International Research Journal of Mathematics, Engineering and IT
ISSN: (2349-0322)
Impact Factor- 5.489, Volume 5, Issue 4, April 2018
Website- www.aarf.asia, **Email** : editor@aarf.asia , editoraarf@gmail.com

TREND ANALYSIS AND FORECASTING OF PERFORMANCE OF STUDENTS IN MATHEMATICS IN CERTIFICATE SECONDARY EDUCATION EXAMINATION IN ZANZIBAR: ARIMA MODELLING APPROACH

Asha Moh'dMussa and K.K.Saxena

Department of Statistics, University of Dodoma, Tanzania

ABSTRACT

It is widely accepted criteria that the mathematics subject in secondary school curriculum is paramount for scientific and human development as it serves both as a tool for academic progress in a chosen career and as a tool for preparing the individual for useful living. It is with this background this study had been undertaken to analyze the trends of students' performance in mathematics subject in secondary education level in Zanzibar. This study analyzed the pattern of students' performance in 123 secondary schools in Zanzibar,who sat for final examination from 1986-2016. The yearly data of students' results were collected from Ministry of Education and Vocational Training, Zanzibar (MoEVTZ 2005,2006) and the yearly time series examination result data for CSEE in Zanzibar from 1986-2016 were compiled for Male and Female students . These data were further analyzed for checking and testing of normality. Normality assumption was validated for male students and female students individually byusing Kolmogorov-Smirnov test and Shapiro-Wilk test. Further, in this study, various Autoregressive Integrated Moving Average (ARIMA) models have been fitted to see the pattern of performance of male students and female students . The best fitted model has been selected on the basis of Akaike Information criterion and Bayesian Information criterion. ACFs and PACFs have also been calculated. Tthe forecast of performance in Mathematics subject for next six years (2017 to 2022) have also been presented for both categories of students.Forecasting analysis has been validated by using mean *absolute percentage error for the accuracy of forecasting the performance of student's result.The results of the present study will provide useful statistical information regarding the performance* of students in mathematics which could be used as a basis for planning towards the realization of future of Tanzania with respect to the development of mathematics subject.

KEYWORDS: Mathematics, Trends, Students' performance, Kolmogorov-Smirnov test, Shapiro-Wilk test, ARIMA Model,Akaike Information criterion and Bayesian Information criterion.

INTRODUCTION

Throughout the world, the mathematics subject in secondary schools is not only considered a very important subject but also considered as a foundation subject for success in further academic endeavor and manpower development. Different researchers have acknowledged the place of mathematics subject in scientific and technological developments of the nation. Jegede and Brown, (1980) stressed that the main effect of education on national development emanate from the areas of science and mathematics. Setidisho(1996) also maintained that mathematics is a fundamental science that is necessary for understanding of most other fields in education.Iji (2007), maintained that any country that aspires for national growth in science and technology must not neglect mathematics subject.

In Tanzania's education curriculum, mathematics is a core subject that every student should study at both primary and ordinary secondary education (ETP, 1995). Because of the importance of mathematics subject, the Tanzanian government is committed to ensure the provision of high quality mathematics education. Since mathematics is a compulsory subject, student's performance in Mathematics in Tanzania is still low for number of years in Certificate of Secondary Education Examinations (CSEE) (Kita, 2004, Mloziat al. 2013), URT, 2008 and SEDP, 2004). For many years the failure rate has been dramatically high in ordinary level (O level)Secondary schools. This may be due to so many factors as lack of resources (such as books, equipment, and classrooms) in the schools (MOEC, 1995).

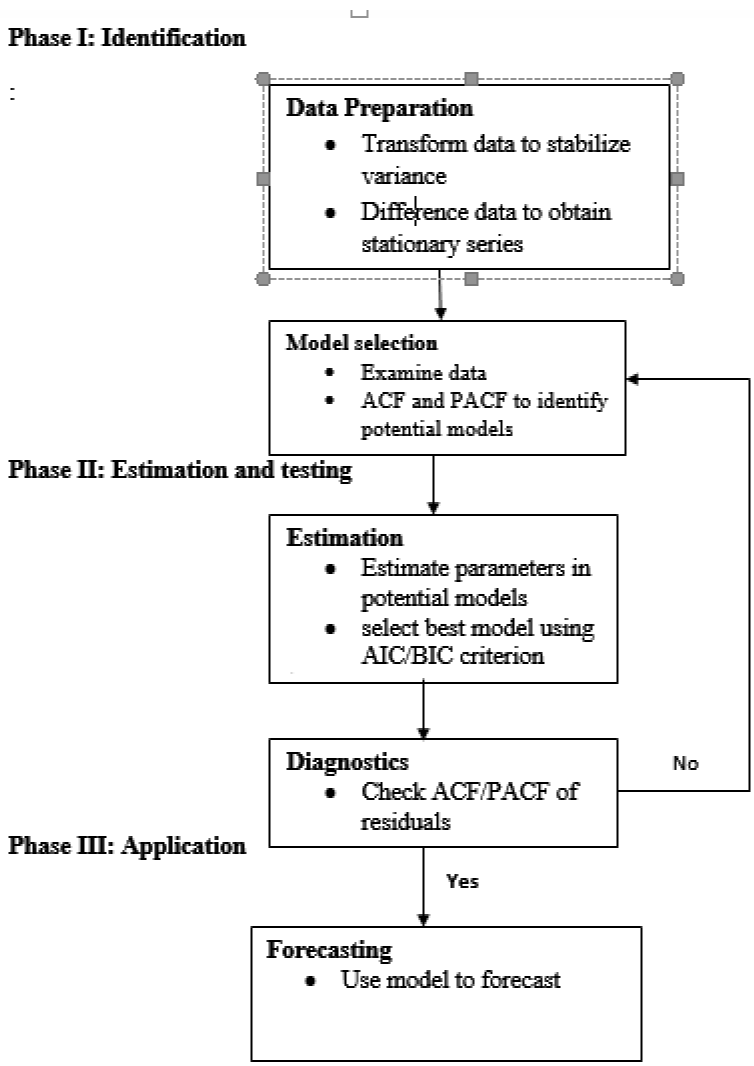
Students "poor performance" in mathematics is globally known, Zanzibar is not being different. Morris and Arora (1992) contend that the problem of students ‘poor performance in mathematics is not confined to any one country but universal. It’s well known, that there is a significance difference in the performance in mathematics with other subjects. In response to this global problem, it is aimed to perform trend analysis and fit various models of the performance of secondary students in mathematics in Zanzibar for the last thirty one years (1986-2016) and forecast the performance of secondary students in mathematics on the basis of selected best model for the next six years. This study may fill up the gap by focusing on the trend of performance of mathematics that influences the situation in schools of Zanzibar, with a reference of the present study and forecasting future years’ performance.

CONCEPTUAL FRAME WORK

The Box-Jenkins methodology is a statistically sophisticated way of analyzing time series data andbuilding a forecasting model which best represents a time series. Following are the steps for conceptual framework used in this study. These steps are also presented in figure 1.

- The first stage is to plot time series and to check the mean and variance. : making sure that the variables are stationary, identifying seasonality in the dependent series (seasonally differencing it if necessary), and using plots of the autocorrelation and partial autocorrelation functions of the dependent time series to decide which (if any) autoregressive or moving average component should be used in the model.
- The second stage is to estimates of the ARIMA model chosen. By using computation algorithms to arrive at coefficients that best fit the selected ARIMA model.
- The third stage is to check the residuals, by testing whether the estimated model confirms to the specifications of a stationary univariate process. In particular, the residuals should be independent of each other and constant in mean and variance over time. If the estimation is inadequate, we have to return to step one and attempt to build a better model, otherwise to forecast and the process ends.

Fig.1: The Box-Jenkins modeling process



REVIEW OF LITERATURE

Various authors like (Hanushek et al. 2000; Barro et al. 2001; Odili, 2006;Iji, 2007; Obioma et al. 2007;) have studied the performance of students in various individual subjects either Mathematics, Physics etc. But these studies have used the primary data to identify the factors effecting performance of students. Russel & Julie (2006) identified the factors affecting the performance in Gate Way courses in Northern Arizona University. Akanle(2007) identified some socio-economic factors influencing students' performance in Nigeria.

Blazenka&Dijana(2009), Erimafa*et al*. (2009), Fagoyinbo*et al*. (2013) and Humera*etal*. (2015) studied the academic performance of students by using discriminant analysis.

Mlambo (2011) studied to determine the effect of learning preference, age, gender, and entryqualifications on academic performance (measured as the final coursework mark obtained) on the basis of a random sample of 66 registered students of AGRI 1013 (representing a 40% samplingfraction) to generate data on demographics (gender and age), learning preference,and entry qualifications.Relationships/associations between gender andlearning styles, gender and entry qualifications, age and learning preferences, andage and entry qualifications were analyzed using Pearson's chi-square test.

Garrett (2012) studied the potential to predict future disease burden basedupon the historical record within public health jurisdictions using Box-Jenkinsforecasting models and the future identification of outbreaks and other disease relatedevents. Garrett utilized NEDSS data (2002 - 2011)obtained from the Utah Departmentof Health, Bureau of Epidemiology.

Aromolaran*et al*. (2013),Angela *et al*.(2013) and Adejumo(2013) studied the students' performance based on logistic regression method .

Sarpong (2013) conducted a study on modeling and forecasting maternal mortality forecasting in Ghana with quarterly 50 observations by using ARIMA models and concluded that ARIMA models were adequately fitted the data than others.

Ebenezer et al.(2013) conducted a study of forecasting municipal solid waste generation in Kumasi Metropolitan Areaby using 72 observations of monthly solid waste generation from January 2005 to December 2010. The mostefficient model was used to forecast solid waste generation for 36 monthly periodsahead up to December 2013 and the most efficient model was ARIMA (1, 1,0).

Ofori et al. (2013) studied to forecast road accident injuries in Ghana and used only 21 annual injuries records from 1991 to 2010 to formulate the best fit ARIMA (1,1,1) that was used to forecast one year period ahead producing 13337 accidents against observed 13272 injuries for the year 2011.

Mamman and Dauda (2014) analyzed trends of students' performance in mathematics in May/June West African Senior Secondary Certificate Examination (WASSCE) by using 10 years data of students' performance in mathematics in Nasarawa State, Nigeria from 2004 - 2013 by using auto regressive model. The data were analyzed using percentages and time series analysis. The estimation technique for the study is the Autoregressive (AR) processes for modeling of time series for short-run forecasts in line with Box-Jenkins approach .

Maureen et al. (2014) studied the factors that cause poor examination passing rates and high dropout rates among primary school girls in Malawi. First hand data were collected by conducting a survey in all the three regions of Malawi. The respondents tothe questionnaire were girls (402) who are repeating the last class in primary schools (Standard 8), primary schoolteachers (481) and Head teachers (82). Secondary data sourced from the Malawi Ministry of Education and the MalawiNational Examinations Board (MANEB) was also analyzed to validate the survey results.

Ugochukwuet al. (2014) studied toidentify factors affecting students' performance in basic technology JuniorSecondary School Certificate Examination.The population of the study consisted of three thousand one hundred and twenty six (3126) basictechnology teachers and students.

