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EXCEL AUTOMATION AND SENSITIVITY ANALYSES OF OPTIMAL MACHINE REPLACEMENT STRATEGIES USING DYNAMIC PROGRAMMING RECURSIONS: A CASE STUDY OF NASCO HOUSEHOLD PRODUCTS LTD., JOS.

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ABSTRACT

This research article is a sequel to an earlier work, by the above authors, which obtained the optimal economic value that might accrue to the industrial system due to machine replacement decisions with respect to packing machines at Nasco Household Products Limited, Jos. The current investigation used the Excel platform, the established results regarding the structure of the sets of feasible machine ages corresponding to the various decision periods, in machine replacement problems, and backward dynamic programming recursions, to obtain the optimal machine replacement strategies corresponding to any desired set of feasible machine starting ages, in one fell-swoop, using only a single solution template outputs. The outputs demonstrated consistency with the manual solutions. The work went further to perform sensitivity analyses on the machine pricing; the results were quite revealing, with the optimal replacement policies encapsulated in the age - transition diagrams. The sets of feasible machine ages provided the needed inputs for the appropriate solution implementation templates. The templates circumvented the inherent cumbersome computations associated with Dynamic programming formulations.

1. INTRODUCTION

The equipment replacement optimization (ERO) is an important topic in operations research, applied mathematics, statistics, economics, industrial engineering and management science. Items which are under constant usage need replacement at appropriate times, as the efficiency of the operating system that uses such items deteriorates, thus resulting in rising operating and maintenance (O&M) costs and decreasing salvage values Taha [1].

The equipment replacement optimization (ERO) problem has been studied by a lot of researchers Bellman [2] and there has been an enormous amount of research on the ERO with finite age horizon using the Deterministic Dynamic Programming (DDP) approach (Hartman & Murphy [3]; Hillier & Liberman [4]). However, it should be noted that almost all the previous research efforts have been devoted to the (DDP) solution formulation and its limited applications to extremely simplified case study/or examples. To the best of our knowledge, there have been no research efforts made so far (except Fan et al. [5][6]) on the application of such DP approaches to solving real-life ERO problems. The DP approach will undoubtedly be the preferred approach to solving the ERO problem because it can explicitly consider the uncertainty in the machine and the annual O&M cost accordingly. Meyer [7] perhaps due to computational constraints, is one among the very few to study the ERO problem technology.

As a case study of machine replacement optimization (ERO) problems, the industrial management needs to determine the optimal revenue that may be accrued in their industry from a packing machine within a ten-year period and to take a decision to either keep or replace the machine. The current investigation included reward functionals that are more helpful in the industry because it uses inputs such as, equipment purchase price, revenue, and salvage value. These functionals are in the form of dynamic programming recursions which are used appropriately to make effective decisions.The study aims at assessing the economic value that an industrial system may accrue due to packing machine replacement, using the Machine Replacement Dynamic Programming (ERDP) model as a decision making tool. This aim shall be achieved by: ascertaining the optimal decision making policy of 'Keeping or Replacing' a machine within ten-year period of its lifetime; calculating the maximum net income that is gained by the industrial system through a 'keep or Replace' decision on packing machine within the ten-year period; determining the most economic age of a packing machine in the industrial system; and exploiting the Excel solution templates of Ukwu [8][9][10] and [11] in computing optimal replacement decisions and associated rewards in Nasco Household Product Ltd. Jos.

2. MATERIAL AND METHODS

The problem data, working definitions, elements of the DP model and the dynamic programming (DP) recursions are laid out as follows:

Equipment Starting age $= t_1$

Equipment Replacement age $= m$

S_i = The set of feasible equipment ages (states) in decision period i (say year i), $i \in \{1, 2, \ldots, n\}$

$s(t)$ = salvage value of a t − year old equipment; $t = 0, 1, \ldots, m$

I = fixed cost of acquiring a new equipment in any year

2.1 The Elements of the DP are the Following:

1. Stage i, represented by year i, $i \in \{1, 2, \ldots, n\}$
2. The alternatives at stage (year) i. These call for keeping or replacing the equipment at the beginning of year i
3. The state at stage (year) i, represented by the age of the equipment at the beginning of year i.

Let $f_i(t)$ be the maximum net income for years $i, i+1, \ldots, n-1, n$ given that the equipment is t years old at the beginning of year i.

Note: The definition of $f_i(t)$ starting from year i to year n implies that backward recursion will be used. Forward recursion would start from year 1 to year i.

