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INEQUALITIES INVOLVING H–FUNCTION OF TWO VARIABLES

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ABSTRACT

The aim of this paper is to establish some new inequalitiesinvolving H–function of two variables.

Keywords:H–function of two variables, Inequality.

1. Introduction:

Recently Mittal and Gupta [2, p. 117] has given the following notation of the H-function of two variables as:

$$H^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$

$$= \frac{-1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}\,d\xi d\eta \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\prod_{j=n_1+1}^{p_1}\Gamma(a_j-\alpha_j\xi-A_j\eta)\prod_{j=1}^{q_1}\Gamma(1-b_j+\beta_j\xi+B_j\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_2}\Gamma(1-c_j+\gamma_j\xi)}{\prod_{j=m_2+1}^{q_2}\Gamma(1-d_j+\delta_j\xi)\prod_{j=n_2+1}^{p_2}\Gamma(c_j-\gamma_j\xi)}$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_3}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_3}\Gamma(1-e_j+E_j\eta)}{\prod_{j=m_3+1}^{q_3}\Gamma(1-f_j+F_j\eta)\prod_{j=n_3+1}^{p_3}\Gamma(e_j-E_j\eta)}$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, $(i = 1, 2, 3; j = 2, 3)$. Also, all the A's, α's, B's, β's, γ's, δ's, E's, and F's are assumed to the positive quantities for standardization purpose.

The contour L_1 is in the ξ-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \delta_j\xi)$ (j = 1, ..., m_2) lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ (j = 1, ..., n_2), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The contour L_2 is in the η-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ (j = 1, ..., m_3) lie to the right, and the poles of $\Gamma(1 - e_j + E_j\eta)$ (j = 1, ..., n_3), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The function, defined by (1), is analytic function of x and y if

$R = \sum_{j=1}^{p_1} \alpha_j + \sum_{j=1}^{p_2} \gamma_j - \sum_{j=1}^{q_1} \beta_j - \sum_{j=1}^{q_2} \delta_j < 0,$

$R = \sum_{j=1}^{p_1} A_j + \sum_{j=1}^{p_3} F_j - \sum_{j=1}^{q_1} B_j - \sum_{j=1}^{q_3} F_j < 0,$

The H-function of two variables given by (1) is convergent if

$$U = -\sum_{j=n_1+1}^{p_1} \alpha_j - \sum_{j=1}^{q_1} \beta_j + \sum_{j=1}^{m_2} \delta_j - \sum_{j=m_2+1}^{q_2} \delta_j + \sum_{j=1}^{n_2} \gamma_j - \sum_{j=n_2+1}^{p_2} \gamma_j > 0, \quad (2)$$

$$U = -\sum_{j=n_1+1}^{p_1} A_j - \sum_{j=1}^{q_1} B_j + \sum_{j=1}^{m_3} F_j - \sum_{j=m_3+1}^{q_3} F_j + \sum_{j=1}^{n_3} E_j - \sum_{j=n_3+1}^{p_3} E_j > 0, \quad (3)$$

and $|\arg x| < \frac{1}{2} U\pi$, $|\arg y| < \frac{1}{2} V\pi$.

2. Result Required:

The following result is required in our present investigation:

From L. Lew, J. Frauenthal and N. Keyfitz [1]:

$$2\Gamma\left(n+\frac{1}{2}\right) \le \Gamma(\tfrac{1}{2})\Gamma(n+1) \le 2^n\Gamma\left(n+\frac{1}{2}\right). \quad (4)$$

3. Main Result:

In this paper we will establish the following inequalities:

$$2H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(\frac{1}{2}-n,u),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$
$$\le \Gamma(\frac{1}{2})H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(-n,u),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$
$$\le 2^n H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2;p_3,q_3}\left[\begin{matrix}x2^u\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(\frac{1}{2}-n,u),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right], \quad (5)$$

provided that $n \ge 1, u > 0$, $|\arg x| < \frac{1}{2} U\pi$, $|\arg y| < \frac{1}{2} V\pi$, where U and V are given in (2) and (3) respectively.

$$2H^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(\frac{1}{2}+n,u),(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$
$$\le \Gamma(\frac{1}{2})H^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(1+n,u),(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right]$$
$$\le 2^n H^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}\left[\begin{matrix}x2^{-u}\\y\end{matrix}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1}:(\frac{1}{2}+n,u),(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}\end{matrix}\right], \quad (6)$$

provided that $n \geq 1, u > 0$,|arg x| < ½ Uπ, |arg y| < ½ Vπ, where U and V are given in (2) and (3) respectively.

Proof:

To prove (5), expressing the H-function on the left-hand side as Mellin-Barnes type integral (1), we have

$$= \frac{(-1)}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\{2\Gamma\left(\frac{1}{2}+n+u\xi\right)\}x^{\xi}y^{\eta}\,d\xi d\eta$$

Now using the inequality (4) and interpreting the result with the help of (1), we obtain the result (5).

Similarly (6), can easily established.

4. Special Cases:

On specializing the parameters in main results, we get following identities in terms of H-function of one variable:

$$2H^{m,n+1}_{p+1,q}[x|^{\left(\frac{1}{2}-n,u\right),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}]$$

$$\leq \Gamma(\frac{1}{2})H^{m,n+1}_{p+1,q}[x|^{(-n,u),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}]$$

$$\leq 2^n H^{m,n+1}_{p+1,q}[x2^u|^{\left(\frac{1}{2}-n,u\right),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}], \qquad (7)$$

provided that $n \geq 1, u > 0$,|argx|<½πA, where A is given by $\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv A > 0$.

$$2H^{m+1,n}_{p,q+1}[x|^{(a_j,\alpha_j)_{1,p}}_{\left(\frac{1}{2}+n,u\right),(b_j,\beta_j)_{1,q}}]$$

$$\leq \Gamma(\frac{1}{2})H^{m+1,n}_{p,q+1}[x|^{(a_j,\alpha_j)_{1,p}}_{(1+n,u),(b_j,\beta_j)_{1,q}}]$$

$$\leq 2^n H^{m+1,n}_{p,q+1}[x2^{-u}|^{(a_j,\alpha_j)_{1,p}}_{\left(\frac{1}{2}+n,u\right),(b_j,\beta_j)_{1,q}}], \qquad (8)$$

provided that $n \geq 1, u > 0$,|argx|<½πA, where A is given by $\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv A > 0$.

References

1. L. Lew, J. Frauenthal and N. Keyfitz: On the average distances in a circular disc in Mathematical Modeling, Classroom notes in applied mathematics, Philadelphia, SIAM, 1987.
2. Mittal, P. K. and Gupta Gupta, K. C.: An integral involving generalized function of two variables, Proc. Indian Acad. Sci., 75 A, p. 117-123.