Miftar et al. (2015) studied the students' performance level in the first year results of the examination of students' performance from University of Vlore by using binary logistic regression model. The data were collected from 240 fresh studentsthrough questionnaires. . The approach via logistic regression is doneto study the results of the students after the first semester according to the variables mentioned above.The analysis was conducted based on variables such as gender,environment where they live, type of private or public school, school location, points taken in high school, the mode ofperception of the social environment.

Sule&Saporu (2015) studied the factors that influence student's performance in Element of Calculus course by using Logistic regression model.

Amon et al. (2015) proposed the different values of minimum data requirement for ARIMA models. It also proposed to use as much data as they are available in formulating ARIMA models. This paper studied the impact of the size ofthe historical data on ARIMA models in forecasting accuracy. The study used 286 weekly records from July 2008 to December 2013 of amount of solid waste generated in Arusha City to formulate four ARIMA models using different data lengths or size. Metrine et al.(2015) studied the time series modeling of rainfall pattern in UasinGishu by applying various ARIMA models. They used 444 monthly observations from January 1977 to December 2014 and forecasted the rainfall for years 2014- 2016 .

Kipyegon (2015) studied the student population of University of Kabianga from the year 2009 to 2013 by using Box-Jenkins Autoregressive Integrated Moving Average Model technique. The student population was further forecasted to the year 2023 using ARIMA (1, 2, 0) model. The model was validated using various tests among them: Akaike Information Criterion (AIC), graphical techniques like time series plots, Schwarz Bayesian Criterion (SBC) and p-values. Results indicate that the student population will grow to 32,421 by the year 2023 when only time is considered as a factor. The results further depict a positive steady increase of student population for University of Kabianga over the next ten years.

Ehab et al. (2016) studied the analysis of time series data on number of incidents of Tuberculosis TB in Sudan from 1995 to 2013 by using ARIMA models. They used four steps namely identification, estimation, diagnostic checking, and forecasting by ARIMA models.

RESEARCH METHODOLOGY

For this study, all the 123 school in Zanzibar have been taken as the target population. The data consisted of annual result of all students in mathematics subject from 1986 – 2016 in the form of grades. Secondary data of all 69,446 students from 123 schools were collected, who sat for final examination from 1986-2016, from MoEVTZ. Since the nature of data consisted of time period, thirty one years' data had been deemed adequate for analyzing trends of students' performance (Rosenberg 1997).Data were cleaned so as to remove apparent errors and missing observations, detecting entry errors and checking for inconsistencies.The year wise (1986-2016) results of all 69,446 students from 123 schools in Zanzibar were cross tabulated according to grades A,B,C,D, and E. Then these grades were categorized into three classes consisting of credits, pass and fail by combining the students falling in grades (A+B+C), pass with grade D and fail with F respectively. Finally the results were further classified in study variables as percentages of all students, henceforth called, as credits percentage, pass percentage and fail percentage by dividing the number of students to total students in each classification. The descriptive and inferential analysis has been performed by using software packages IBM SPSS version 20 and MS Excel 2007.

A time series is a sequence of observations (of some quantity) taken over an extended period of time. Usually these observations are made at a settime interval hourly; daily, monthly, yearly etc.

The series may be denoted by Y_1 , $Y_2, \dots \dots \dots Y_t$ where t refers to the time period and Y refers to the value.

Autoregressive (AR) :

Moving average (MA)In an auto regressive AR (*p*) model, the future value of a variable is assumed tobe a linear combination of *p* past observations and a random error together with a constantterm. Mathematically the AR(*p*) model can be expressed as

$$Y_t = c + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots \varphi_p Y_{t-p} + \varepsilon_t \dots \dots \dots \dots \dots (i)$$

Moving average (MA):

The moving average MA (*q*) model uses past errors as the explanatory variables. The order of the MA (q) process is given by the autocorrelation function(ACF). The ACF cuts off after lag q and the partial autocorrelation function decay to zero. Mathematically, the MA (*q*) model can be expressed as

$$Y_t = \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-p} + \varepsilon_t \dots \dots \dots \dots \dots (ii)$$

The Autoregressive Moving Average (ARMA) Models

AnAutoregressive Moving Average model (ARMA) (*p*, *q*) model is a combination of AR (*p*) and MA (*q*) models and is suitable for univariate time series modeling. The parameters p and q are called the autoregressive and the moving average orders respectively. A mixed autoregressive moving average or ARMA(p,q,) model is expressed as

$$Y_t = \delta + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots \theta_q \varepsilon_{t-q} \dots \dots \dots .. (iii)$$

Differencing:

Differencing is a process of computing the difference between every two successive values in a time series. Often differencing is used to account for non-stationarity that occurs in the form of trend or seasonality or both. Differencing that accounts for trend is referred to as regular (non-seasonal) differencing and when it accounts for seasonality it is known as seasonal differencing.

Thus if Y_t(t=1, 2, 3….n) denotes the previous series the first order non seasonal difference is

$$\delta Y_t = Y_t - Y_{t-1}$$

The difference $Y_t - Y_{t-1}$can be expressed as $(1 - B)Y_t$ where $B$is back shift operator.

Autoregressive Integrated Moving Average (ARIMA):

An Autoregressive Integrated Moving AverageARIMA model is characterized by the notation ARIMA (p,d,q) where p, d and q denote orders of auto-regression, integration(differencing) and moving average respectively. In ARIMA, time series is a linear function of past actual values and random shocks.

Seasonal and Non seasonal ARIMA model:

Seasonality in time series ARIMA model is defined as a pattern that repeats itself over fixed interval of time otherwise it is called as non- seasonal.

An observable time series $Y_t$in which successive values are highly dependent can frequently be regarded as generated from a series of independent“shocks” a_t. These shocks are randomdrawings from a fixed distribution, usually assumed normal and having mean zero and variance $\sigma^2{}_a$. Such a sequence of independent random variables a_t,at_{-1}, a_{t-2}, . . . is called a white noise process

Building ARIMA model:

Autoregressive integrated moving average (ARIMA) modeling, also known as the Box-Jenkins (1970) modeling approach, requires significantly large data set and the development of ARIMA model for any variable involves several steps; model identification, model estimation, Diagnostic checking and then forecasting.

Model identification:

Model identification starts with preliminary efforts in understanding thetype of process from which the data are coming and how it is collected.ARIMA model is estimated only after transforming the time series into a stationary series. A stationary series is one whose values vary over time only around a constant mean, constant variance and constant autocorrelation.

The graph of the data and structure of the autocorrelation and partial correlation coefficients may provide a direction for the presence of stationarity.Non stationary in mean is the one who’s mean of the series changes over time.

Partial Autocorrelations (PACF):

The autocorrelation function is the correlation which gives the correlations between different lags of a series.

According to Box -Jenkins (1976), autocorrelation function can be used for the following two purposes:

- To detect non-randomness in data
- To identify an appropriate time series model if the data are not random.

Given measurements, $Y_1, Y_2 \dots Y_N$ at a time $X_1, X_2, \dots X_N$, the lag k autocorrelation is defined as:

$$r_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

Partial autocorrelations are used to measure the degree of association between Y_t and Y_{t-k}, when the effects of other time lag$-1, 2, 3 \dots k-1-$are removed.

In other words, partial autocorrelation is somewhat similar to autocorrelation, except that when calculating it, the (auto) correlations with all elements within the lag are deleted (Box and Jenkins, 1976).

The partial autocorrelation coefficient of order k is denoted by α_k and can be calculated by regressing Y_t against$Y_{t-1}, \dots Y_{t-k}$.

$$Y_t = b_0 + b_1 Y_{t-k} + b_2 Y_{t-k} + \cdots b_k Y_{t-k}$$

The partial autocorrelation, α_k, is the estimated coefficient b_k from the multiple regression.

Parameter estimation:

Parameters are estimated using numerical minimization proceduressuch as methods of moments, maximum likelihood, and least squares that can be employed to estimate the parameters in the tentatively identified model.

Estimation is usually automatically performed by sophisticated software packages such as Mini tab, JMP, SAS, SPSS. Most software use maximum likelihood estimation methods to find the estimates. The researcher may have the choice of estimation method and can accordingly choose the most appropriate method based on the problem specifications.

Information Criterion methods:

The Akaike information criterion (AIC) is a measure of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Hence, AIC provides a means for model selection.AIC is found on information theory as it offers a relative estimate of the information lost when a given model is used to represent the process that generates the data.

Let L be the maximum value of the likelihood functionfor the model and k be the number of estimated parameters in the model then*AIC* value of the model can be calculated as following:

$$AIC = 2K - 2logL$$

Where K is the number of estimated parameters included in the statistical model and L is the maximum value of the likelihood function for the estimated statistical model

The minimum *AIC* value for a model among various AICs is an indicator of the best model. .AIC gives goodness of fit (by the likely-hood function), but it also includes a penalty that is an increasing function of the number of estimated parameters, hence increasing the number of parameters in the model always gives the calculated goodness of the fit.

Bayesian Information Criterion (BIC):

The Bayesian information criterion (BIC) can be written as

BIC=AIC+ (log (T)-2)(p+q+k+1).

The objective is to minimize the AIC or BIC values for a good model. The lower the values of one of these criteria for a range of models being investigated, the better the model will suit the data. It should be noted however that the AIC and the BIC are used for two completely different purposes. Whilst the AIC tries to approximate models towards the reality of the situation, the BIC attempts to find the perfect fit. The BIC approach is often criticized as there never is a perfect fit to real-life complex data; however, it is still a useful method for selection as it penalizes models more heavily for having more parameters than the AIC would.

AIC can only be used to ARIMA models with the same orders of differencing. For ARIMA with different orders of differencing, RMSE can be used for model comparison.

Diagnostic checking:

Diagnostic checking of a model is done through residual analysis; the fitted model ischecked forstudying the autocorrelation plots of the residuals to see if further structure (large correlation values) can be found. If all the autocorrelations and partial autocorrelations are small and should not differ significantly from zero,the model is considered adequate and forecast series are generated.

The best model is obtained with the following criterion

- Low Akaike Information Criteria (AIC) / Bayesian Information Criteria (BIC)
- Non significances of auto correlations of residuals based on Ljung-Box tests.

Forecasting:

The main objective of ARIMA model is forecasting. Once an appropriate time series model has been fit, it may be used to generate forecasts of future time. The standard criterion to use in obtaining the best forecast is the mean squared error for which the expected value of the squared forecast errors is minimized. The important measures of forecast errors such as mean absolute percentage error (MAPE) are used to judge the forecasting ability of the fitted model.