The following theorem is applicable to the backward recursive procedure:

2.2 Theorem 1: Theorem on Dynamic Programming (DP) Recursion

$$f_i(t) = \max \begin{cases} r(t) - c(t) + f_{i+1}(t+1); \text{ IF KEEP} \\ r(0) + s(t) - I - c(0) + f_{i+1}(1); \text{ REPLACE} \end{cases}$$

$$f_{n+1}(x) = s(x), \quad i = 0, 1, \ldots, n-1, \quad x = \text{age of machine at the start of period } n+1$$

2.2.1 Pertinent Remarks on the DP Recursions

For $i \in \{1, 2, \cdots, n\}$, $f_i(t)$ may be identified as $f_i(t) = \max_{\{K,R\}} \{f_i^K(t), f_i^R(t)\}$, where

$f_i^K(t) = r(t) - c(t) + f_{i+1}(t+1)$ and $f_i^R(t) = r(0) + s(t) - I - c(0) + f_{i+1}(1)$

For $i \in \{1, 2, \cdots, n\}$ and $t \in S_i$, the optimal decision may be identified as $D_i(t)$, where

$$D_i(t) = \underset{\{K,R\}}{\operatorname{argmax}}\, g_i(t, K, R);\ g_i(t, K, R) = \begin{cases} f_i^K(t), & \text{if Decision is KEEP} \\ f_i^R(t), & \text{if Decision is REPLACE} \end{cases}$$

Define

$$x_i = \begin{cases} 1, & \text{if decision is REPLACE in stage } i \text{ (start of decision year } i) \\ 0, & \text{if decision is KEEP in stage } i \text{ (start of decision year } i) \end{cases}$$

Then

$$g_i(t, K, R) = f_i(t) = (1 - x_i) f_i^K(t) + x_i f_i^R(t),\ i \in \{1, 2, \cdots, n\}$$

2.3 Theorem 2: Theorem on Analytic Determination of the Set of Feasible Ages at Each Stage

Let S_i denote the set of feasible equipment ages at the start of the decision year i. Let t_1 denote the age of the machine at the start of the decision year i, that is, $S_1 = \{t_1\}$. Then for $i \in \{1, 2, \ldots, n\}$,

$$S_i = \begin{cases} \left\{\min\limits_{2 \le j \le i}\{j-1, m\}\right\} \cup \left\{1 + (i - 2 + t_1)\operatorname{sgn}\left(\max\{m + 2 - t_1 - i, 0\}\right)\right\}, & \text{if } t_1 < m \\ \left\{\min\limits_{2 \le j \le i}\{j-1, m\}\right\}, & \text{if } t_1 \ge m \end{cases}$$

2.3.1 Corollary 1

$S_1 = \{t_1\}$. If $m \le n$ and $t_1 \in \{0,1\}$, then for $i \in \{2, \ldots, n\}$,

$$S_i = \left\{\min_{2 \le j \le i+1}\{j-1, m\}\right\}$$

2.3.2 Corollary2

If $t_1 < m$ and $m > n$, then for $i \in \{2, \ldots, n\}$,

$$S_i = \begin{cases} \{1, \ldots, i-1, i-1+t_1\}, & \text{if } i \le m + 1 - t_1 \\ \{1, \ldots, i-1\}, & \text{if } i > m + 1 - t_1 \end{cases}$$

2.3.3 Corollary 3

If the mandatory replacement age restriction is waived, then

$$S_i = \{1, 2, \cdots, i-1, i-1+t_1\}, \; i \in \{2, \cdots, n\}.$$

3. RESULTS AND DISCUSSION

3.1 Case Study Application and the Implementation of the Solution Templates

NASCO HOUSEHOLD PRODUCTS LTD. JOS needs to determine the optimal replacement policy for a Pakona (pk70 packing) machineover the next ten years. The following table gives the pertinent data for the problem. The company does not prescribe any mandatory replacement age. The cost of a new Pakona (pk70 packing) machine is ₦8,608,000.

