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MODIFIED NEW ALGEBRAIC STRUCTRES ON IMAGINABLE FUZZY SOFT RING HOMOMORPHISM AND ISOMORPHISM

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ABSTRACT

Most of the scholars did their research in fuzzy soft sets. In this article by using basic concepts of fuzzy soft sets we develop and introduce new definition of algebraic structures especially in ring structure. This article also explained briefly regarding fuzzy soft rings homomorphism of fuzzy soft rings and pre – image of fuzzy soft rings. We prove selective properties of imaginable fuzzy soft homomorphism

Key Words: Fuzzy soft rings, pre – image of fuzzy soft rings, supremum property, soft groups and rough groups

1. Introduction

Researchers in Economics, Engineering, Environmental Science, Sociology, Medical Science and many other fields daily with complexities of modeling uncertain data. Classical methods are not always successful, because the uncertainties appearances in these domain may be of various types.

Most of the existing mathematical tools for formal modeling, reasoning and computing are Crisp, deterministic and precise in character. But in real life situation, the problems in Economics, Engineering, Environmental Science, Sociology, Medical Science, etc., do not always involve crisp data. Consequently, we cannot successfully by using the traditional classical methods because of various uncertainties in this problem.

2. Previous Research

While Probability Theory, Fuzzy sets Zadeh, 1965 [24], Rough Set, Pei and Miac, 2005 [20] and other mathematical tools are well – known and often useful approaches to describe uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsove, 1999, [18]

Jun, et. al 2009, [9] analyzed the applications of soft sets in $d - algebra$. Aygunoglu and Aygun , 2009 [3] introduced soft normal groups. Xu, et. al, 2010, [21] discussed the notion of vague soft set, which is an extension to the soft set and vague set, explained the basic properties of vague soft sets

In real world, the parameters and variables in fuzzy matrix may be almost uncertain data. Hence, fuzzy matrix has been discussed recently.

Zadeh L.A. (1965) [23] introduced fuzzy sets, information and control, Lewis F. C. introduced Cayley – Hamiltonian Theorem and Fader's Model.

Dengfeng and Chuntian, 2002 [6] introduced the concept of the degree similarity between intuitionistic fuzzy sets, and they presented several new similarity measure for measuring the degree of similarity between intuitionistic fuzz sets, which may be finite or continuous, and they gave corresponding between intuitionistic fuzzy sets to recognize the pattern problems.

Burillo and Bustince, 1996 [4] showed that the notion of vague sets coincides with that of intuitionistic fuzzy sets, and considered the fuzzy sets in ordered groupoids. De et. al 2001 [5] studies the Sanchez's approach for medical diagnosis and extended this concepts with the notion of intuitionistic fuzzy set theory.

Kim, et. al. 2000, [11] introduced the notion of an intuitionistic fuzzy sub quasi group of a quasi group. Kehayopulu and Tsingelis, 2002 [10] first considered the fuzzy sets in ordered groupoids. Atin and Lee, 2004, [2] discussed the concepts of fuzzy sub – algebra of BG – algebra, and studied union, intersection and other basic properties of fuzzy sub – algebra of BG – algebra.

In case of intuitionistic fuzzy sets, there were several attempts to define intuitionistic fuzzy rings by Jun. et. al. 1996 [8] and Le – mei Yan, 2008 [12] by generalized the approach unused by Liu, 1982 [12] to define fuzzy ring. Dip et. al. 1996 [5] obtained another new formulation for fuzzy rings and fuzzy ideals.

Mohammed F. Marashdeh and Abdul Razak Salleh 2011, [17] presented a new formulation of intuitionistic fuzzy rings based on the notion of intuitionistic fuzzy spaces, and a relation between intuitionistic fuzzy rings based on intuitionistic fuzzy spaces and ordinary ring is obtained in terms of induction and correspondence.

There are several theories, for example, theory fuzzy sets Zadeh, 1965 [23], theory of intuitionistic fuzzy sets Atannassov, 2000 [1] vague sets Gau and Buehrer, 1993, [7a]. Interval sets, Yang et. al. 2009 [22], and Rough sets, Pawlek, 1982, [19], which can be considered as mathematical tools for dealing with uncertainties, but all these theories have this inherent difficulties pointed out by Molodtsov, in 1999 [18] . The reason for these difficulties is possibly the inadequacy of the parameterization tool of the theories.

