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Website- www.aarf.asia, **Email** : editor@aarf.asia , editoraarf@gmail.com

DIVISION OF MATRICES WITH SAME AND DIFFERENT ORDERS

Kwenge Erasmus
St. Mary's College of Education, P.O. Box 420210, Mbala – Zambia.

ABSTRACT

Many teachers of mathematics are unable to perform the division of matrices with different orders because there is no formula or method. For the 2 x 2 matrices, there is only one method that of using inverse method which can be used to perform the division and yet we encourage students or learners to think of using different methods of finding the solutions to the problem. The problem of division of matrices has made some teachers and some mathematicians to be saying that division of matrices with different orders is ***undefined****. This situation has contributed to the curriculum of mathematics especially on division of matrices to be narrow and does not support development of critical thinking because teachers are unable to facilitate the process of division of matrices with different and same orders. The need to find the solution to this problem made the researcher to undertake the study on the relationship between matrices of the same order and different orders in terms of division and multiplication. The researcher derived the formula for division of the matrices such as 2 x 2 ÷ 2 x 2 , 2 x 1 ÷ 2 x 2 , 3 x1 ÷ 3 x 3 etc, considering that matrices are not commutative under multiplication and there exists the inverse of the divisor.*

Keywords: division of matrices, same order, different orders, undefined, multiplication.

Introduction

Matrices is one of the topics being taught at secondary school level. Many teachers have challenges on division of matrices because much has not been done to find methods on how matrices can be divided especially if they have different orders. It is easy to add, subtract and multiply but when it comes to division teachers and learners do face some challenges because there is only one common method of finding the quotient. A number of teachers will say that division for matrices is undefined because some teachers are not aware of the inverse method of dividing the matrices. The teacher can only be effective if he has the subject or content knowledge. It is difficult for someone to teach what he does know. It is for this reason that the researcher decided to undertake the study on division of matrices so that another method can be found so that both teachers and learners become knowledgeable on division of matrices with same and different orders. If a teacher does not have an adequate understanding of the subject-matter, he/she will not be guide learners in performing division of matrices. In order to understand and appreciate mathematics since it is full of abstract; the teacher must have sound content subject matter knowledge and pedagogical knowledge (Shulman, 1987) so that he can be resourceful, creative and be able to make models and teaching aids that can facilitate the teaching and learning of mathematics with joy. In mathematics, teachers with sound content knowledge are likely to present problems to the learners in a context that they are familiar with and make a link to the problems to what they already know while those with less mathematics knowledge tend to put emphasis on calculations based on memorizations of procedures rather than on the underlying concepts (Kwenge, 2015). According to Zuya E. H (2014), mathematics teachers' ability to probe students' thinking processes is important to effective teaching of mathematics. Teachers can only do this if they have been equipped with pedagogical content knowledge because the quality of the questions teacher asks, plays an important role in identifying learners' difficulties.

The Purpose

The purpose of the study was to find the alternative method that can enable the teacher and the learner to be able to divide matrices with same and different orders, and become more knowledgeable about matrices.

Literature Review

According to (http://www.mathcaptain.com/algebra/dividing-matrices.html), there is no significant method for matrix division. We cannot divide two matrices. But, we can multiply one matrix by the inverse of the other in order to divide the matrices. To solve the system of linear equations, we can use division property of matrices.

A matrix may be divided by a scalar and also by a matrix. Here, we will learn how to divide matrices with the help of this example extracted from the above stated web site.

If A = $\begin{pmatrix} 6 & 2 \\ 3 & 2 \end{pmatrix}$ and B =$\begin{pmatrix} 6 & -4 \\ 2 & 1 \end{pmatrix}$, find A ÷ B.

Solution.