TABLE 1: Pertinent Data for Optimal Policy and Reward Determination

Age: t yrs.	Revenue: $r(t)$ (₦)	Operating cost: $c(t)$	Salvage value: $s(t)$
0	2,330,000	240,000	-
1	2,320,000	253,000	8,177,600
2	2,210,000	257,000	7,768,720
3	2,090,000	272,000	7,380,284
4	1,895,000	274,000	7,011,269
5	1,770,000	301,000	6,310,142
6	1,720,000	311,000	5,679,127
7	1,655,000	361,000	5,111,215
8	1,590,000	396,000	4,600,093
9	1,345,000	403,000	3,910,079
10	1,029,000	415,000	3,323,567

The above problem is solved in one fell-swoop, for $t_1 \in \{0, 1, \cdots, 8\}$, Theorem 2 using the dynamic programming of Theorem 2, the structure of the feasible ages in Corollary 3, and the solution template developed on Microsoft Excel platform, in [11]. Furthermore, sensitivity analyses on the purchase price is carried out using the Excel solution templates. The template outputs are organized in groups of stages of cardinality at most six, so that each group can fit onto a page, in portrait style.

Figure 1: Template Outputs for the Optimal Strategies and Rewards for Stages 17 to 12 using index zero batch starting ages.

Equipment Replacement Problem Solution Template					*n*	**Batch Index**					
Replacement Age =			99999	**years**	17	0					
	Given Data			**Stage**	17						
	***I* =**	8608000	***V(0) = r(0)-c(0) - I =***		-6518000						
Age *t* (yrs.)	0	1	2	3	4	5	6	7	8	9	10
Revenue: *r(t)* ($)	2330000	2320000	2210000	2090000	1895000	1770000	1720000	1655000	1590000	1345000	1029000
Mnt. cost, *c(t)* ($)	240000	253000	257000	272000	274000	301000	311000	361000	396000	403000	415000
Salvage value, *s* (t)		8177600	7768720	7380284	7011269	6310142	5679127	5111215	4600093	3910079	3323567
K		9835720	9333284	8829269	7931142	7148127	6520215	5894093	5104079	4265567	614000
R		9837200	9428320	9039884	8670869	7969742	7338727	6770815	6259693	5569679	4983167
Opt. value: *f* (t)		9837200	9428320	9039884	8670869	7969742	7338727	6770815	6259693	5569679	4983167
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	16						
K		11495320	10992884	10488869	9590742	8807727	8179815	7553693	6763679	5925167	614000
R		11496800	11087920	10699484	10330469	9629342	8998327	8430415	7919293	7229279	6642767
Opt. value: *f* (t)		11496800	11087920	10699484	10330469	9629342	8998327	8430415	7919293	7229279	6642767
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	15						
K		13154920	12652484	12148469	11250342	10467327	9839415	9213293	8423279	7584767	614000
R		13156400	12747520	12359084	11990069	11288942	10657927	10090015	9578893	8888879	8302367
Opt. value: *f(t)*		13156400	12747520	12359084	11990069	11288942	10657927	10090015	9578893	8888879	8302367
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	14						
K		14814520	14312084	13808069	12909942	12126927	11499015	10872893	10082879	9244367	614000
R		14816000	14407120	14018684	13649669	12948542	12317527	11749615	11238493	10548479	9961967
Opt. value: *f(t)*		14816000	14407120	14018684	13649669	12948542	12317527	11749615	11238493	10548479	9961967
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	13						
K		16474120	15971684	15467669	14569542	13786527	13158615	12532493	11742479	10903967	614000
R		16475600	16066720	15678284	15309269	14608142	13977127	13409215	12898093	12208079	11621567
Opt. value: *f(t)*		16475600	16066720	15678284	15309269	14608142	13977127	13409215	12898093	12208079	11621567
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	12						
K		18133720	17631284	17127269	16229142	15446127	14818215	14192093	13402079	12563567	614000
R		18135200	17726320	17337884	16968869	16267742	15636727	15068815	14557693	13867679	13281167
Opt. value: *f(t)*		18135200	17726320	17337884	16968869	16267742	15636727	15068815	14557693	13867679	13281167
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10

Figure 2: Template Outputs for the Optimal Strategies and Rewards for Stages 11 to 6 using index zero batch starting ages.