Furthermore Maji et. al 2003 [14] worked on soft set theory. Also he discussed in [15,16] the definition of fuzzy soft set and in [14,15,16] he presented some applications of these notion to decision making problems. The algebraic structures of set theories dealing with uncertainties have also studied by some scholars.

Consequently, Modlodtsov 1999 [18] proposed a completely new approach for modeling vagueness and uncertainty. This is so called Soft Set Theory, that s free from the difficulties affecting related problems simply does not arise. This makes the theory very convenient and easy to apply in practice soft set theory that has potential applications in many different fields, including the smoothness of functions, game theory, operation research, Riemann integration, Person integration, Probability theory and Measurement theory. Most of these applications have already been demonstrated in Molodtsov's book 1999 [18]

At present work on the soft set theory is progressing rapidly. Maji et. al [15] described the applications of soft set theory to decision making problem using rough sets, and also he gave the detailed theoretical study on soft sets. The algebraic structure set theories dealing with uncertainties has also been studied by him. The main purpose of the work is to introduce a basic version of soft group theory, which extends the notion of a group to include the algebraic structure of soft set

Our definition of soft groups is similar to the definition of rough groups, but is constructed using different methods. This chapter begins by introducing the basic concepts fuzzy soft set theory, and then a basic version of fuzzy soft group theory is discussed, and it is extended the notion of a group to the algebraic structures of fuzzy soft sets. A fuzzy soft group is a parameterized family of fuzzy sub groups. Recently many scholars have discussed the soft set. For example, the concept of soft semi ring, soft group, soft BCK/BCI algebra, soft BL – algebras and fuzzy soft groups.

In this chapter we begin by introducing the basic concepts of fuzzy soft set theory, and then we introduce the basic version of fuzzy soft ring theory, which extends the notion of the ring to the algebraic structure of fuzzy soft set. In this discussion, Molodotsov notion of soft set is studied and fuzzy soft set is considered so that the parameters are mostly fuzzy hedges or fuzzy parameters. The algebraic structures of fuzzy soft sets are discussed, and given the definition of fuzzy soft ring. Operations are defined on fuzzy soft rings and proved some new results on them. Finally image pre – image and fuzzy soft homomorphism on fuzzy soft sets are discussed with their algebraic properties.

3. Preliminaries and Basic Needed Definitions

First of all, throughout this article, R denotes the commutative ring and all fuzzy soft sets are considered over R

3.1.Definition for Complement Fuzzy Subset

A fuzzy subset μ, in a non – empty set X, it is a function $\mu: X \longrightarrow [0,1]$. The complement $\mu^c\ of\ \mu$, it is the fuzzy set in X, given by $\mu^c(x) = 1 - \mu(x), \forall\ x$

3.2.Definition for Lower – level cut of μ

A mapping $\mu: X \longrightarrow [0,1]$, it is a fuzzy set in X, and any $X \in [0,1]$, the set $L(\mu:\alpha) = \{x \in X | \mu(x) \geq \alpha\}$, it is called a lower – level cut of μ

3.3.Definition for upper Q −fuzzy subgroup

Let $Q\ and\ G$, they are a set and group respectively. A mapping $A: G \times Q \longrightarrow [0,1]$, it is a Q −fuzzy set in G. If a Q −fuzzy set A , it is a upper $Q -$ fuzzy – subgroup of G it satisfies

QFG1: $A(xy,q) \leq \max\{A(x,q), A(y,q)\}$

QFG2: $A(x^{-1},q) = A(x,q)$

QFG2: $A(e,q) = 1$

For all $x, y \in G\ and\ q \in Q$

3.4.Definition for Soft Set

A pair (f, A), it is called a soft set over X, if $f: A \longrightarrow P(X), where\ X$, it is the initial universe and A, it is a set (Consists of parameters) and $P(X)$, denotes the power set of X

3.5.Definition for Fuzzy Soft Set

A pair (f, A), it is called a fuzzy soft set over X, if $f: A \longrightarrow 1^{\times}$, and $1^{\times}$ denotes the set of all fuzzy sets on X, implies $f(a) = f_a: X \longrightarrow [0, 1]$, it is a fuzzy set in $X, \forall\, a \in A$