RESULTS AND DISCUSSION

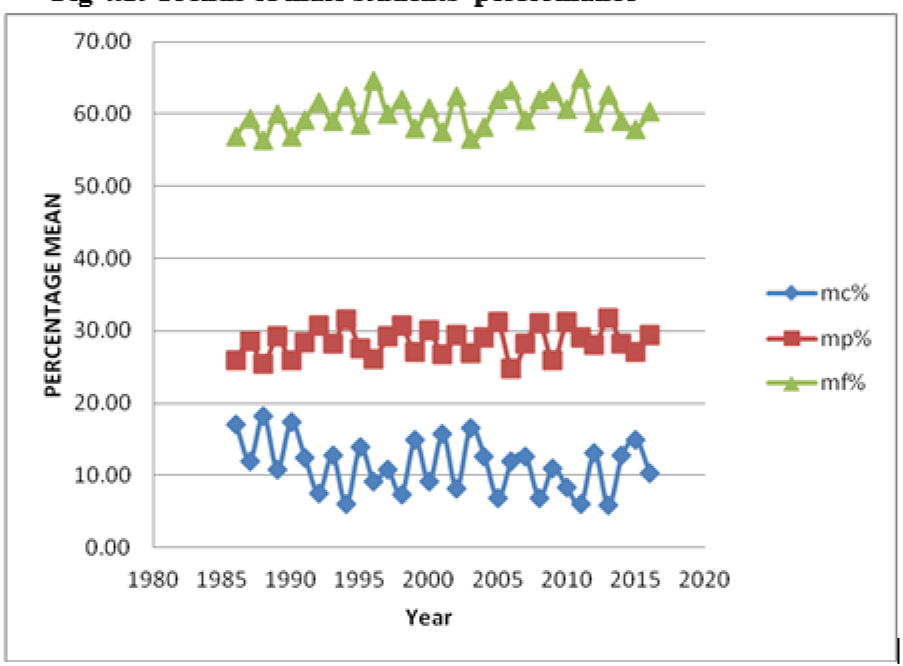
The data have been categorized in the form of male students, female students and total students (male and female). The performance of students has been categorized in terms of percentage credit, percentage pass and percentage fails.

Male students' results:

MaleStudents'performance trend

The trend performance of male students by percent credit, percent pass and percent fail students' in Zanzibar have been presented in Figure 4.1. It showsthat the peaks have different intensitywhich means that trends of performance patterns have varying means and variances. Also the trend showed up and down pattern over time as shown in figure 4.1.

Fig 4.1: Trends of male students' performance



Normality test for male student's performance:

Kolmogorov-Smirnov and Shapiro- Wilk's tests have been used for testing the normality of male students' performance by percent credit, percent pass and percent fail. The results have been presented in table 4.1. The results show that the male percentage credit, percentage pass and fail were normally distributed, since the p-value was greater than 0.05 significant level.

Table 4.1 : Normality test for male students' percentage credit, pass and fail

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Male students' percentage credit	0.092	31	0.205	0.959	31	0.268
Male students' percentage pass	0.092	31	0.210	0.963	31	0.345
Male students' percentage fail	0.107	31	0.320	0.965	31	0.392

Model identification for male students:

The data have been analyzed by Box and Jenkins approach for ARIMA model. The original data for male percentage credit, percentage pass and percentage fail were not stationary due to the peaks of different intensity. Thetransformation of natural logarithm and first differencing have been applied to attain stationarity as indicated in figure 4.2, 4.3, 4.4 for male students' percentage credit, pass, and fail respectively.

Fig 4.2: Male students'percentage credit

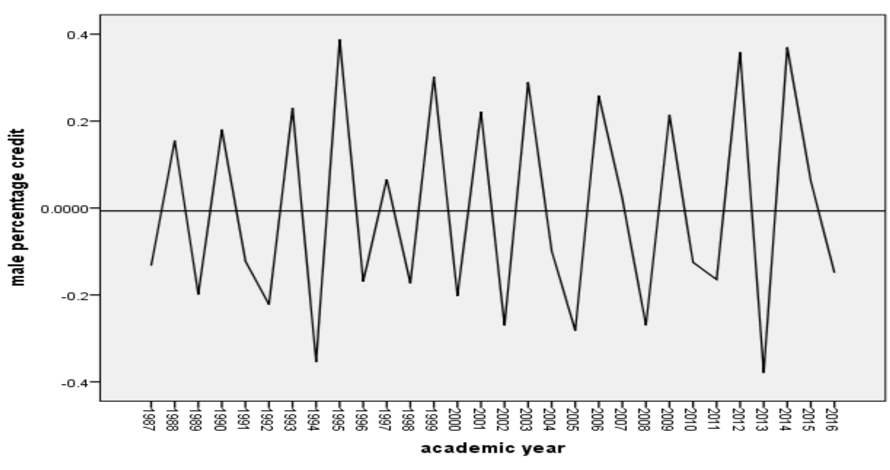


Fig 4.3: Male students' percentage pass

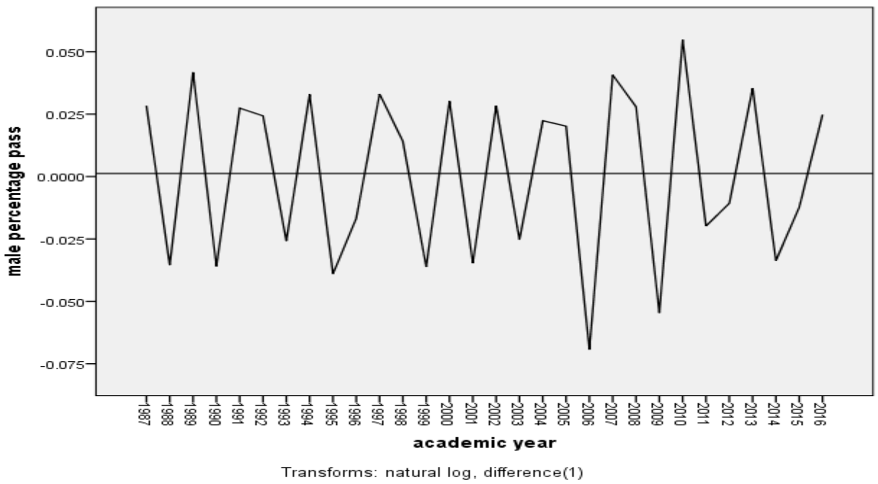
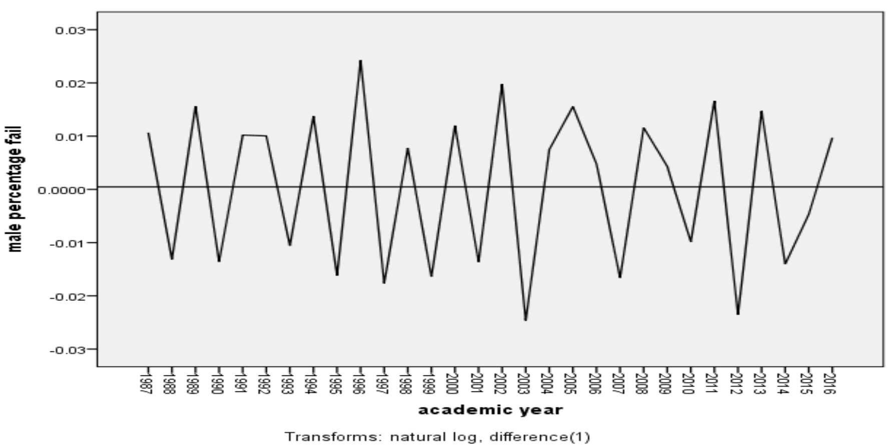


Fig 4.4: Male students' percentage fail



The stationarity of the data have been confirmed by ACF and PACF plots as shown in figure 4.5 and 4.6 for male students' percentage credit, 4.7 and 4.8 for male students' percentage pass 4.9 and 4.10 for male students' percentage fail respectively.