				Stage	11						
K		19793320	19290884	18786869	17888742	17105727	16477815	15851693	15061679	14223167	614000
R		19794800	19385920	18997484	18628469	17927342	17296327	16728415	16217293	15527279	14940767
Opt. value: *f(t)*		19794800	19385920	18997484	18628469	17927342	17296327	16728415	16217293	15527279	14940767
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	10						
K		21452920	20950484	20446469	19548342	18765327	18137415	17511293	16721279	15882767	
R		21454400	21045520	20657084	20288069	19586942	18955927	18388015	17876893	17186879	
Opt. value: *f(t)*		21454400	21045520	20657084	20288069	19586942	18955927	18388015	17876893	17186879	
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	
State		1	2	3	4	5	6	7	8	9	
				Stage	9						
K		23112520	22610084	22106069	21207942	20424927	19797015	19170893	18380879		
R		23114000	22705120	22316684	21947669	21246542	20615527	20047615	19536493		
Opt. value: *f(t)*		23114000	22705120	22316684	21947669	21246542	20615527	20047615	19536493		
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*		
State		1	2	3	4	5	6	7	8		
				Stage	8						
K		24772120	24269684	23765669	22867542	22084527	21456615	20830493			
R		24773600	24364720	23976284	23607269	22906142	22275127	21707215			
Opt. value: *f(t)*		24773600	24364720	23976284	23607269	22906142	22275127	21707215			
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*			
State		1	2	3	4	5	6	7			
				Stage	7						
K		26431720	25929284	25425269	24527142	23744127	23116215				
R		26433200	26024320	25635884	25266869	24565742	23934727				
Opt. value: *f(t)*		26433200	26024320	25635884	25266869	24565742	23934727				
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*				
State		1	2	3	4	5	6				
				Stage	6						
K		28091320	27588884	27084869	26186742	25403727					
R		28092800	27683920	27295484	26926469	26225342					
Opt. value: *f(t)*		28092800	27683920	27295484	26926469	26225342					
Opt. Decision		*R*	*R*	*R*	*R*	*R*					
State		1	2	3	4	5					

Figure 3: Template Outputs for the Optimal Strategies and Rewards for Stages 5 to 1 using index zero batch starting ages.

				Stage	5						
K		29750920	29248484	28744469	27846342						
R		29752400	29343520	28955084	28586069						
Opt. value: *f(t)*		29752400	29343520	28955084	28586069						
Opt. Decision		*R*	*R*	*R*	*R*						
State		1	2	3	4						
				Stage	4						
K		31410520	30908084	30404069							
R		31412000	31003120	30614684							
Opt. value: *f(t)*		31412000	31003120	30614684							
Opt. Decision		*R*	*R*	*R*							
State		1	2	3							
				Stage	3						
K		33070120	32567684								
R		33071600	32662720								
Opt. value: *f(t)*		33071600	32662720								
Opt. Decision		*R*	*R*								
State		1	2								
				Stage	2						
K		34729720									
R		34731200									
Opt. value: *f(t)*		34731200									
Opt. Decision		*R*									
State		1									
				Stage	1						
K	36821200										
R	28213200										
Opt. value: *f(t)*	36821200										
Opt. Decision	*K*										
State	0										

3.2 Determination of the Optimal Replacement Policies for Feasible Starting Ages

Let n_2 denote the extended horizon length for the n-horizon problem. For $t \in S_i$, $i \in \{1, 2, \cdots, n_2\}$, let $\hat{f}_i(t, q)$ denote the optimal reward for the q-stage (q-horizon) problem. Then

$$f_1(0) = \hat{f}_1(0, n_2) - \hat{f}_2(1, n_2) + \hat{f}_{2+n_2-n}(1, n_2),$$

$$f_i(t) = \hat{f}_{i+n_2-n}(t, n_2), t \in \{1, 2, \cdots, \min\{n, i-1+n_2-n\}\}$$

$$\Rightarrow f_i(t) = \begin{cases} \hat{f}_{i+n_2-n}(t, n_2), t \in \{1, 2, \cdots, i-1+n_2-n\}, i \leq 1+2n-n_2 \\ \hat{f}_{i+n_2-n}(t, n_2), t \in \{1, 2, \cdots, n\}, i \geq 1+2n-n_2 \end{cases}$$

Starting Age 0

$n = 10, n_2 = 17 \Rightarrow f_1(0) = \hat{f}_1(0, n_2) - \hat{f}_2(1, n_2) + \hat{f}_{2+n_2-n}(1, n_2)$

$= \hat{f}_1(0,17) - \hat{f}_2(1,17) + \hat{f}_9(1,17) = 36821200 - 34731200 + 23114000 = 25,204,000$

3.2.1 Age transition diagram for the determination of the optimal policy prescriptions and the corresponding returns:

$$0K1R1R1R1R1R1R1R1R1R1S$$

$f_1(0) =$ (The maximum net income from years 1 to 10) = ₦ 25,240,000

Note that in general, there are $2(n+1)$ concatenated age and decision symbols corresponding to the horizon length n. The above diagrams are consistent with $n=10$.