3.6.Definition for Soft Set over the Lattice

A pair (f, A), it is called a fuzzy soft set over the lattice L. If $f: A \longrightarrow P(L), where\ L$, it is the initial universe and A, it is a set (Consists of parameters) and $P(L)$, denotes the power set of L

3.7.Definition for Soft Set over the Ring

A pair (f, A), it is a non – null fuzzy soft set over a ring R. Then (f, A), it is said to be a soft ring over R, if and only if $f(a)$, it is a sub ring of $R, \forall\, a \in A$

3.8.Definition for Fuzzy Soft ring

A pair (f, A), it is a non – null fuzzy soft set over a ring R. Then (f, A), it is said to be a soft ring over R, if and only if $f(a) = f_a$, it is a fuzzy Soft ring of $R, \forall\, a \in A$, satisfies the following axioms

FSR1: $f_a(x + y) \geq \{f_a(x), f_a(y)\}$

FSR2: $f_a(-x) \geq f_a(x)$

FSR3: $f_a(xy) \geq T\{f_a(x), f_a(y)\}$

For all $x, y \in R\ and\ \forall\, a \in A$

3.9.Definition for α −level soft set

A pair (f, A), it is a fuzzy soft set over a lattice L. The soft set $(f, A)_\alpha = \{(f_a)_\alpha | a \in A\}$, and for each $\alpha \in [0, 1]$, it is called α −level soft set

3.10. Definition for Fuzzy Soft Function

Let $\varphi: X \longrightarrow Y\ and\ \psi: A \longrightarrow B$, they are two functions where $A\ and\ B$, they are parameter sets for the crisp sets $X\ and\ Y$ respectively. Then the pair (φ, ψ), it is called a fuzzy soft function from $X\ to\ Y$

3.11. Definition for Fuzzy Soft Set (Another Approach)

The pre – image of (g, B), under the fuzzy soft function (φ, ψ), denoted by $(\varphi, \psi)^{-1}$, it is a fuzzy soft set defined by $(\varphi, \psi)^{-1}(g, B) = \left(\varphi^{-1}(g), \psi^{-1}(B)\right)$

3.12. Definition for Fuzzy Soft Homomorphism – Fuzzy Soft Isomorphism

Let $(\varphi, \psi): X \longrightarrow Y$, it is a fuzzy soft unction. If φ, it is a homomorphism from $X \longrightarrow Y$, then (φ, ψ), it is said to be fuzzy soft homomorphism

If φ, it is an isomorphism from $X \longrightarrow Y$ and ψ, it is one – to – one mapping from $A \; onto \; B$, and then(φ, ψ), it is said to be fuzzy soft isomorphism

Note:

Let f_a, it is a fuzzy soft ring in R. Let $\theta: R \longrightarrow R^1$, it is a map and defines $f_a^{\theta}(x) = f_a(\theta x)$

By defining $f_a^{\theta}: R \longrightarrow [0, 1]$

In this study, homomorphic image of fuzzy soft ring is introduced by Acar et. al. 2010, and it is a generalization of soft rings. Some properties are studied.

Our Contributions are start below:

3.13. Theorem – 1:

Let $R \; and \; R^1$, they are two rings and $\theta: R \longrightarrow R^1$, it is a soft homomorphism. If f_b, it is a fuzzy soft ring of R, and then the pre – image $\theta^{-1}(f_b)$, it is a fuzzy soft ring of R^1

Proof:

Given that f_b, it is a fuzzy soft ring of R , and let $x, y \in R$

FSR1:

$\mu_{\theta^{-1}(f_b)}(x + y) = \mu_{f_b}\theta(x + y) = \mu_{f_b}(\theta x + \theta y)$

$\Longrightarrow \mu_{\theta^{-1}(f_b)}(x + y) \geq \left\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\right\} \geq \left\{\mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)\right\}$

FSR2:

$\mu_{\theta^{-1}(f_b)}(-x) = \mu_{f_b}\theta(-x) \geq \mu_{f_b}\theta(x) = \mu_{\theta^{-1}(f_b)}(x)$

FSR3:

$\mu_{\theta^{-1}(f_b)}(xy) = \mu_{f_b}\theta(xy) = \mu_{f_b}\left((\theta x)(\theta y)\right) \geq T\left\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\right\}$