Figure 4.5: ACF of malestudents' percentage credit

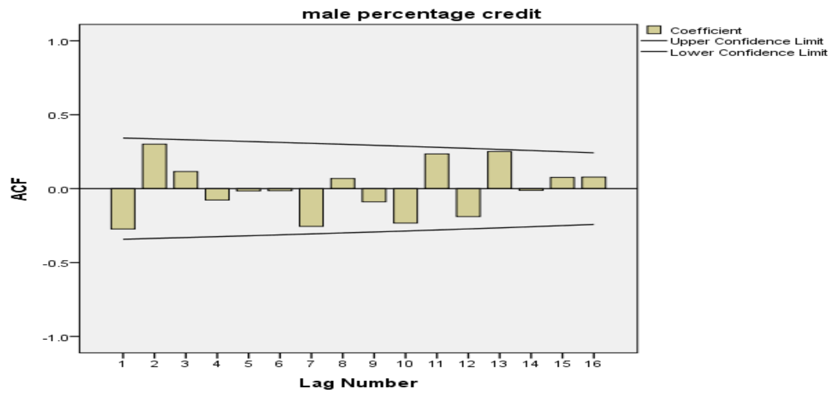


Figure 4.6: PACF of male students' percentage credit

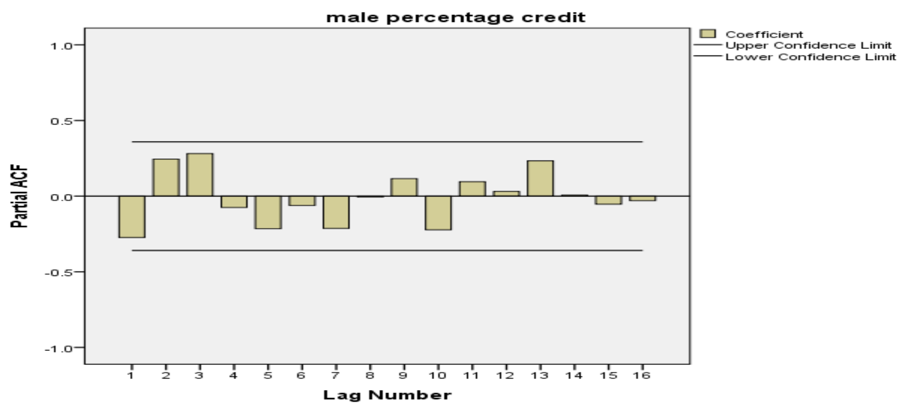


Figure 4.7: ACF of male students' percentage pass

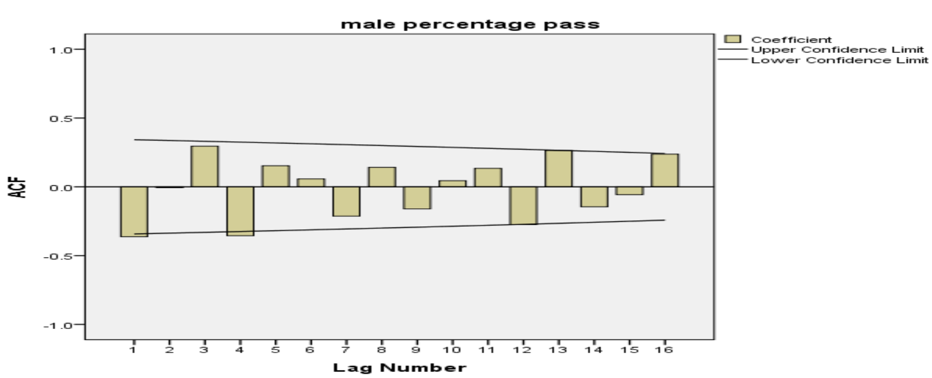


Figure 4.8: PACF of male students' percentage pass

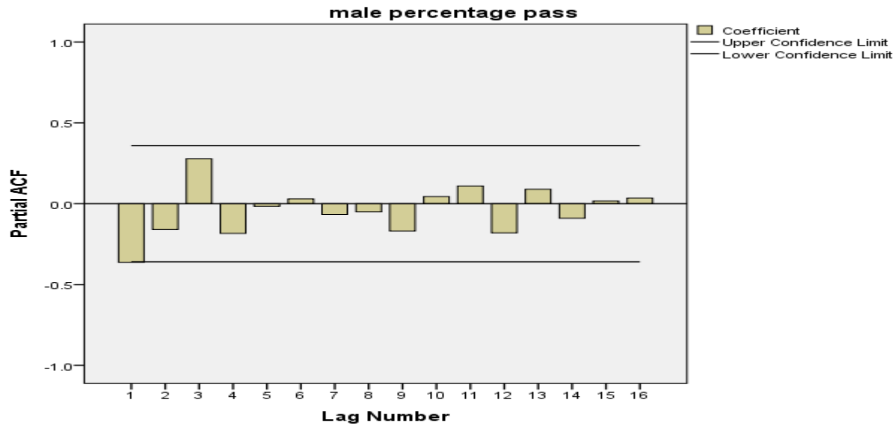


Figure 4.9: ACF of male students' percentage fail

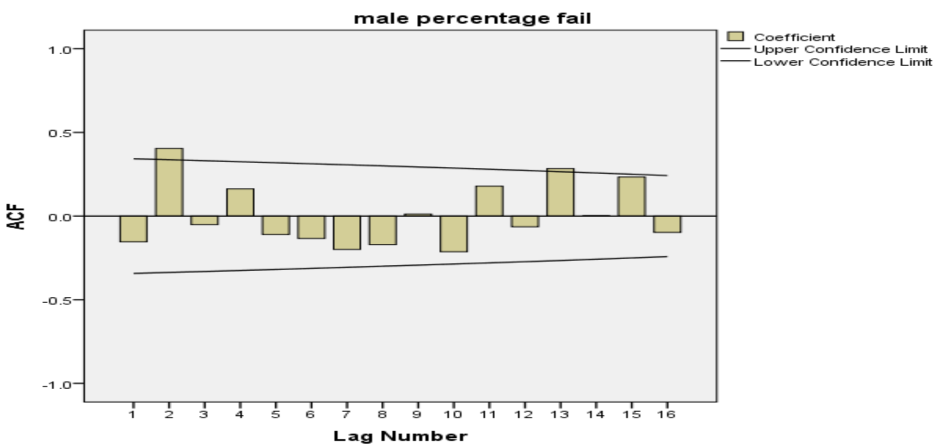
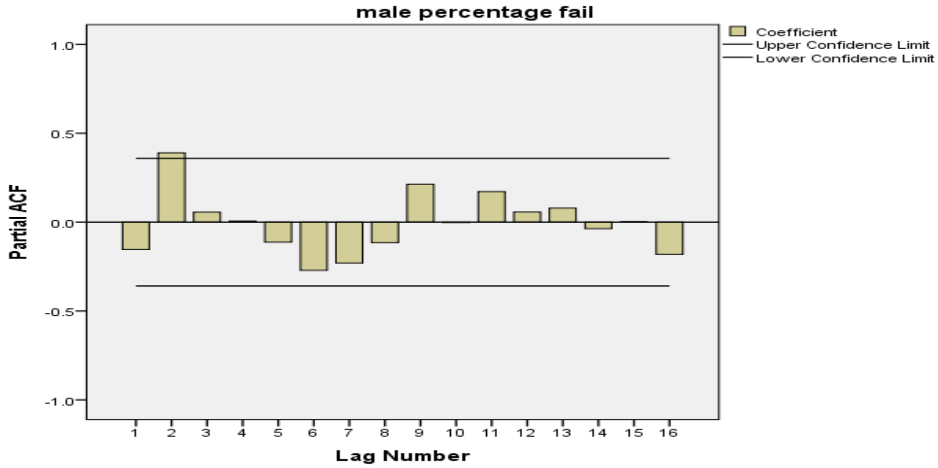


Figure 4.10: PACF of male students' percentage fail



Model selection :

The best models have been selected on the basis of the least AIC and BIC amongst different models as shown in table 4.2 as follows:

- ARIMA (1, 1, 2) for male percentage credit with least AIC and BIC as 1.104 and -1.825 respectively.
- ARIMA (2, 1, 0) for male percentage pass with least AIC and BIC as 1.121 and -4.999 respectively.
- ARIMA (1, 1, 0) for male percentage pass with least AIC and BIC as 1.048 and -6.233 respectively.

Table 4.2: ARIMA models for male students' percentage credit, percentage pass and percentage fail

Model	Male students' percent credit		Male students' percent pass		Male students' percent fail	
	AIC	BIC	AIC	BIC	AIC	BIC
ARIMA(0,1,0)	1.113	-1.04	1.098	-4.225	1.089	-5.507
ARIMA(0,1,1)	1.178	-1.813	1.157	-4.965	1.149	-6.136
ARIMA(0,1,2)	1.243	-1.786	1.220	-4.934	1.212	-6.226
ARIMA(1,1,0)	1.178	-1.664	1.159	-4.565	**1.048**	**-6.233**
ARIMA(2,1,0)	1.243	-1.823	**1.021**	**-4.999**	1.202	-6.098
ARIMA(1,1,1)	1.243	-1.786	1.220	-4.984	1.312	-6.115
ARIMA(2,1,1)	1.307	-1.676	1.268	-4.872	1.277	-5.964

ARIMA(1,1,2)	**1.104**	**-1.825**	1.286	-4.836	1.280	-5.965
ARIMA(2,1,2)	1.372	-1.525	1.349	-4.779	1.342	-5.812

Table 4.3 presents the model fit statistics and Ljung-Box Q statistics for male students' percentage credit, percentage pass and percentage fail. The p-values for male students' percentage credit, percentage pass and percentage fail for Ljung-Box Q statistics are 0.541, 0.709 and 0.908 respectively, which shows that the models ARIMA (1, 1, 2)for male students' percentage credit, ARIMA (2, 1, 0) for male students'percentage pass and ARIMA (1, 1, 0) for male students' percentage failare best fit.

Table 4.3: Model fit statistics for male students' performance

Model	Model Fit statistics		Ljung-Box Q(18)			
	Stationary R-squared	Normalized BIC	Statistics	DF	Sig.	RMSE
Male students' Percentage credit	0.638	-1.825	14.774	16	0.541	0.339
Male students' Percentage pass	0.635	-4.999	12.494	16	0.709	0.720
Male students' Percentage fail	0.555	-6.233	9.886	16	0.908	0.041

Estimation of parameters:

The results of maximum likelihood estimation of the parameters of the models and their significance have been obtained using IBM SPSS V-20 software and are presented in table 4.4. The results reveal that the estimates for parameters of the selected models for male students' percentage credit consist of AR(1) and MA(2) and first difference ,male students' percentage pass consist of AR(2) and first difference and male students' percentage fail consist of AR(1) and first difference, are significant as the p values are less than 0.05 .

Table 4.4 ARIMA model parameters estimates

Model description			Estimate	SE	t	Sig
Male students' percentage credit	constant		-0.004	0.028	-0.133	0.001
	AR	Lag 1	-0.336	0.348	-0.967	0.002
	difference		1			
	MA	Lag 1	0.769	0.349	2.204	0.007

		Lag 2	-0.372	0.328	-1.135	0.003
Male students' percentage pass	constant		0.002	0.005	0.365	0.008
	AR	Lag 1	-1.015	0.146	-6.928	0.000
		Lag 2	-0.631	0.145	-4.337	0.000
	difference		1			
Male students' percentage fail	constant		0.001	0.004	0.179	0.009
	AR	Lag 1	-0.740	0.124	-5.946	0.000
	difference		1			

Male students'percentage credit fitted model equation:

By using general equation of non-seasonal

$\varphi(B)\nabla^{d}Y_t = \delta + \theta(B)w_t$………………… (i)

Where $\varphi(B)$ and $\theta(B)$ are the regular autoregressive and moving average ,∇ is the difference linear operator, (d=1) is the ordinary differencing,$\nabla \mathrm{Y_t} = (1 - \mathrm{B})\mathrm{Y_t}$B is the back shift operator, $\delta$is the constant value.

From equation(i), it can be written as

$$(1 - \varphi_1 B)(1 - B)Y_t = \delta + (1 + \theta_1 B + \theta_2 B^2)w_t$$

But $\theta_1 B w_t = \theta_1 w_{t-1}$,$\theta_2 B^2 w_t = \theta_2 w_{t-2}$and hence the equation for model can be written as

$Y_t = \delta + Y_{t-1}(1 + \varphi_1) - \varphi_1 Y_{t-2} + \theta_1 w_{t-1} + \theta_2 w_{t-2} + w_t$………..(ii)

Where$\delta = -0.004$, $\varphi_1 = -0.336$, $\theta_1 = 0.769$, $\theta_2 = -0.372$. Putting these values in equation (ii) , the fitted model equation can be written as

$$\boldsymbol{Y_t = -0.004 + 0.664Y_{t-1} + 0.336Y_{t-2} + 0.769w_{t-1} - 0.372w_{t-2} + w_t}$$

Male students'percentage pass fitted model equation:

$\boldsymbol{Y_t = \delta + Y_t + \varphi_1(Y_{t-1} - Y_{t-2}) + \varphi_2(Y_{t-2} - Y_{t-3}) + w_t}$……….. (iii)

Where $\delta = 0.002$, $\varphi_1 = -1.015$, $\varphi_2 = -0.631$

Thus equation (iii) can be written as

$$\boldsymbol{Y_t = 0.002 + Y_{t-1} - 1.015(Y_{t-1} - Y_{t-2}) - 0.631(Y_{t-2} - Y_{t-3}) + w_t}$$

Male students'percentage fail fitted model equation:

$\boldsymbol{Y_t = \delta + Y_{t-1} + \varphi_1(Y_{t-1} - Y_{t-2}) + w_t}$………… (iv)

Where $\delta = 0.001$, $\varphi_1 = -0.740$

Hence equation (iv) can be written as

$$\boldsymbol{Y_t = 0.001 + Y_{t-1} - 0.740(Y_{t-1} - Y_{t-2}) + w_t}$$

Diagnostic checking:

Model diagnostic checking of the fitted models has been performed through residual analysis to determine the adequacy of the model or to indicate potentials improvement.

4.2.6.1: Diagnostic checking for Male students' percentage credit

The residual plots of ACF and PACF against lags have been presented in figure 4.11 and 4.12. The figures display that the residuals from ARIMA (1, 1, 2) fitted model for the male students' percentage credit are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilk test , which have been presented in table 4.5 .It can be concluded that the residuals are normally distributed as the p values are greater than 0.05.

