open in X. Hence A is bĝ–semi minimal closed in X.

THEOREM 4.6: Let B be a bĝ–semi-minimal closed set of X if Int(B) $\subset$ A $\subset$ B, then A is also bĝ–semi-minimal closed in X.

PROOF: Let B be a b$\hat{g}$–semi-minimal closed set of *X*. Then there exists a minimal b$\hat{g}$–closed set U such that Int(U) $\subset$ B $\subset$ U and since Int(B) $\subset$ A $\subset$ B, we have Int(U) $\subset$Int(B) $\subset$A $\subset$ B $\subset$U.That is Int(U) $\subset$ A $\subset$U. Therefore A is a b$\hat{g}$–semi-minimal closed set in *X*.

THEOREM 4.7: Let Y be any open subspace of *X* and A $\subset$ Y . If A is a b$\hat{g}$–semi-maximal open set of *X*, then A is also a b$\hat{g}$–semi-maximal open set of Y .

PROOF: Since A is b$\hat{g}$–semi-maximal open set of *X*, there exists a maximal b$\hat{g}$–open set U such that U $\subset$A $\subset$ Cl(U). Hence U is subset of Y. Since U is maximal b$\hat{g}$–open in *X*, Y $\cap$ U = U is maximal b$\hat{g}$–open in Y and U = Y $\cap$U $\subset$ Y $\cap$ A $\subset$Y $\cap$Cl((U). That is U $\subset$ A $\subset$$Cl_Y$ (U). Hence A is a b$\hat{g}$–semi-maximal open set of Y.

THEOREM 4.8: Let A_i is a b$\hat{g}$–semi-maximal open set of X_i (i = 1, 2), then $A_1 \times A_2$ is a b$\hat{g}$–semi-maximal open set of $X_1 \times X_2$.

PROOF: For i = 1, 2 there exists a maximal b$\hat{g}$–open set U_i such that $U_i \subset A_i \subset Cl_{X_i}(U_i)$. Therefore $U_1 \times U_2 \subset A_1 \times A_2 \subset Cl_{X1}(U_1) \times Cl_{X2}(U_2) = Cl_{X1\times X2}(U_1 \times U_2)$. Hence $A_1 \times A_2$ is b$\hat{g}$–semi-maximal open in $X_1 \times X_2$.

5 CONCLUSION

In this paper the concepts of minimal b$\hat{g}$–closed, maximal b$\hat{g}$–closed, minimal b$\hat{g}$–open and maximal b$\hat{g}$–open sets are introduced and studied. Also we studied the concepts of b$\hat{g}$–semi-maximal open sets and b$\hat{g}$–semi-minimal closed sets. Many other new types of sets can be formed from this set which may be very helpful in applied field of science and for further research work.

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