(i) Let B/A = BA^{-1}

(ii) Calculate the inverse of A, we know that $A^{-1} = \frac{AdjA}{IAI}$

(iii) Cofactor matrix of A = = $\begin{pmatrix} 2 & -3 \\ -2 & 6 \end{pmatrix}$= C

(iv) C^T = = $\begin{pmatrix} 2 & -2 \\ -3 & 6 \end{pmatrix}$

(v) Determinant of A = 12 – 6 = 6

(vi) $\frac{A}{B}$ = D = BA^{-1}

$$= \begin{pmatrix} 6 & -4 \\ 2 & 1 \end{pmatrix} \text{ x } \frac{1}{6}\begin{pmatrix} 2 & -2 \\ -3 & 6 \end{pmatrix}$$

$$= \frac{1}{6}\begin{pmatrix} 24 & -36 \\ 1 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 4 & -6 \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

$$\therefore A \div B = \begin{pmatrix} 4 & -6 \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

Example 2

Given that A = $\begin{pmatrix} 2 & 4 \\ 6 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 3 \\ 2 & 8 \end{pmatrix}$, find A ÷ B

Solution

A ÷ B = $\frac{A}{B}$ = AB^{-1}

Determinant of B = (1 x 8) – (2 x 3)

$= 8 - 6$

$= 2$

$B^{-1} = \frac{1}{2}\begin{pmatrix} 8 & -3 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4 & \frac{-3}{2} \\ -1 & \frac{1}{2} \end{pmatrix}$

$A \div B = B^{-1}A$

$= \begin{pmatrix} 4 & \frac{-3}{2} \\ -1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 6 & 5 \end{pmatrix}$

$= \begin{pmatrix} 8 - 9 & 16 - 7.5 \\ -2 + 3 & -4 + 2.5 \end{pmatrix}$

$B^{-1}A = \begin{pmatrix} -1 & \frac{17}{2} \\ 1 & \frac{-3}{2} \end{pmatrix}$

Show that $A = B \times B^{-1}A$

$= \begin{pmatrix} 1 & 3 \\ 2 & 8 \end{pmatrix}\begin{pmatrix} -1 & \frac{17}{2} \\ 1 & \frac{-3}{2} \end{pmatrix}$

$= \begin{pmatrix} -1 + 3 & \frac{17-9}{2} \\ 19 - 13 & \frac{34-24}{2} \end{pmatrix}$

$= \begin{pmatrix} 2 & \frac{8}{2} \\ 6 & \frac{10}{2} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 5 \end{pmatrix}$

Theoretical Framework

The study was based on constructivism theory of leaning which states thatlearning is an active process in which learners construct new ideas or concepts based upon their current/past knowledge. According to social constructivist scholars learning is viewed as an active process where learners learn to discover principles, concepts and facts for themselves, hence the importance of encouraging guesswork and intuitive thinking in learners (Brown T., McNamara O., Olwen H., & Jones L., 1989).The learner selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so. Cognitive structure (i.e., schema, mental models) provides meaning and organization to experiences and allows the individual to go beyond the information given. As far as instruction is concerned, the instructor should try and encourage learners to discover principles by themselves. The instructor and learner should engage in an active dialogue (Socratic learning/question and answer). The task of the instructor is to translate information to be learned into a format appropriate to the learner's current state of understanding and this can only happen if the instructor is more knowledgeable than the learner. Constructivist approach has a primary goal: helping students learn "How to learn" which fosters critical thinking and learners are more motivated and independent. A constructivist view of knowledge implies that knowledge is continuously created and reconstructed so that there can be no template for constructivist teaching (Peterson & Knapp, 1993).

Method

Procedure of dividing matrices using Kwenge's method.

Example 1

Given that A = $\begin{pmatrix} 2 & 4 \\ 6 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 3 \\ 2 & 8 \end{pmatrix}$, find A ÷ B using kwenge's method.

Solution

Interchange the elements in the leading diagonal of matrix A and change the signs of elements in the secondary diagonal of the same matrix to get the matrix $\begin{pmatrix} 5 & -4 \\ -6 & 2 \end{pmatrix}$ and multiply this matrix by B.

$$\begin{pmatrix} 5 & -4 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 5+(-8) & 15+(-32) \\ -6+4 & -18+16 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -17 \\ -2 & -2 \end{pmatrix}$$, divide this matrix by the determinant of B

$$= \begin{pmatrix} -3 & -17 \\ -2 & -2 \end{pmatrix} \div 2$$

$$= \frac{1}{2}\begin{pmatrix} -3 & -17 \\ -2 & -2 \end{pmatrix}$$

$= \begin{pmatrix} \frac{-3}{2} & \frac{-17}{2} \\ -1 & -1 \end{pmatrix}$, interchange the elements in the leading diagonal and change the signs for the elements in the secondary diagonal to get matrix C = $\begin{pmatrix} -1 & \frac{17}{2} \\ 1 & \frac{-3}{2} \end{pmatrix}$