Figure 4.11 : ACF of male percentage credit residuals

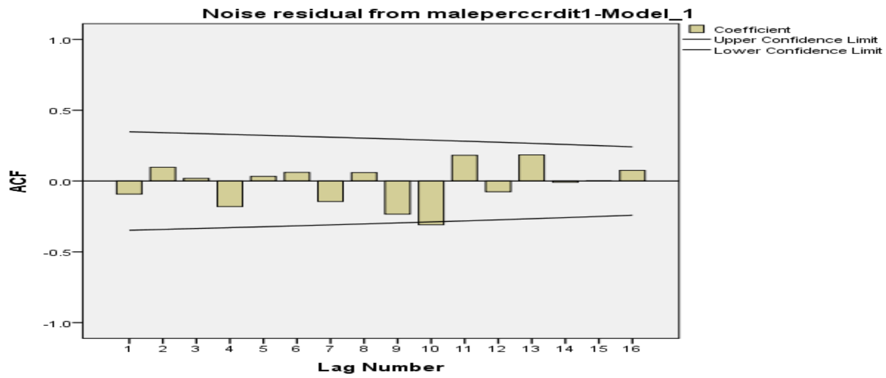


Figure 4.12 : PACF of male percentage credit residuals

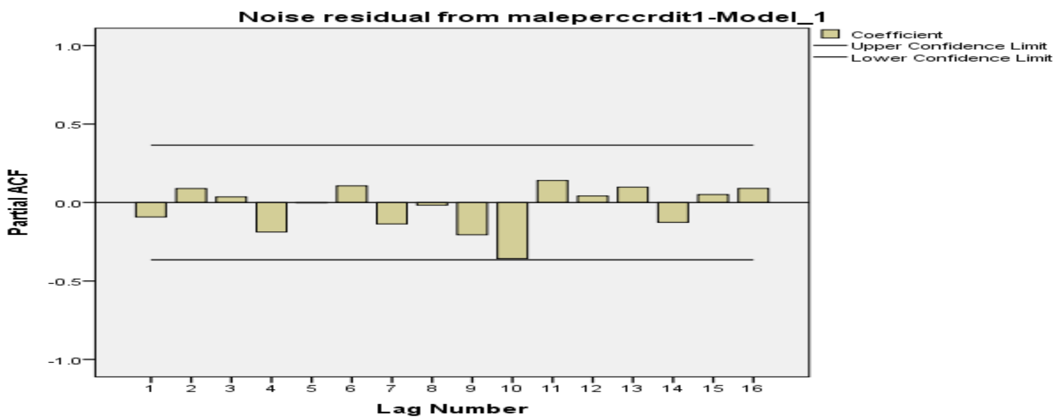


Table 4.5: Normality test of male residual for percentage credit

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Noise residual from male student's percentage credit	0.117	30	0.300	0.971	30	0.553

Diagnostic checking for Male students' percentage pass:

The residual plots of ACF and PACF against lags have been presented in figure 4.13 and 4.14. The figures display that the residuals from ARIMA (2, 1, 0) fitted model for the male students' percentage pass are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilk test , which have been presented in table 4.6 .It can be concluded that the residuals are normally distributed as the p values are greater than 0.05.

Figure 4.13: ACF of male percentage pass residual

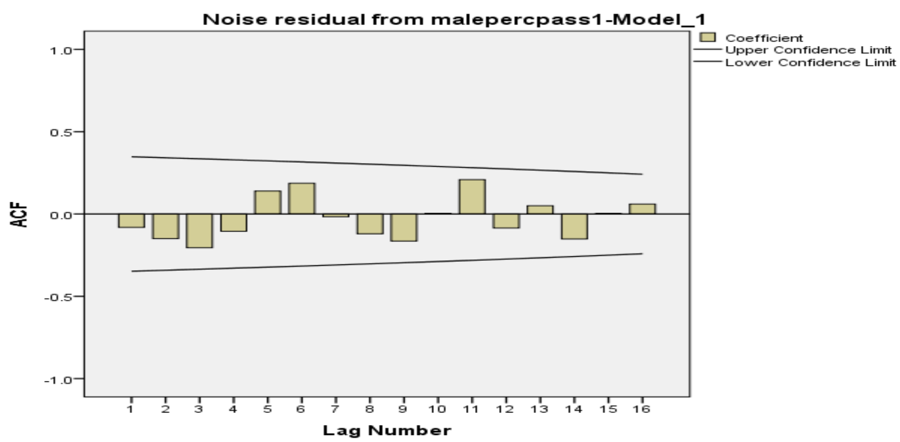


Figure 4.14: PACF of male percentage pass residual

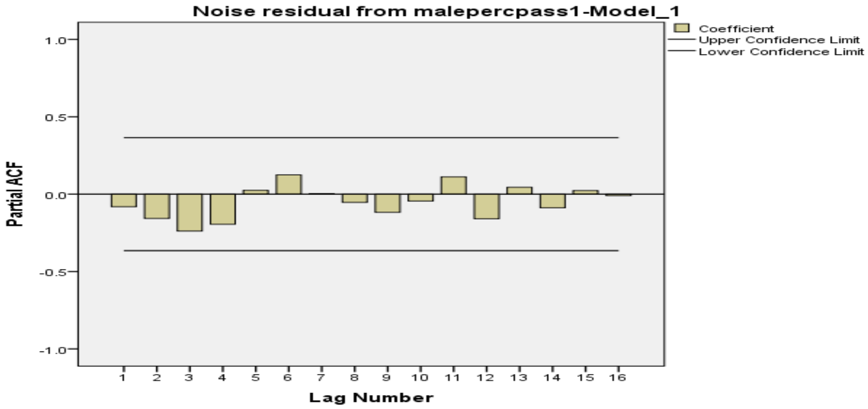


Table 4.6: Normality test of male residual for percentage pass

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Noise residual from male student's percentage pass	0.124	30	0.200	0.960	30	0.313

Diagnostic checking for Male student's percentage fail :

The residual plots of ACF and PACF against lags have been presented in figure 4.15 and 4.16. The figures display that the residuals from ARIMA (1, 1, 0) fitted model for the male students' percentage fail are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilk test , which have been presented in table 4.7 .It can be concluded that the residuals are normally distributed as the p values are greater than 0.05.

Figure 4.15: ACF of male percentage fail residual

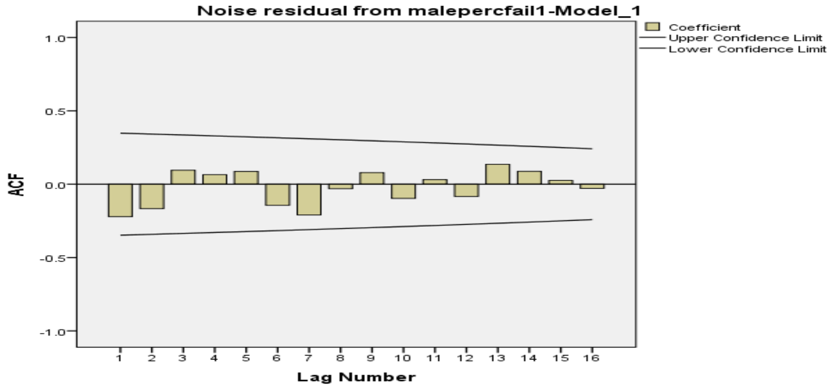


Figure 4.16: PACF of male percentage fail residual

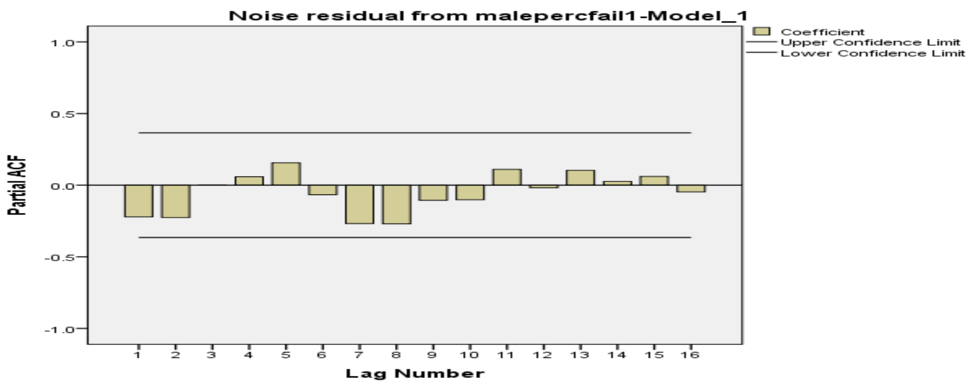


Table 4.7: Normality test of male residual for percentage fail

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Noise residual from male percentage fail	0.149	30	0.088	0.957	30	0.263

Forecasting for Male students' performance:

The table 4.8presents the forecasted values formale students'percentage credit, percentage pass and percentage fail from 2017-2022. On the basis of best ARIMAfitted model forecast, it is found that in next 6 years the performance of students in mathematics in secondary school examination in Zanzibar will lie between 12.31 % in (2017) to 21.51% in (2022) for percentage credit, 27.94% in (2017) to 33.50% in (2021) for percentage pass and 56.74% in (2020) to 60.15% in (2021) for percentage fail.

Table 4.8: Forecasted performance of male students' on the basis of fitted models

Year	Percentage credit ARIMA(1,1, 2)	percentage pass ARIMA(2, 1, 0)	Percentage fail ARIMA(1, 1,0)
2017	12.31	27.94	58.56
2018	14.45	28.22	57.78
2019	16.15	29.08	59.15
2020	19.78	31.22	56.74
2021	18.52	33.50	60.15
2022	21.51	32.99	59.74

Forecast error analysis:

The forecast error analysis for male students' percentage credit, percentage pass and percentage fail has been done and the results are presented in table 4.9. It can be seen from the results that mean absolute percentage error for percentage credit, percentage pass and percentage fail are respectively 11.73 %, 8.75 % and 9.77 % .This shows the accuracy of the forecast as 88.27 %, 91.25 % and 90.23 %respectively .

Table 4.9: Forecast error analysis

	Percentage credit	Percentage pass	Percentage fail
Cumulative forecast	0.17	0.02	0.05
Mean error	0.057	0.07	0.017
Mean square error	0.1029	0.047	0.0154
Root mean sq. error	0.3208	0.0687	0.0392
Mean absolute deviation	0.2737	0.0587	0.03167
Mean absolute percent. error	11.73	8.75	9.77

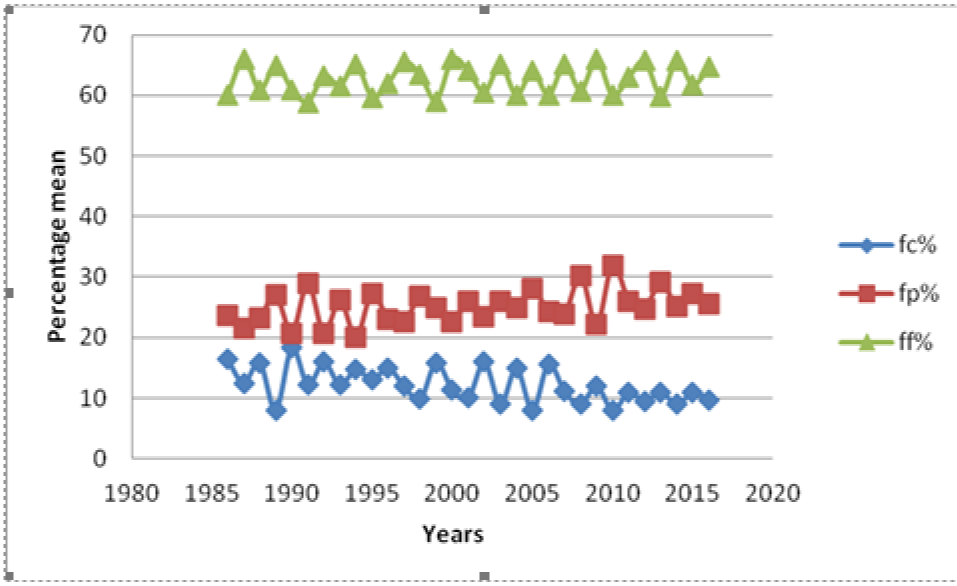
'

Female students' results

Female students' performance trend:

The trend performance of female students by percent credit, pass and fail in Zanzibar has been presented in figure 4.17. It shows that the peaks have different intensity which means that trends of performance patterns have varying means and variances. Also the trend showed up and down pattern over time as shown in figure 4.17.