Interpretation

Start with a new Pakona (pk70 packing) machine; keep the machine for the next one year, and then replace each resulting 1-year machine at the beginning of the succeeding year. Sell off the last replacement machine at the end of the 10-year period.

3.2.2 Set of Starting Ages {1, 2,…, 7}

The optimal replacement policy prescriptions and returns corresponding to the above set of starting are determined in one fell-swoop from stage 8 to 17 – a total of 10 stages starting from the top (stage 17) and are encapsulated in the age-transition diagrams starting from stage 8 of the extended 17-stage problem. In general, for an n-stage process with extended horizon length n_2, the optimal returns and age-transition diagrams are secured from stage $1+n_2-n$ to n_2. In particular, the optimal returns and initial decision corresponding to $n \in \{10, 11, \cdots, 16\}$ are summarized as follows, with the (last state − 1) corresponding to the starting stage:

Table 2: ***n* = 10**

Opt. value: *f(t)*		24773600	24364720	23976284	23607269	22906142	22275127	21707215
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7

Table 3: ***n* = 11**

Opt. value: *f(t)*		26433200	26024320	25635884	25266869	24565742	23934727
Opt. Decision		*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6

Table 4: *n* = 12

Opt. value: *f(t)*		28092800	27683920	27295484	26926469	26225342
Opt. Decision		*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5

Table 5: *n* = 13

Opt. value: *f(t)*		29752400	29343520	28955084	28586069
Opt. Decision		*R*	*R*	*R*	*R*
State		1	2	3	4

Table 6: *n* = 14

Opt. value: *f(t)*		31412000	31003120	30614684
Opt. Decision		*R*	*R*	*R*
State		1	2	3

Table 7: *n* = 15

Opt. value: *f(t)*		33071600	32662720
Opt. Decision		*R*	*R*
State		1	2

Table 8: *n* = 16

Opt. value: *f(t)*		34731200
Opt. Decision		*R*
State		1

3.2.3 Age transition diagram for the determination of the optimal policy prescriptions and the corresponding returns

$1R1R1R1R1R1R1R1R1R1S$

$f_1(1) =$ (The maximum net income from year 1 to 10) = ₦ 24,773,600

Interpretation for the Age Transition Diagram (The Concatenated Symbol)

Start with a 1- year old Pakona (pk70 packing) machine. Replace the packing machine every year and sell off the last Replacement machine at the end of the 10- year Replacement decision process.

The remaining Age transition diagrams can be interpreted in a similar fashion. The optimal policy is to replace the machine each year and then sell the last machine at the end of the horizon. The maximum net income for each starting year and for each horizon length is as given by **Opt. value** in the furnished summary tables above.

Price - Sensitivity Analysis, with new *I* = 9,000000.

Figure 4: Price Sensitivity Analysis: Template Outputs for the Optimal Strategies and Rewards for Extended Stages 17 to 10 using index zero batch starting ages