$\therefore \quad A \div B = \begin{pmatrix} -1 & \frac{17}{2} \\ 1 & \frac{-3}{2} \end{pmatrix}$

Show that BC = A

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 8 \end{pmatrix}\begin{pmatrix} -1 & \frac{17}{2} \\ 1 & \frac{-3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -1+3 & \frac{17-9}{2} \\ -2+8 & \frac{34-24}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & \frac{8}{2} \\ 6 & \frac{10}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 6 & 5 \end{pmatrix}$$

Remember that BC ≠ CB becauseunder multiplication matrices are not commutative.

Example 2

Given that A = $\begin{pmatrix} 17 & -8 \\ 7 & -14 \end{pmatrix}$ B =$\begin{pmatrix} -1 & 3 \\ -5 & 2 \end{pmatrix}$, find A ÷ B using Kwenge’s method.

Solution

Interchange the elements in the leading diagonal of matrix A and change the signs of elements in the secondary diagonal of the same matrix to get the matrix $\begin{pmatrix} -14 & 8 \\ -7 & 17 \end{pmatrix}$ and multiply this matrix by B.

$\begin{pmatrix} -14 & 8 \\ -7 & 17 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 14-40 & -42+16 \\ 7-85 & -21+34 \end{pmatrix}$

$= \begin{pmatrix} -26 & -26 \\ -78 & 13 \end{pmatrix}$, divide this matrix by the determinant of B

$= \begin{pmatrix} -26 & -26 \\ -78 & 13 \end{pmatrix} \div 13$

$= \frac{1}{13}\begin{pmatrix} -26 & -26 \\ -78 & -13 \end{pmatrix}$

$= \begin{pmatrix} -2 & -2 \\ -6 & 1 \end{pmatrix}$, interchange the elements in the leading diagonal and change the signs in the secondary diagonal to get matrix C = $\begin{pmatrix} 1 & 2 \\ 6 & -2 \end{pmatrix}$

$\therefore$ A ÷ B = $\begin{pmatrix} 1 & 2 \\ 6 & -2 \end{pmatrix}$

Show that A = BC

$= \begin{pmatrix} -1 & 3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 6 & -2 \end{pmatrix}$

$= \begin{pmatrix} -1+18 & -2-6 \\ -5+12 & -10-4 \end{pmatrix}$

A = $\begin{pmatrix} 17 & -8 \\ 7 & -14 \end{pmatrix}$

To show that A is equal to the given matrix, multiply B by $\frac{A}{B}$ given that A ÷ B.

Example 3

Given that P = $\begin{pmatrix} -1 & 2 \\ -3 & -5 \end{pmatrix}$ and Q = $\begin{pmatrix} 6 & -4 \\ 2 & 9 \end{pmatrix}$, find P ÷ Q using kwenge's method.

Solution

Interchange the elements in the leading diagonal of matrix P and change the signs of elements in the secondary diagonal of the same matrix to get the matrix A = $\begin{pmatrix} -5 & -2 \\ 3 & -1 \end{pmatrix}$ and multiply this matrix by Q

$\begin{pmatrix} -5 & -2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 6 & -4 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} -30-4 & 20-18 \\ 18-2 & -12-9 \end{pmatrix}$

$=\begin{pmatrix} -34 & 2 \\ 16 & -21 \end{pmatrix}$, divide this matrix by the determinant of Q

$$= \begin{pmatrix} -33 & 2 \\ 16 & -21 \end{pmatrix} \div 62$$

$$= \frac{1}{62}\begin{pmatrix} -34 & 2 \\ 16 & -21 \end{pmatrix}$$

$=\begin{pmatrix} \frac{-34}{62} & \frac{2}{62} \\ \frac{16}{62} & \frac{-21}{62} \end{pmatrix}$, interchange the elements in the leading diagonal and change the signs for the elements in the secondary diagonal to get A = $\begin{pmatrix} \frac{-21}{62} & \frac{-2}{62} \\ \frac{-16}{62} & \frac{-34}{62} \end{pmatrix}$