$\Longrightarrow \mu_{\theta^{-1}(f_b)}(xy) \geq T\{\mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)\}$

$\Longrightarrow$ The pre – image $\theta^{-1}(f_b)$, it is a fuzzy soft ring of R^1

Hence complete the proof of the theorem – 1

3.14. Theorem – 2:

Let $\theta: R \longrightarrow R^1$, it is an epimorphism and f_b, it is a fuzzy soft set in R. If $\theta^{-1}(f_b)$, it is a fuzzy soft ring of R^1, and then f_b, it is a fuzzy soft ring in R

Proof:

Let $x, y \in R$, and then there exist $k, l \in R^1$ such that $\theta(x) = k\ and\ \theta(y) = l$, and it follows that

FSR1:

$\mu_{(f_b)}(x + y) = \mu_{(f_b)}\theta(k + l) = \mu_{\theta^{-1}(f_b)}(k + l) \geq \{\mu_{\theta^{-1}(f_b)}(k), \mu_{\theta^{-1}(f_b)}(l)\}$

$\Longrightarrow \mu_{(f_b)}(x + y) \geq \{\mu_{(f_b)}(x), \mu_{(f_b)}(y)\}$

FSR2:

$\mu_{(f_b)}(-x) = \mu_{(f_b)}\theta(-k) = \mu_{(f_b)}\big(\theta(-k)\big) \geq \mu_{(f_b)}\big(\theta(k)\big) = \mu_{\theta^{-1}(f_b)}\big(\theta(k)\big) \geq \mu_{(f_b)}(x)$

FSR3:

$\mu_{(f_b)}(xy) = \mu_{(f_b)}\theta(kl) = \mu_{(f_b)}\big(\theta(k)\theta(l)\big) = \mu_{\theta^{-1}(f_b)}(kl) \geq T\{\mu_{\theta^{-1}(f_b)}(k), \mu_{\theta^{-1}(f_b)}(l)\}$

$\Longrightarrow \mu_{(f_b)}(xy) \geq T\ T\{\mu_{(f_b)}(x), \mu_{(f_b)}(y)\}$

$\Longrightarrow$ f_b, it is a fuzzy soft ring in R

Hence complete the proof of theorem - 2

3.15. Theorem – 3:

If f_a, it is a fuzzy soft ring of R, and $\theta: R \longrightarrow R^1$, it is a fuzzy soft homomorphism of R, and then the fuzzy soft set $f_a^{\theta} = \left\{\left(x, f_a^{\theta}(x)\right) \middle| x \in R\right\}$, it is a soft ring of R

Proof:

Let $x, y \in R$. It gives the following implications:

FSR1;

$f_a^{\theta}(x+y) = f_a\theta(x+y) = f_a(\theta x + \theta y) \geq \{f_a(\theta x), f_a(\theta x)\} \geq \{f_a^{\theta}(x), f_a^{\theta}(y)\}$

FSR2:

$f_a^{\theta}(-x) = f_a\big(\theta(-x)\big) \geq f_a\big(\theta(x)\big) \geq f_a^{\theta}(x)$

FSR3:

$f_a^{\theta}(xy) = f_a\theta(xy) = f_a\big((\theta x)(\theta y)\big) \geq T\{f_a\big((\theta x)\big), f_a\big((\theta y)\big)\} \geq T\{f_a^{\theta}(x), f_a^{\theta}(y)\}$

$\Longrightarrow f_a^{\theta}$, it is a fuzzy soft ring in R

Hence complete the proof of theorem – 3

4. Level Cut – Set of Fuzzy Soft Set over Lattice

4.1.Theorem – 4:

Let f_a, it is a fuzzy soft set over the lattice L, and then f_a, it is a fuzzy soft ring over the lattice L, if and only if for all $a \in A$, and for any arbitrary $\alpha \in [0,1]\ with\ (f_a)_\alpha \neq 0$, and then level soft set $(f_a)_\alpha$, it is a fuzzy soft ring over the lattice L

Proof:

Assume that f_a, it is a fuzzy soft set over the lattice L, and then for each $a \in A\ and\ f_a$, it is a fuzzy sub ring over the lattice L

For any arbitrary $\alpha \in [0,1]\ with\ (f_a)_\alpha \neq 0$

Let $x, y \in (f_a)_\alpha\ and\ f_a(x) \geq \alpha\ and\ f_a(x) \geq \alpha$

FSR1:

$(f_a)_\alpha(x+y) \geq \{(f_a)_\alpha(x), (f_a)_\alpha(y)\} \geq \{f_a(x), f_a(y)\} \geq \{\alpha, \alpha\} \geq \alpha$

$\Longrightarrow (x+y) \in (f_a)_\alpha$

FSR2:

$(f_a)_\alpha(-x) \geq (f_a)_\alpha(x) \geq \alpha \Longrightarrow (-x) \in (f_a)_\alpha$

FSR3:

$(f_a)_\alpha(xy) \geq T\{(f_a)_\alpha(x), (f_a)_\alpha(y)\} \geq T\{f_a(x), f_a(y)\} \geq T\{\alpha, \alpha\} \geq \alpha$

$\Longrightarrow (xy) \in (f_a)_\alpha$

$\Longrightarrow (f_a)_\alpha$, it is a fuzzy soft ring of R

Hence complete the proof of theorem – 4

4.2.Theorem – 5:

Every imaginable fuzzy soft ring $\mu\ of\ R$, it is a fuzzy soft ring of R

Proof:

Assume that μ, it is imaginable fuzzy soft ring of R

Then we have $\mu(x+y) \geq \{\mu(x), \mu(y)\}$, and also

$\mu(-x) \geq \mu(x)$

$\mu(xy) \geq T\{\min\{\mu(x), \mu(y)\}, \min\{\mu(x), \mu(y)\}\} \geq T\{\mu(x), \mu(y)\}$

$\Longrightarrow\ T\{\mu(x), \mu(y)\} = \min\{\mu(x), \mu(y)\}$

$\Longrightarrow\ \mu(xy) \geq T\{\mu(x), \mu(y)\}, \forall\, x, y \in R$

$\Longrightarrow\ \mu$, it is a fuzzy soft ring of R. Hence complete the proof of theorem – 5

4.3.Theorem – 6:

If μ, it is a fuzzy soft ring of $R\ and\ \theta$, it is endomorphism of R, and the $\mu[\theta]$, it is a fuzzy soft ring of R

Proof:

For any $x, y \in R$, we have

FSR1:

$\mu\big([\theta](x+y)\big) = \mu\big(\theta(x+y)\big) = \mu(\theta x + \theta y) \geq \{\mu(\theta x), \mu(\theta y)\} \geq \big\{\mu\big([\theta](x)\big), \mu\big([\theta](y)\big)\big\}$

FSR2:

$\mu\big([\theta](-x)\big) = \mu\big(\theta(-x)\big) \geq \mu(\theta x) \geq \mu\big([\theta](x)\big)$

FSR3:

$\mu\big([\theta](xy)\big) = \mu\big(\theta(xy)\big) = \mu(\theta x)\mu(\theta y) \geq T\{\mu(\theta x), \mu(\theta y)\} \geq \big\{\mu\big([\theta](x)\big), \mu\big([\theta](y)\big)\big\}$

$\Longrightarrow\ \mu[\theta]$, it is a fuzzy soft ring of R

Hence complete the proof of theorem - 6

4.4.Theorem – 7:

Let T, it is continuous $t-$norms and f , it is a soft homomorphism on R. If μ, it is a fuzzy soft ring of R , and then μ^f, it is a fuzzy soft ring of $f(R)$

Proof:

Let $A_1 = f^{-1}(y_1)\ , A_2 = f^{-1}(y_2)\ and\ A_{12} = f^{-1}(y_1 + y_2)\ where\ \ y_1, y_2 \in f(R)$

Consider the set $A_1 + A_2 = \{x \in R | x = a_1 + a_2, for\ some\ a_1 \in A_2, for\ some\ a_2 \in A_2\}$

If $x \in A_1 + A_2, and\ then\ x = x_1 + x_2, for\ some\ x_1 \in A_1, for\ some\ x_2 \in A_2$

So that it follows that

$f(x) = f(x_1 + x_2) = f(x_1) + f(x_2) = y_1 + y_2 \; and \; x \in f^{-1}(y_1 + y_2) = A_{12}$

$\Longrightarrow A_1 + A_2 \in A_{12}$

FSR1:

$\mu^f(y_1 + y_2) = \sup\{\mu(x)|x \in f^{-1}(y_1 + y_2)\} = \sup\{\mu(x)|x \in A_{12}\}$

$\Longrightarrow \mu^f(y_1 + y_2) \geq \sup\{\mu(x)|x \in A_1 + A_2\} \geq \sup\{\mu(x_1 + x_2)|x_1 \in A_1, x_2 \in A_2\}$

Since T, it is continuous t −norms

⟹It sees that for every $\varepsilon > 0$, there exists $\delta > 0$ such that

$\sup\{\mu(x_1)|x_1 \in A_1\} \leq \delta \; and \; \sup\{\mu(x_2)|x_2 \in A_2\} \leq \delta$, and also

$T\{\sup\{\mu(x_1)|x_1 \in A_1\}, \sup\{\mu(x_2)|x_2 \in A_2\}\} + T\{\mu(a_1), \mu(a_2)\} \leq \varepsilon$

Consequently, we can find that

$\mu^f(y_1 + y_2) \geq T\{\sup \mu(x_1), \sup \mu(x_2)| \; x_1 \in A_1, x_2 \in A_2 \}$

$\Longrightarrow \mu^f(y_1 + y_2) \geq T\{\sup \mu(x_1), |x_1 \in A_1, \sup \mu(x_2)| \; x_2 \in A_2\} \geq T\{\mu^f(y_1), \mu^f(y_2)\}$

Like this we will get the result $\mu^f(x + y) \geq \{\mu^f(x), \mu^f(y)\}$

Similarly we can easily prove that

$\mu^f(-x) \geq \mu^f(x)$

$\mu^f(xy) \geq T\{\mu^f(x), \mu^f(y)\}$

$\Longrightarrow \mu^f$, it is a fuzzy soft ring over $f(R)$. Hence complete the proof of theorem - 7

4.5.Theorem – 8:

Onto homomorphic image of a fuzzy soft ring with supremum property, it is again a fuzzy soft ring

Proof:

Let $f: R \longrightarrow R^1$, it is an onto homomorphism of rings and also

μ, it has the supremum property of fuzzy soft ring of R

Let $x^1, y^1 \in R^1 \; and \; x_0 \in f^{-1}(x^1), y_0 \in f^{-1}(y^1)$, such that

$\mu(x_0) = \sup_{h \in f^{-1}(x^1)} \mu(h) \; and \; \mu(y_0) = \sup_{k \in f^{-1}(y^1)} \mu(k)$, respectively and then it can be deduced that

FSR1:

$$\mu^f(x^1+y^1)=\sup_{z\in f^{-1}(x^1+y^1)}\mu(z)\geq\{\sup\mu(h),\sup\mu(k), for\ h\in f^{-1}(x^1), k\in f^{-1}(y^1)\}$$

$$\Rightarrow \mu(x^1+y^1)\geq\{\mu^f(x^1),\mu^f(y^1)\}$$

FSR2:

$$\mu^f(-x^1)=\sup_{h\in f^{-1}(-x^1)}\mu(h)\geq\mu(x_0)\geq\sup_{h\in f^{-1}(x^1)}\mu(h)\geq\mu^f(x^1)$$

FSR3:∈

$$\mu^f(x^1y^1)=\sup_{z\in f^{-1}(x^1y^1)}\mu(z)\geq T\{\mu(x_0),\mu(y_0)\}\geq T\left\{\sup_{h\in f^{-1}(x^1)}\mu(h), \sup_{k\in f^{-1}(x^1)}\mu(k)\right\}$$

$$\Rightarrow \mu^f(x^1y^1)\geq T\{\mu^f(x^1),\mu^f(y^1)\}$$

$\Rightarrow \mu^f$, it is a fuzzy soft ring of R^1

Hence complete the proof of theorem - 8

4.6.Theorem – 9:

Let f_a, it is a fuzzy soft ring over R, and (φ,ψ), it is a fuzzy soft homomorphism from $R\ to\ R_1$. Then $(\varphi\psi)f_a$ it is a fuzzy soft ring over R_1

Proof:

Let $k\in(\psi)f_a\ and\ y_1,y_2\in y$, and if $\varphi^{-1}(y_1)=\varphi=\varphi^{-1}(y_2)$

Let us assume that, there exists $x_1,x_2\in X\ such\ that\ \varphi(x_1)=y_1\ and\ \varphi(x_2)=y_2$