Equipment Replacement Problem Solution Template					***n* Extended**	***n***	**Batch Index**				
Replacement Age =			99999	**years**	17	10	0				
	Given Data			**Stage**	17						
	***I* =**	9000000	***V(0) = r(0)-c(0) - I =***		-6910000						
Age *t* (yrs.)	0	1	2	3	4	5	6	7	8	9	10
Revenue: *r(t)* ($)	2330000	2320000	2210000	2090000	1895000	1770000	1720000	1655000	1590000	1345000	1029000
Mnt. cost, *c(t)* ($)	240000	253000	257000	272000	274000	301000	311000	361000	396000	403000	415000
Salvage value, *s* (t)		8177600	7768720	7380284	7011269	6310142	5679127	5111215	4600093	3910079	3323567
K		9835720	9333284	8829269	7931142	7148127	6520215	5894093	5104079	4265567	614000
R		9445200	9036320	8647884	8278869	7577742	6946727	6378815	5867693	5177679	4591167
Opt. value: *f* (t)		9835720	9333284	8829269	8278869	7577742	6946727	6378815	5867693	5177679	4591167
Opt. Decision		*K*	*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
Applicable State		1	2	3	4	5	6	7	8	9	10
				Stage	16						
K		11400284	10782269	10096869	9198742	8415727	7787815	7161693	6371679	5533167	614000
R		11103320	10694440	10306004	9936989	9235862	8604847	8036935	7525813	6835799	6249287
Opt. value: *f* (t)		11400284	10782269	10306004	9936989	9235862	8604847	8036935	7525813	6835799	6249287
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	15						
K		12849269	12259004	11754989	10856862	10073847	9445935	8819813	8029799	7191287	614000
R		12667884	12259004	11870568	11501553	10800426	10169411	9601499	9090377	8400363	7813851
Opt. value: *f(t)*		12849269	12259004	11870568	11501553	10800426	10169411	9601499	9090377	8400363	7813851
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	14						
K		14326004	13823568	13319553	12421426	11638411	11010499	10384377	9594363	8755851	614000
R		14116869	13707989	13319553	12950538	12249411	11618396	11050484	10539362	9849348	9262836
Opt. value: *f(t)*		14326004	13823568	13319553	12950538	12249411	11618396	11050484	10539362	9849348	9262836
Opt. Decision		*K*	*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	13						
K		15890568	15272553	14768538	13870411	13087396	12459484	11833362	11043348	10204836	614000
R		15593604	15184724	14796288	14427273	13726146	13095131	12527219	12016097	11326083	10739571
Opt. value: *f(t)*		15890568	15272553	14796288	14427273	13726146	13095131	12527219	12016097	11326083	10739571
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	12						
K		17339553	16749288	16245273	15347146	14564131	13936219	13310097	12520083	11681571	614000
R		17158168	16749288	16360852	15991837	15290710	14659695	14091783	13580661	12890647	12304135
Opt. value: *f(t)*		17339553	16749288	16360852	15991837	15290710	14659695	14091783	13580661	12890647	12304135
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	11						
K		18816288	18313852	17809837	16911710	16128695	15500783	14874661	14084647	13246135	614000
R		18607153	18198273	17809837	17440822	16739695	16108680	15540768	15029646	14339632	13753120
Opt. value: *f(t)*		18816288	18313852	17809837	17440822	16739695	16108680	15540768	15029646	14339632	13753120
Opt. Decision		*K*	*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10
				Stage	10						
K		20380852	19762837	19258822	18360695	17577680	16949768	16323646	15533632	14695120	
R		20083888	19675008	19286572	18917557	18216430	17585415	17017503	16506381	15816367	
Opt. value: *f(t)*		20380852	19762837	19286572	18917557	18216430	17585415	17017503	16506381	15816367	
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	
State		1	2	3	4	5	6	7	8	9	

Figure 5: Price Sensitivity Analysis: Template Outputs for the Optimal Strategies and Rewards for Extended Stages 9 to 1 using Index Zero Batch Starting Ages.

				Stage	9						
K		21829837	21239572	20735557	19837430	19054415	18426503	17800381	17010367		
R		21648452	21239572	20851136	20482121	19780994	19149979	18582067	18070945		
Opt. value: *f(t)*		21829837	21239572	20851136	20482121	19780994	19149979	18582067	18070945		
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*		
State		1	2	3	4	5	6	7	8		
				Stage	8						
K		23306572	22804136	22300121	21401994	20618979	19991067	19364945			
R		23097437	22688557	22300121	21931106	21229979	20598964	20031052			
Opt. value: *f(t)*		23306572	22804136	22300121	21931106	21229979	20598964	20031052			
Opt. Decision		*K*	*K*	*K/R*	*R*	*R*	*R*	*R*			
State		1	2	3	4	5	6	7			
				Stage	7						
K		24871136	24253121	23749106	22850979	22067964	21440052				
R		24574172	24165292	23776856	23407841	22706714	22075699				
Opt. value: *f(t)*		24871136	24253121	23776856	23407841	22706714	22075699				
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*				
State		1	2	3	4	5	6				
				Stage	6						
K		26320121	25729856	25225841	24327714	23544699					
R		26138736	25729856	25341420	24972405	24271278					
Opt. value: *f(t)*		26320121	25729856	25341420	24972405	24271278					
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*					
State		1	2	3	4	5					
				Stage	5						
K		27796856	27294420	26790405	25892278						
R		27587721	27178841	26790405	26421390						
Opt. value: *f(t)*		27796856	27294420	26790405	26421390						
Opt. Decision		*K*	*K*	*K/R*	*R*						
State		1	2	3	4						
				Stage	4						
K		29361420	28743405	28239390							
R		29064456	28655576	28267140							
Opt. value: *f(t)*		29361420	28743405	28267140							
Opt. Decision		*K*	*K*	*R*							
State		1	2	3							
				Stage	3						
K		30810405	30220140								
R		30629020	30220140								
Opt. value: *f(t)*		30810405	30220140								
Opt. Decision		*K*	*K/R*								
State		1	2								
				Stage	2						
K		32287140									
R		32078005									
Opt. value: *f(t)*		32287140									
Opt. Decision		*K*									
State		1									
				Stage	1						
K	34377140										
R	25377140										
Opt. value: *f(t)*	34377140										
Opt. Decision	*K*										
State	0										