$\therefore$ P ÷ Q = $\begin{pmatrix} \frac{-21}{62} & \frac{-2}{62} \\ \frac{-16}{62} & \frac{-34}{62} \end{pmatrix}$

Show that P = QA

$$= \begin{pmatrix} 6 & -4 \\ 2 & 9 \end{pmatrix}\begin{pmatrix} \frac{-21}{62} & \frac{-2}{62} \\ \frac{-16}{62} & \frac{-34}{62} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-126+64}{62} & \frac{-12+136}{62} \\ \frac{-42-144}{62} & \frac{-4-306}{62} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-62}{62} & \frac{124}{62} \\ \frac{-186}{62} & \frac{-310}{62} \end{pmatrix}$$

P = $\begin{pmatrix} -1 & 2 \\ -3 & -5 \end{pmatrix}$

Alternative kwenge's method

Example 1

Given that A = $\begin{pmatrix} 10 & 5 \\ 6 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 1 \\ 4 & 0 \end{pmatrix}$, find A ÷ B, using kwenge's alternative method.

Solution

(i) Interchange the elements in the leading diagonal of matrix A to get matrix C = $\begin{pmatrix} 3 & 5 \\ 6 & 10 \end{pmatrix}$

(ii) Change the signs of elements in the secondary diagonal of matrix B to get D = $\begin{pmatrix} 2 & -1 \\ -4 & 0 \end{pmatrix}$

(iii) Multiply C by D to get E = $\begin{pmatrix} 3 & 5 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -4 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 6-20 & -3+0 \\ 12-40 & -6+0 \end{pmatrix}$$

$= \begin{pmatrix} -14 & -3 \\ -28 & -6 \end{pmatrix}$, divide this matrix by the determinant of B

$$= \begin{pmatrix} -14 & -3 \\ -28 & -6 \end{pmatrix} \div -4$$

$$= \frac{1}{-4}\begin{pmatrix} -14 & -3 \\ -28 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{14}{4} & \frac{3}{4} \\ \frac{28}{4} & \frac{6}{4} \end{pmatrix},$$

$$=\begin{pmatrix} \frac{7}{2} & \frac{3}{4} \\ 7 & \frac{3}{2} \end{pmatrix}$$, interchange the elements in the leading diagonal.

$$= \begin{pmatrix} \frac{3}{2} & \frac{3}{4} \\ 7 & \frac{7}{2} \end{pmatrix}$$

$$\therefore A \div B = \begin{pmatrix} \frac{3}{2} & \frac{3}{4} \\ 7 & \frac{7}{2} \end{pmatrix}$$

(iv) Show that A = BE

$$= \begin{pmatrix} 2 & 1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} & \frac{3}{4} \\ 7 & \frac{7}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 3+7 & \frac{6}{4}+\frac{7}{2} \\ 6+0 & 3+0 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & \frac{6+14}{4} \\ 6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & \frac{20}{4} \\ 6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 5 \\ 6 & 3 \end{pmatrix}$$

Example 2

Given that P = $\begin{pmatrix} 110 & 20 \\ 40 & -70 \end{pmatrix}$ and Q =$\begin{pmatrix} 4 & 2 \\ -7 & -6 \end{pmatrix}$, find P ÷ Q using kwenge's method.

Solution

(i) Interchange elements in the leading diagonal of P to get matrix A =$\begin{pmatrix} -70 & 20 \\ 40 & 110 \end{pmatrix}$

(ii) Change the signs of elements in the secondary diagonal of Q to get matrix B = $\begin{pmatrix} 4 & -2 \\ 7 & -6 \end{pmatrix}$

(iii) Find AC to get matrix D

D = AC

$= \begin{pmatrix} -70 & 20 \\ 40 & 110 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 7 & -6 \end{pmatrix}$

$= \begin{pmatrix} -280 + 140 & 140 - 120 \\ 160 + 770 & -80 - 660 \end{pmatrix}$

$=\begin{pmatrix} -140 & 20 \\ 930 & -740 \end{pmatrix}$, divide this matrix by the determinant of Q

$= \begin{pmatrix} -140 & 20 \\ 930 & -740 \end{pmatrix}$ ÷ -10

$= \frac{1}{-10}\begin{pmatrix} -140 & 20 \\ 930 & -740 \end{pmatrix}$

$= \begin{pmatrix} 14 & -2 \\ -93 & 74 \end{pmatrix}$, interchange the elements in the leading diagonal of this matrix.