Fig 4.17: Trends of female students' performance



Normality test for female student's performance:

Kolmogorov-Smirnov and Shapiro- Wilk's tests have been used for testing the normality of female students' performance by percent credit, percent pass and percent fail. The results have been presented in table 4.10. The results showed that the male percentage credit, percentage pass and fail were normally distributed, since the p-value was greater than 0.05 significant level for both the tests.

Table 4.10: Normality test of female percentage credit, pass and fail

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Female students' percentage credit	0.133	31	0.172	0.934	31	0.480
Female students' percentage pass	0.160	31	0.224	0.985	31	0.340
Female students' percentage fail	0.158	31	0.470	0.893	31	0.205

Model identification:

The data have been analyzed by Box and Jenkins approach for ARIMA model.The original data for female percentage credit, percentage pass and percentage fail was not stationary due to the peaks of different intensity. The transformation of natural logarithm and first differencing have been applied to attain stationarity as indicated in figure 4.18, 4.19 and 4.20 for female students' percentage credit, pass, and fail respectively.

Figure 4.18: Time series plot for female students' percentage credit

Figure 4.19: Time series plot for female students' percentage pass

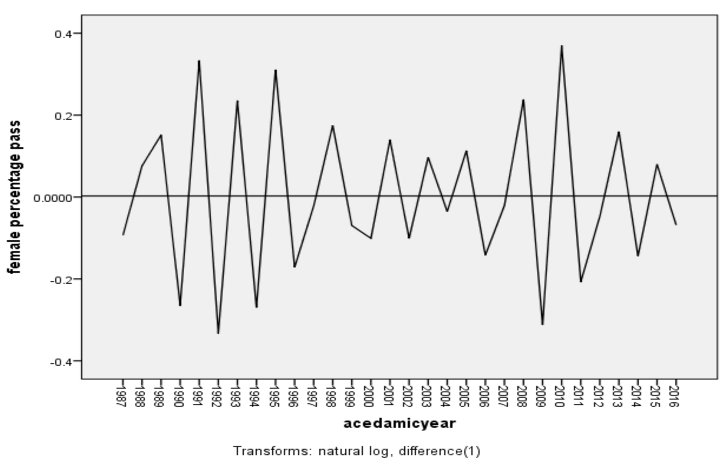
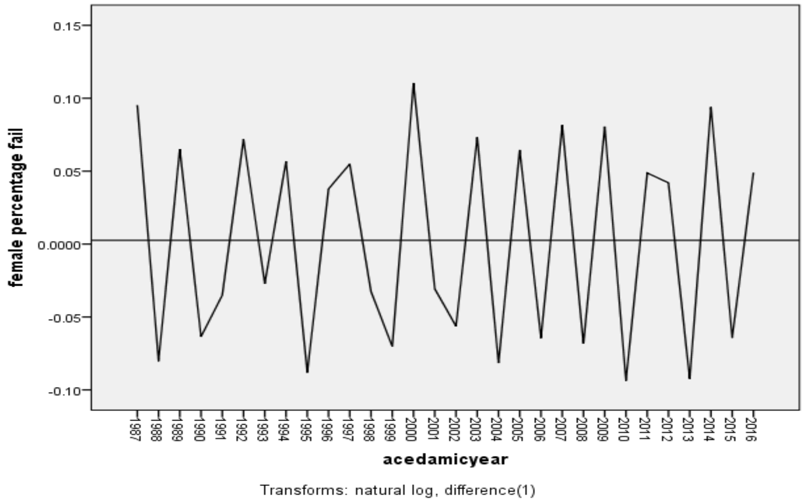


Figure 4.20: A time series plot for female students’ percentage fail



The stationarity of the data have been confirmed by ACF and PACF plots as shown in figures 4.21 and 4.22 for female students' percentage credit, in figures 4.23 and 4.24 for female students' percentage pass and in figures 4.25 and 4.26 for female students' percentage fail respectively.

Figure 4.21: ACF of female students’ percentage credit

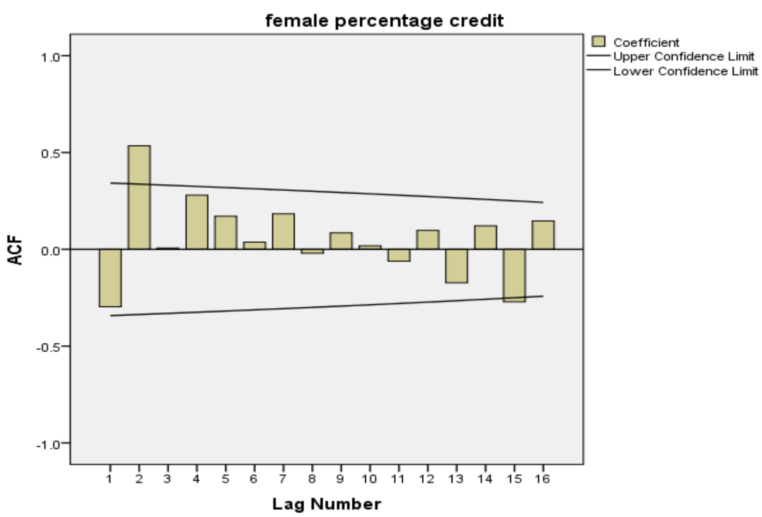
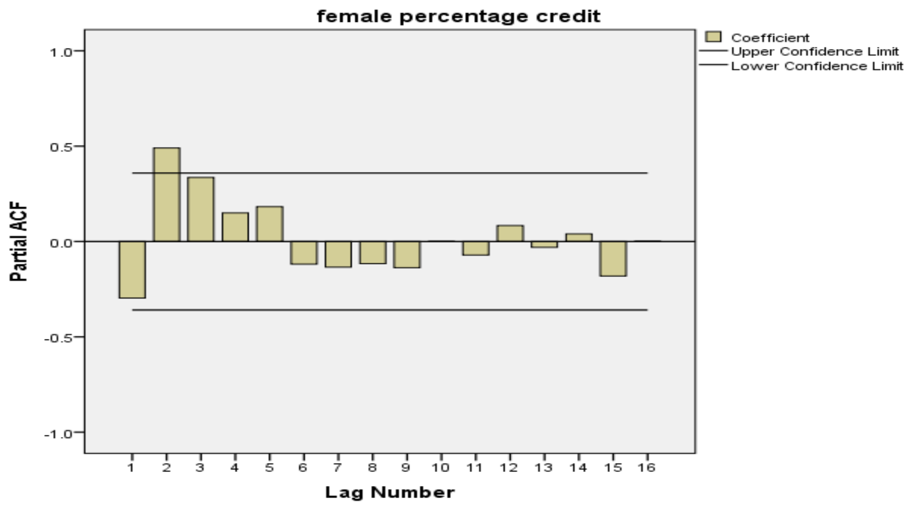


Figure 4.22: PACF of female students' percentage credit



Similarly, ACF in figure 4.23 suggest moving average of order one and PACF in figure 4.24 suggest autoregressive of order one for percentage pass.

Figure 4.23: ACF of female students' percentage pass

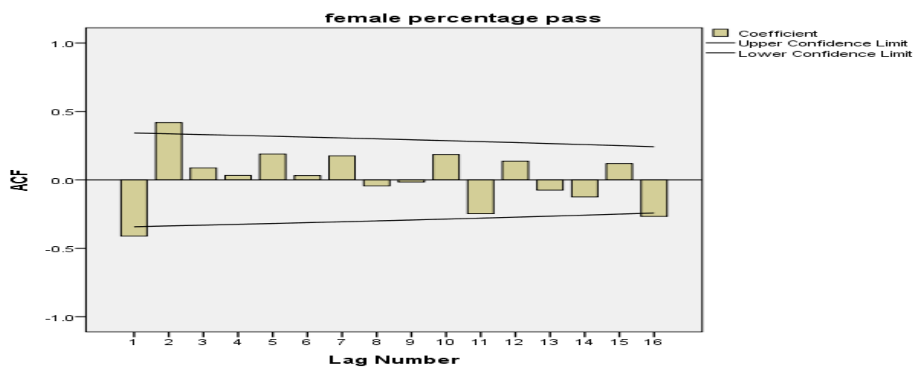
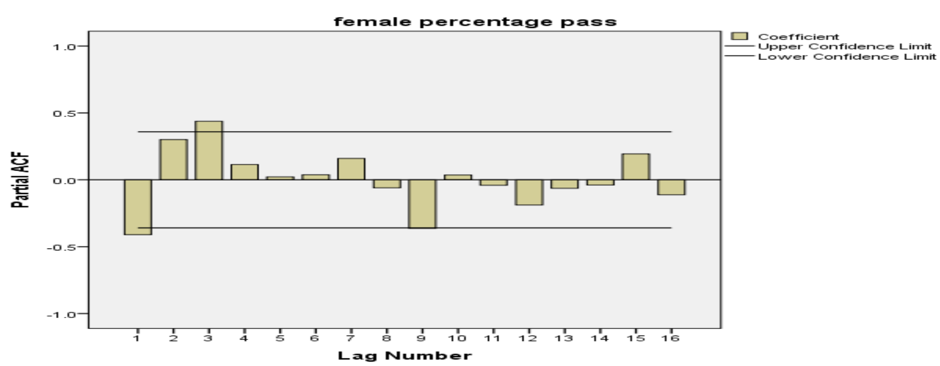


Figure 4.24: PACF of female students' percentage pass



ACF in figure 4.25 suggest moving average of order one and PACF in figure 4.26 suggest autoregressive of order one for percentage fail.