And then we have

$$\varphi(f_a)(y_1+y_2)=\left\{\underset{\varphi(t)=y_1+y_2}{V}\quad\underset{\psi(a)=kf_a(x_1+x_2)}{V}\right\}$$

$$\Rightarrow\ \varphi(f_a)(y_1+y_2)\geq\underset{\psi(a)=k}{V}f_a(x_1+x_2)$$

$$\Rightarrow\ \varphi(f_a)(y_1+y_2)\geq\underset{\psi(a)=k}{V}\{f_a(x_1),f_a(x_2)\}$$

$$\Rightarrow\ \varphi(f_a)(y_1+y_2)\geq V\{f_a(x_1)+f_a(x_2), for\ \psi(a)=k\}$$

This inequality satisfies for each $x_1,x_2\in X\ such\ that\ \varphi(x_1)=y_1\ and\ \varphi(x_2)=y_2$

Thus it becomes that

FSR1:

$$\varphi(f_a)(y_1+y_2)\geq\left\{\begin{pmatrix}V & Vf_{a_1}(t_1)\\ \varphi(t_1)=y_1 & \psi(a)=k\end{pmatrix},\begin{pmatrix}V & Vf_{a_2}(t_2)\\ \varphi(t_2)=y_2 & \psi(a)=k\end{pmatrix}\right\}$$

$\Rightarrow \varphi(f_a)(y_1 + y_2) \geq \{\varphi(f_a)_k(y_1), \quad \varphi(f_a)_k(y_2)\}$

Similarly we can prove that

$\varphi(f_a)(-y) \geq \varphi(f_a)_k(y)$, and also

$\varphi(f_a)(y_1 y_2) \geq T\{\varphi(f_a)_k(y_1), \quad \varphi(f_a)_k(y_2)\}$

$\Rightarrow (\varphi\ \psi)f_a$, it is a fuzzy soft ring over R_1

Hence completes the proof of theorem – 9

4.7. Theorem – 10:

Let g_a, it is a fuzzy soft ring over R_1, and $(\varphi\psi)$, it is a fuzzy soft homomorphism from $R\ toR_1$. Then $(\varphi\psi)^{-1}g_a$ it is a fuzzy soft ring over R

Proof:

Let $a \in (\psi)^{-1}(a)\ and\ x_1, x_2 \in X$, and then we have

$(\varphi)^{-1}(g_a)(x_1 + x_2) = g_{\psi(a)}\big(\varphi(x_1 + x_2)\big) = g_{\psi(a)}\big(\varphi(x_1) + \varphi(x_2)\big)$

$\Rightarrow (\varphi)^{-1}(g_a)(x_1 + x_2) \geq \big\{g_{\psi(a)}\big(\varphi(x_1)\big), g_{\psi(a)}\big(\varphi(x_2)\big)\big\}$

$\Rightarrow (\varphi)^{-1}(g_a)(x_1 + x_2) \geq \{(\varphi)^{-1}(g_a)(x_1), (\varphi)^{-1}(g_a)(x_2)\}$

Similarly, we can prove that

$\Rightarrow (\varphi)^{-1}(g_a)(-x) \geq (\varphi)^{-1}(g_a)(x)$, and also

$\Rightarrow (\varphi)^{-1}(g_a)(x_1.x_2) \geq T\{(\varphi)^{-1}(g_a)(x_1), (\varphi)^{-1}(g_a)(x_2)\}$

$\Rightarrow (\varphi\psi)^{-1}g_a$, it is a fuzzy soft ring over R

Hence complete the proof of theorem – 10

5. Conclusion

By using basic concepts of fuzzy soft sets, we extended the theoretical part of fuzzy soft sets briefly. By using those concepts we introduced new algebraic structures in fuzzy soft rings. We also highlighted on fuzzy soft rings homomorphism of fuzzy soft rings and pre – image of fuzzy soft rings.

We also by using the concept of fuzzy soft ring homomorphism and fuzzy soft ring isomorphism we prove related new theorem, by the concept of supremum property we develop some theorems and explain the properties of continuous homomorphic fuzzy soft rings.

For any other interested scholars can study the properties of fuzzy soft sets in other algebraic structures such as near rings, groups, ideals, fields and G – modules.

6. References:

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