$= \begin{pmatrix} 74 & -2 \\ -93 & 14 \end{pmatrix}$

∴ P ÷ Q = $\begin{pmatrix} 74 & -2 \\ -93 & 14 \end{pmatrix}$

Show that P = QD

$$P = \begin{pmatrix} 4 & 2 \\ -7 & -6 \end{pmatrix}\begin{pmatrix} 74 & -2 \\ -93 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} 296 - 186 & -8 + 28 \\ -518 + 558 & 14 - 84 \end{pmatrix}$$

$$= \begin{pmatrix} 110 & 20 \\ 40 & -70 \end{pmatrix}$$

Division of matrices using Kwenge's method when the matrices have different orders

Example 1

Given that P = $\begin{pmatrix} 21 \\ 37 \end{pmatrix}$ and Q =$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, find P ÷ Q.

$$P \div Q = \begin{pmatrix} 21 \\ 37 \end{pmatrix} \div \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -3 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 21 \\ 37 \end{pmatrix}$$

$$= \begin{pmatrix} 105 - 111 \\ -84 + 74 \end{pmatrix}$$

$= \begin{pmatrix} -6 \\ -10 \end{pmatrix}$, divide this matrix by the determinant of Q

$$= \begin{pmatrix} -6 \\ -10 \end{pmatrix} \div -2$$

$$= \begin{pmatrix} -6 \\ -10 \end{pmatrix} \text{ x } \frac{1}{-2}$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Kwenge's formula on division of 2 x 1 by 2 x 2.

If A =$\begin{pmatrix} a \\ b \end{pmatrix}$, and B = $\begin{pmatrix} c & d \\ e & f \end{pmatrix}$

Then A ÷ B = $\begin{pmatrix} f & -d \\ -e & c \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$ ÷ (cf – ed)

$= \begin{pmatrix} af - bd \\ -ae + bc \end{pmatrix} \div$ (cf – ed)

$= \begin{pmatrix} af - bd \\ -ae + bc \end{pmatrix}$ x $\frac{1}{cf - ed}$

$= \begin{pmatrix} \frac{af - bd}{cf - ed} \\ \frac{-ae + bc}{cf - ed} \end{pmatrix}$

Given that A = $\begin{pmatrix} 13 \\ 20 \end{pmatrix}$ and B = $\begin{pmatrix} 3 & 5 \\ 4 & 8 \end{pmatrix}$, find A ÷ B using Kwenge's formula.

A ÷ B = $\begin{pmatrix} \frac{af - bd}{cf - ed} \\ \frac{-ae + bc}{cf - ed} \end{pmatrix}$

$= \begin{pmatrix} \frac{104 - 100}{24 - 20} \\ \frac{-52 + 60}{24 - 20} \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{4} \\ \frac{8}{4} \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

To check whether the answer is correct, multiply B by (A ÷ B) **not** (A ÷ B) by B because under multiplication, matrices are not commutative.

Example 2

Division of 1 x 2 matrix by 2 x 2 matrix

(i) Given that A = (4 10) and B = $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$, find A ÷ B.

A ÷ B = (4 10) $\begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix}$

= (16 – 20 -12 + 10) divide by the determinant of B.

= (-4 -2) ÷ -2

= (-4 -2) x $\frac{1}{-2}$

= (2 1)

Example 2

Given that $A = \begin{pmatrix} 14 \\ 10 \\ 11 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}$, find $A \div B$.