Figure 4.25: ACF of female students' percentage fail

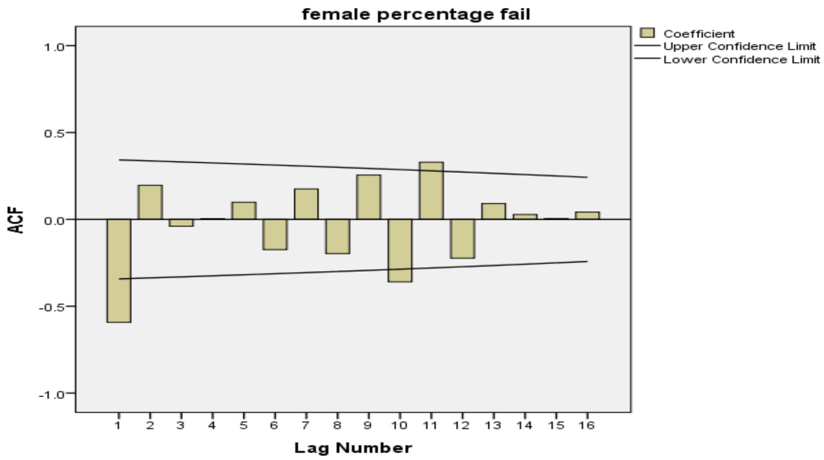
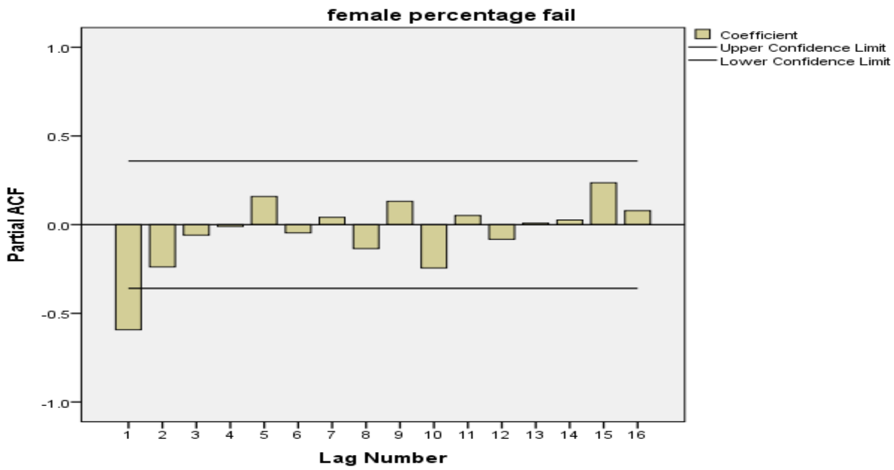


Figure 4.26: PACF of female students' percentage fail



Model selection:

The best models have been selected on the basis of the least AIC and BIC from different models as shown in table 4.11 as follows:

- ARIMA (1, 1, 1) for female percentage credit with least AIC and BIC of 1.021 and 1.830 respectively.

- ➢ ARIMA (2, 1, 1) for female percentage pass with least AIC and BIC of 1.181 and 1.856 respectively.
- ➢ ARIMA (2, 1, 2) for female percentage fail with least AIC and BIC of 1.121 and 1.279 respectively.

Table 4.11: ARIMA model for female students' percentage credit, pass and fail

Model	Female students' percent credit		Female students' percent pass		Female students' percent fail	
	AIC	BIC	AIC	BIC	AIC	BIC
ARIMA(0,1,0)	1.131	3.223	2.981	3.290	1.891	3.081
ARIMA(0,1,1)	1.811	2.342	1.572	2.374	1.494	2.260
ARIMA(0,1,2)	1.342	2.108	2.204	1.997	2.123	1.989
ARIMA(1,1,0)	1.781	2.236	1.593	2.395	1.482	2.472
ARIMA(2,1,0)	1.342	2.077	1.212	1.995	2.027	2.334
ARIMA(1,1,1)	**1.021**	**1.830**	2.205	1.952	3.122	2.047
ARIMA(2,1,1)	1.370	1.971	**1.181**	**1.856**	2.770	2.176
ARIMA(1,1,2)	1.043	1.971	2.860	2.078	2.800	2.145
ARIMA(2,1,2)	1.372	2.124	3.491	3.742	**1.121**	**1.279**

Table4.12 Model fit statistics for female students' performance

Model	Model Fit statistics		Ljung-Box Q(18)			
	Stationary R-squared	Normalized BIC	Statistics	DF	Sig.	RMSE
Female students' percentage credit	0.816	1.830	14.202	16	0.584	2.107
Female students' percentage pass	0.836	1.856	11.653	16	0.705	2.095
Female students' percentage fail	0.648	1.279	18.972	16	0.270	2.710

Estimation of parameters:

The results of maximum likelihood estimation of the parameters of the models and their significance have been obtained using IBM SPSS V-20 software and are presented in table 4.15. The results reveal that the estimates for parameters of the selected models for female students' percentage credit consist of AR(1) and MA(1) with first difference, female students' percentage pass consist of AR(2), MA(1)with first difference and female percentage fail consist of AR(2), MA(2) with first difference as shown in table 4.13.

Table 4.13: ARIMA model parameters estimates

Model description			Estimate	SE	t	Sig
Female percentage credit	constant		-0.162	0.023	-7.019	0.000
	AR	Lag 1	-0.682	0.136	-5.004	0.000
	difference		1			
	MA	Lag 1	0.995	2.659	0.374	0.002
Female percentage pass	Constant		0.136	0.019	7.012	0.001
	AR	Lag 1	-0.869	0.202	-4.307	0.000
		Lag 2	-0.264	0.201	-1.313	0.003
	difference		1			
	MA	Lag 1	0.992	1.328	0.747	0.004
Female percentage fail	constant		0.000	0.000	1.024	0.001
	AR	Lag 1	-1.524	0.187	-8.165	0.000
		Lag 2	-0.638	0.150	-4.260	0.000
	difference		1			
	MA	Lag 1	0.048	0.578	2.001	0.005
		Lag 2	0.951	0.554	3.127	0.002

Female students' percentage credit fitted model equation:

Thus considering the general equation of the model

$$\varphi(B)\nabla^{d}Y_t = \delta + \theta(B)w_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(i)}$$

Where $\varphi(B)$ and $\theta(B)$ are the regular autoregressive and moving average, ∇ is the difference linear operator, (d=1) is the ordinary differencing, $\nabla \mathrm{Y_t} = (1 - \mathrm{B})\mathrm{Y_t}$ where B is the back shift operator, δ is the constant value

Equation (i) can written as

$$Y_t = \delta + Y_{t-1} + \varphi_1(Y_{t-1} - Y_{t-2}) + \theta_1 w_{t-1} + w_t \ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}$$

But $\delta = constant\ value(-0.162)\theta_1 = 0.995\varphi_1 = -0.682$

$$\boldsymbol{Y_t = -0.162 + Y_{t-1} - 0.682(Y_{t-1} - Y_{t-2}) + 0.995w_{t-1} + w_t}$$

Femalestudents' percentage pass fitted model equation:

From general equation of non seasonal

$\varphi(B)\nabla^d Y_t = \delta + \theta(B)w_t$………………… (iii)

$$Y_t = \delta + Y_{t-1} - \varphi_1(Y_{t-1} - Y_{t-2}) + \varphi_2(Y_{t-2} - Y_{t-3}) + \theta_1 w_{t-1} + w_t$$

Where $\delta = 0.136$ $\varphi_1 = -0.869$ $\varphi_2 = -0.264$ $\theta_1 = 0.992$

Hence equation (iii) can be written as

$$\boldsymbol{Y_t = 0.136 + Y_{t-1} + 0.869(Y_{t-1} - Y_{t-2}) - 0.264(Y_{t-2} - Y_{t-3}) + 0.992w_{t-1} + w_t}$$

Femalestudents'percentage fail fitted model equation

$$\varphi(B)\nabla^d Y_t = \delta + \theta(B)w_t$$

$Y_t = \delta + Y_{t-1} - \varphi_1(Y_{t-1} - Y_{t-2}) + \varphi_2(Y_{t-2} - Y_{t-3}) + \theta_1 w_{t-1} + \theta_2 w_{t-2} + w_t$…(iv)

Where$\delta = 0.00$, $\varphi_1 = -1.524$, $\varphi_2 = -0.638$, $\theta_1 = 0.048$, $\theta_2 = 0.951$

Hence equation (iv) can be written as

$$\boldsymbol{Y_t = Y_{t-1} + 1.524(Y_{t-1} - Y_{t-2}) - 0.638(Y_{t-2} - Y_{t-3}) + 0.048w_{t-1} + 0.951w_{t-2} + w_t}$$

Diagnostic checking:

Model diagnostic checking of the fitted models has been performed through residual analysis to determine the adequacy of the model or to indicate potentials improvement.

Diagnostic checking for female students' percentage credit:

The diagnostic check is a procedure that is used to check residuals. The residual should fulfill the models assumption of being independent and normally distributed. The residual plots of ACF and PACF against lags have been presented in figure 4.27 and 4.28. The figures display that the residuals from ARIMA (1, 1, 1) fitted model for the female students' percentage credit are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilks' test, which have been presented in table 4.14. It can be concluded that the residuals are normally distributed as the p values are greater than 0.05 for Kolmogorov-Smirnov (0.107>0.05) andfor Shapiro-Wilk's test (0.151>0.05).

Figure 4.27: ACF of female students' percentage credit residuals

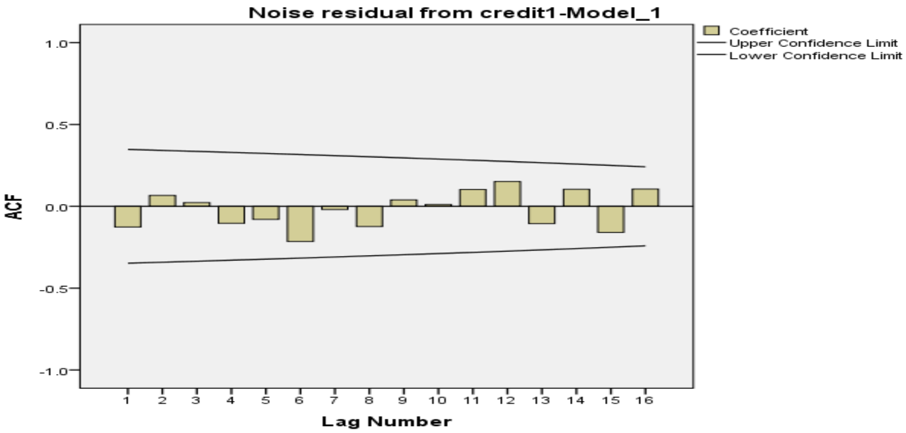


Figure 4.28: PACF of female students' percentage credit residuals

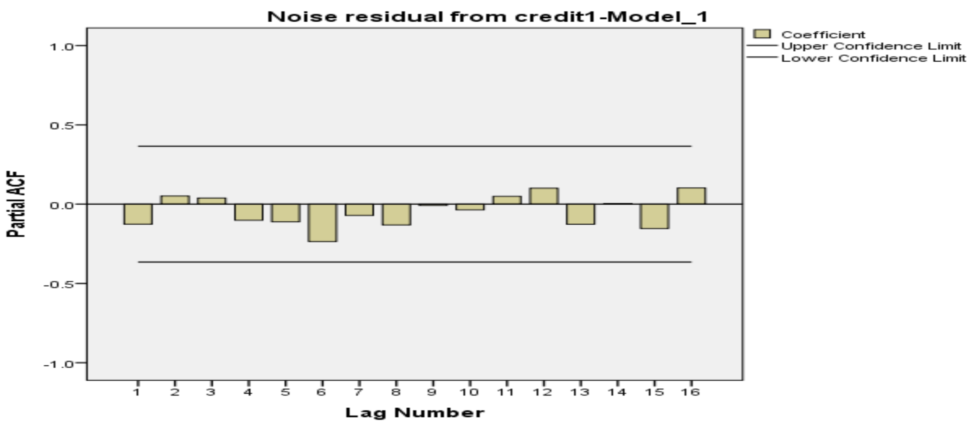


Table 4.14: Normality test of female students 'residuals for percentage credit

	Kolmogorov-Smirnov			Shapiro-Wilk's		
	Statistic	df	Sig.	Statistic	df	Sig.
Noise residual from credit	0.145	30	0.107	0.931	30	0.151

Diagnostic checking for female students' percentage pass:

The residual plots of ACF and PACF against lags have been presented in figure 4.29 and 4.30. The figures display that the residuals from ARIMA (2, 1, 1) fitted model for the female students' percentage pass are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilk test, which have been presented in table 4.15. It can be concluded that the residuals are normally distributed as the p values are greater than 0.05 for Kolmogorov-Smirnov (0.123>0.05) and for Shapiro-Wilk test (0.143>0.05).