3.2.4 Determination of the Optimal Replacement Policies for Feasible Starting Ages

Starting Age 0

$$I = 9,000,000,\ n = 10, n_2 = 17 \Rightarrow f_1(0) = \hat{f}_1(0, n_2) - \hat{f}_2(1, n_2) + \hat{f}_{2+n_2-n}(1, n_2)$$

$$= \hat{f}_1(0,17) - \hat{f}_2(1,17) + \hat{f}_9(1,17) = 34377140 - 32287140 + 21829837 = 23919837,$$

obtained from the template using ' = B123 – C116 + C67 ', without the quotes.

3.2.5 Age transition diagram for the determination of the optimal policy prescriptions and the corresponding returns:

$$0K1K2K3K4R1K2K3R1K2K3S$$

$$0K1K2K3R1K2K3K4R1K2K3S$$

$$0K1K2K3R1K2K3R1K2K3K4S$$

$f_1(0) =$ (The maximum net income from years 1 to 10) = ₦ 23,919,837

Starting Age 1

There are a number of alternate optimal policies; one alternative is

$$1K2K3K4K4R1K2K3R1K2K3S$$

$f_1(1) =$ (The maximum net income from years 1 to 10)

$= \hat{f}_8(1) = 23,306,572$ ₦ 23,919,837.

The process continues for the remaining starting ages.

The optimal returns and initial decision corresponding to $n \in \{10, 11, \cdots, 16\}$ are summarized as follows, with the (last state − 1) corresponding to the starting stage:

Table 9: ***n* = 10**

Opt. value: *f(t)*		23306572	22804136	22300121	21931106	21229979	20598964	20031052
Opt. Decision		*K*	*K*	*K/R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7

Table 10: ***n* = 11**

Opt. value: *f(t)*		24871136	24253121	23776856	23407841	22706714	22075699
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6

Table 11: ***n* = 12**

Opt. value: *f(t)*		26320121	25729856	25341420	24972405	24271278
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*
State		1	2	3	4	5

Table 12: ***n* = 13**

Opt. value: *f(t)*		27796856	27294420	26790405	26421390
Opt. Decision		*K*	*K*	*K/R*	*R*
State		1	2	3	4

Table 13: ***n* = 14**

Opt. value: *f(t)*		29361420	28743405	28267140
Opt. Decision		*K*	*K*	*R*
State		1	2	3

Table 14: ***n* = 15**

Opt. value: *f(t)*		30810405	30220140
Opt. Decision		*K*	*K/R*
State		1	2

Table 15: ***n* = 16**

Opt. value: *f(t)*	34377140
Opt. Decision	*K*
State	0

4. CONCLUSION

Based on the pertinent data from Nasco Household Products, Limited, Jos, the optimal policy prescription for the given horizon length is to replace the machine every year. This would attract installation costs not factored into the template. Further investigations could be carried out depending on the specification of the installation costs. The batch-starting age form of the machine replacement templates are the most suitable mechanism for determining the optimal replacement strategies for the Pakona packing machine. Any other replacement policy is sub-optimal.

This work appropriated the computational formulas in [8] for the sets of feasible machine ages for each decision year in place of network diagrams which are unwieldy, prone to errors, and unsuitable for electronic implementation. These sets of machine ages served as inputs to the prototypical templates in [9][10] and [11] deployed for the determination of the optimal replacement strategies for Pakona Packing machine. The work went further to perform price sensitivity analysis on the optimal replacement policies and corresponding rewards, with illuminating revelations on the KEEP and REPLACE decisions. In general, the template can be used to solve this and other machine replacements problems, as well as conduct sensitivity analyses on the process parameters in less than 0.5 percent of the time required for manual generation of the optimal solutions and returns.

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