Step 1. Write down the co - factors of B to get C.

$$C = \begin{pmatrix} -1 & 1 & 5 \\ -7 & -5 & -1 \\ -4 & -8 & -4 \end{pmatrix}$$

Step 2. Write down the transpose of C to get D

$$D = \begin{pmatrix} -1 & -7 & -4 \\ 1 & -5 & -8 \\ 5 & -1 & -4 \end{pmatrix}$$

Step 3. Apply $\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$ to D to get matrix E

$$E = \begin{pmatrix} -1 & 7 & -4 \\ -1 & -5 & 8 \\ 5 & 1 & -4 \end{pmatrix}$$

Step 4. Find the determinant of B. Determinant of B = 12

Step 5. Divide 1 by the determinant to F

$$F = \frac{1}{12}$$

$$A \div B = EAF$$

$$= \begin{pmatrix} -1 & 7 & -4 \\ -1 & -5 & 8 \\ 5 & 1 & -4 \end{pmatrix} \begin{pmatrix} 14 \\ 10 \\ 11 \end{pmatrix} x \frac{1}{12}$$

$$= \begin{pmatrix} -14 + 70 - 44 \\ -14 - 50 + 88 \\ 70 + 10 - 44 \end{pmatrix} x \frac{1}{12} = \begin{pmatrix} 12 \\ 24 \\ 36 \end{pmatrix} x \frac{1}{12} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Example 3

Given that $P = \begin{pmatrix} 29 \\ 29 \\ 20 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 5 & 3 \\ 6 & -1 & 4 \\ 3 & 2 & 1 \end{pmatrix}$, find $P \div Q$

Step 1. Write down the co-factors of Q to get A.

$$A = \begin{pmatrix} -9 & -6 & 15 \\ -1 & -7 & -11 \\ 23 & -10 & -32 \end{pmatrix}$$

Step 2. Write down the transpose of A to get B

$$B = \begin{pmatrix} -9 & -1 & 23 \\ -6 & -7 & -10 \\ 15 & -11 & -32 \end{pmatrix}$$

Step 3. Apply $\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$ to B to get C

$$C = \begin{pmatrix} -9 & 1 & 23 \\ 6 & -7 & 10 \\ 15 & 11 & -32 \end{pmatrix}$$

Step 4. Find the determinant of Q. Determinant of $Q = 2(-9) - 5(-6) + 3(15)$
$= -18 + 30 + 45$
$= 57$

Step 5. Divide 1 by the determinant to get D

$$D = \frac{1}{57}$$

$$P \div Q = C\,P\,D$$

$$= \begin{pmatrix} -9 & 1 & 23 \\ 6 & -7 & 10 \\ 15 & 11 & -32 \end{pmatrix} \begin{pmatrix} 29 \\ 29 \\ 20 \end{pmatrix} \times \frac{1}{57} = \begin{pmatrix} -261 + 29 + 460 \\ 174 - 203 + 200 \\ 435 + 319 - 640 \end{pmatrix} \times \frac{1}{57} = \begin{pmatrix} 228 \\ 171 \\ 114 \end{pmatrix} \times \frac{1}{57} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$$

Using this method one can find the answer to matrices with different orders being divided, provided multiplication can be performed and both matrices have inverses.

Discussion.

The knowledge about the method of dividing matrices of same and different orders can help the teachers to be able to engage learners into the process of mathematical thinking (Kwenge, 2014). We need to know that even when we are multiplying we do involve addition and subtraction. We cannot find the product of 2 matrices without involving the addition and subtraction. The same principles applies when it comes to division. You cannot divide matrices without involving multiplication because multiplication and division are interrelated and division is the reverse of multiplication. The number 9 is unique and has some powers and characteristics which other numbers do not have. To understand Kwenge's method, it needs reasoning and thinking that can help the learners develop analytical and critical thinking and appreciate the role that division and multiplication plays in dealing with problems involving matrices (Kwenge, 2017).

There is need for the learners to see that there is a special relationship between multiplication and division of matrices. This relationship can help us to divide matrices because if one can multiply the matrices then he can also divide matrices either directly or use other alternatives provided that the determinant of one of the matrices is not equal to zero. It is this special relationship that the researcher used to come up with the method known as **Kwenge's method for division of matrices.** This method can be used to divide matrices with same or different orders provided that the determinant of one of the matrices is not equal to **zero** and also bearing in mind that matrices are not commutative under multiplication.

Conclusion

It is important for the learners to know that certain problems cannot be found by applying direct methods. What is important is to find the solution to the problem and have the solution justified. We cannot continue to say that matrices under division are undefined when we can find the quotient. Using Kwenge's method of division for matrices can help both teachers and learners to be more knowledgeable on matrices, and can help them to develop critical thinking.

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