Figure 4.29: ACF of female students' percentage pass residuals

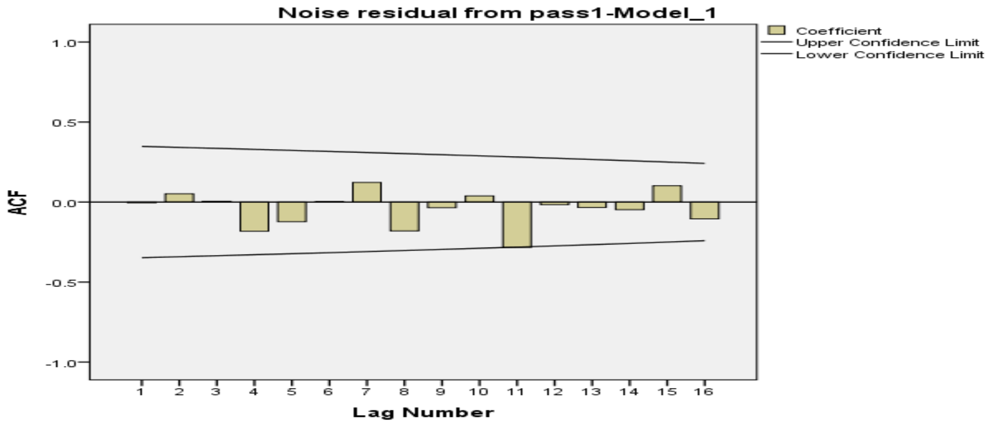


Figure 4.30: PACF of female students' percentage pass residuals

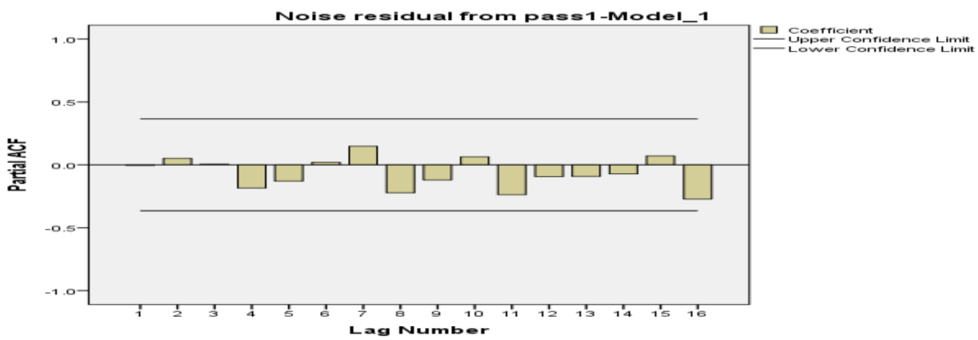


Table 4.15: Normality test of female residual for percentage pass

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Noise residual from female student's percentage pass	0.205	30	0.123	0.947	30	0.143

Diagnostic checking for female students' percentage fail:

The residual plots of ACF and PACF against lags have been presented in figure 4.31 and 4.32. The figures display that the residuals from ARIMA (2, 1, 2) fitted model for the female students' percentage fail are not far from zero. The results are supported by using Kolmogorov-Smirnov and Shapiro-Wilk test, which have been presented in table 4.16. It can be concluded that the residuals are normally distributed as the p values are greater than 0.05 for Kolmogorov-Smirnov (0.251>0.05) and for Shapiro-Wilk's test (0.872 >0.05).

Figure 4.31: ACF of female students' percentage fail residuals

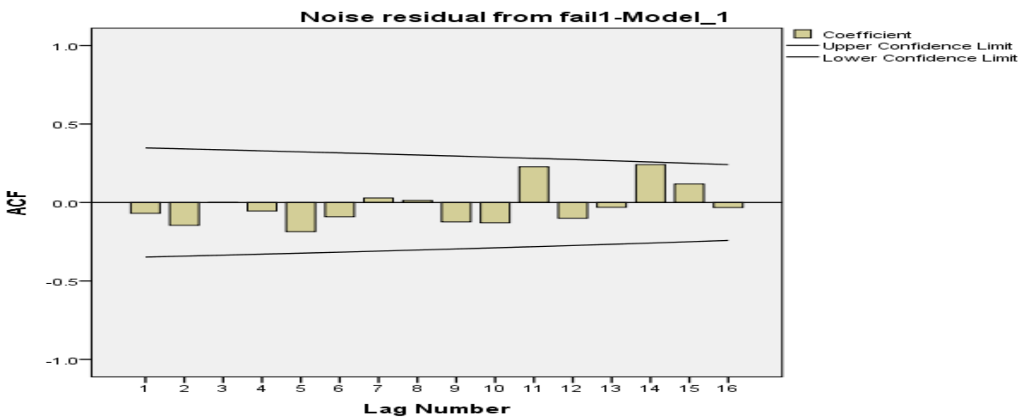


Figure 4.32: PACF of female students' percentage fail residuals

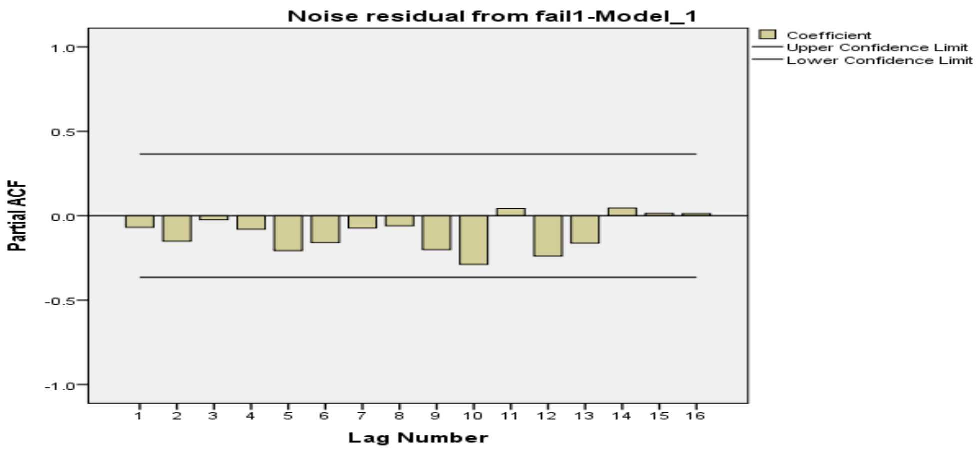


Table 4.16: Normality test of female residual for percentage fail

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Residual from female percentage fail	0.065	30	0.251	0.982	30	0.872

Forecasting for female students' performance:

The table 4.17 presents the forecasted values for femalestudents' performance according to percentage credit, percentage pass and percentage fail from 2017-2022. On the basis of best ARIMA model forecast, it is found that in next 6 years the performance of female students in mathematics in secondary school examination in Zanzibar will lie between 10.22 % in (2017) to 19.42% in (2022) for credit pass, 24.76% in (2017) to 30.32% in (2021) for percentage pass and 58.5% in (2020) to 61.91 % in (2021) for percentage fail.

Table 4.17: Forecasted performance of female students on the basis of fitted models

Year	Percentage credit ARIMA(1,1, 1)	percentage pass ARIMA(2, 1, 1)	Percentage fail ARIMA(2, 1,2)
2017	10.22	24.76	60.32
2018	12.36	25.04	59.54
2019	14.06	25.90	60.91
2020	17.69	28.04	58.50
2021	16.43	30.32	61.91
2022	19.42	29.81	61.50

Forecast error analysis:

The forecast error analysis for female students' percentage credit, percentage pass and percentage fail has been done and the results are presented in table 4.18. It can be seen from the results that mean absolute percentage error for percentage credit, percentage pass and percentage fail are respectively 12.051, 8.985 and 10.33.This shows the accuracy of the forecast as 87.949, 91.015 and 89.67 respectively.

Table 4.18: forecast error analysis

	Percentage credit	Percentage pass	Percentage fail
Cumulative forecast	0.491	0.255	0.606
Mean error	0.378	0.305	0.573
Mean square error	0.4239	0.282	0.5714
Root mean sq. error	0.6418	0.3037	0.5952
Mean absolute deviation	0.5947	0.2937	0.58767
Mean abs. percent. error	12.051	8.985	10.33

CONCLUSION

The following findings have been obtained from the research regarding the performance of male students and female students in mathematics in secondary school examination in Zanzibar on the basis of analysis of data from 1986 to 2016.

Male students' performance:

The pattern of 31 years of data in respect of performance of male students in mathematics in secondary school examination in Zanzibar shows that the mean percentage with credit is 5.80, mean percentage pass is 25.90 and mean percentage fail is 59.05 over the years 1986-2016. Various models have been fitted by Box Jenkins method to the data of performance of students in mathematics in secondary school examination in Zanzibar and among them the following models have been selected as the best fitted model by using two model information criterions, namely Aikaike Information Criterion and Bayesian Information Criterion:

- ARIMA (1, 1, 2) for male percentage credit
- ARIMA (2, 1, 0) for male percentage pass
- ARIMA (1, 1, 0) for male percentage fail.

The parameters of these models have been fitted by maximum likelihood estimation procedure and the equations for each model have been evaluated. Forecast analysis has been done on the basis of best ARIMA model .It has been found that in next 6 years the performance of male students in mathematics in secondary school examination will lie between 12.31 % in (2017) to 21.51% in (2022) for credit pass, 27.94% in (2017) to 33.50% in (2021) for percentage pass and 56.74% in (2020) to 60.15% in (2021) for percentage fail. Forecast error analysis has also been done and by using mean absolute percentage error, it is concluded that the accuracy of these forecasts are 88.27 %, 91.25 % and 90.23 %respectively for percentage credits, percentage pass and percentage fail of male students.

Female students' performance:

The pattern of 31 years of data in respect of performance of female students in mathematics in secondary school examination in Zanzibar shows that the mean percentage with is 12.21, mean percentage pass is 25.08 and mean percentage fail is 62.70 over the years 1986-2016.Various models have been fitted by Box Jenkins method to the data of performance of female students in mathematics in secondary school examination in Zanzibar and among them the following models have been selected as the best fitted model by using two model information criterions, namely Aikaike Information Criterion and Bayesian Information Criterion:

- ARIMA (1, 1, 1) for female percentage credit
- ARIMA (2, 1, 1) for female percentage pass
- ARIMA (2, 1, 2) for female percentage fail.

The parameters of these models have been fitted by maximum likelihood estimation procedure and the equations for each model have been evaluated. Forecast analysis has been done on the basis of best ARIMA model chosen.It has been found that in next 6 years the performance of female students in mathematics in secondary school examination will lie between 10.22 % in (2017) to 19.42% in (2022) for credit pass,24.76% in (2017) to 30.32% in (2021) for percentage pass and 58.5% in (2020) to 61.91 % in (2021) for percentage fail. Forecast error analysis has also been done and by using mean absolute percentage error, it is concluded that the accuracy of these forecasts are as 87.949%, 91.015% and 89.67 %respectively for percentage credits, percentage pass and percentage fail of male students.

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