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Two Dimensional Fractional Fourier-Mellin Transform of Some Signals

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Abstract: Integral transform are recognized as useful tools for quantitative studies of geophysical anomalies. Two dimensional fractional Fourier-Mellin transform are used in agriculture, medical stream, detection of watermark in images regardless of the scaling and rotation.

In this work we have calculated two dimensional fractional Fourier-Mellin transform of some signals in the range of $-\infty$ to ∞.

Keywords: Two-Dimensional Fractional Fourier Transform, Two-Dimensional Fractional Mellin Transform, Two-Dimensional Fractional Fourier-Mellin Transform, Testing Function Space, Generalized function.

I. Introduction

During the second half of the twentieth century, considerable amount of research in fractional calculus was published in engineering literature. The method of defining fractional transform is in the terms of Eigen decomposition choosing some eigen function, the R.S. Pathak has defined fractional Fourier transform which is depend on parameter α [4].

The first investigators to use a version of the Fourier-Mellin transform were Brousil and Smith in 1967. The Fourier –Mellin moments were firstly proposed by Sheng and Davernoy in 1986, it is defined in polar coordinate system [6]. Image matching methods should be invariant to translation, rotation and scale. B Dasgupta & B N Chatterji presented in their work that image matching algorithm based on Fourier-Mellin transform [9]. J. R. Martínez-de Diosy A. Ollero developed robust real-time image stabilization system based on the Fourier-Mellin transform. The system is capable of performing image capture-stabilization-display at a rate of standard video on a general Pentium III at 800 MHz without any specialized hardware and the use of any particular software platforms [1].

Fourier-Mellin transform is used to identify plant leaves at various life stages based on the leaves shape or contour. Fourier-Mellin transform is also used in estimation of optical flow [8, 5]. Fourier-Mellin transform used the for detection of watermark in images regardless of the scaling and rotation [8]. Using this transform human face also detected [2]. G.S. Page used this method for comparing of distorted objects [3].

In our previous work we define some terminology [5,9,10] is as follows. In this paper we have calculated two dimensional fractional Fourier-Mellin transform of some signals in the range of $-\infty$ to ∞.

II. Definitions

2.1 Definition of two-dimensional fractional Fourier-Mellin transform

The two-dimensional fractional Fourier-Mellin transform with parameters α and θ of $f(x,y,t,q)$ denoted by FRFMT{ $f(x,y,u,v)$} performs a linear operation, given by the integral transform.

FRFMT{ $f(x,y,u,v)\} = F_{\alpha,\theta}(p,q,r,s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)K_{\alpha,\theta}(x,y,u,v,p,q,r,s)dxdydudv$ ----(2.1.1)

where $K_{\alpha,\theta}(x,y,u,v,p,q,r,s)$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}$$

$$= C_{1\alpha}e^{i[(x^2+y^2+p^2+q^2)C_{2\alpha}-2(xp+yq)C_{3\alpha}}u^{C_{1\theta}ir-1}v^{C_{1\theta}is-1}e^{C_{2\theta}i[r^2+s^2+log^2u+log^2v]}$$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{cot\alpha}{2}, C_{2\alpha} = \frac{cosec\alpha}{2}, C_{1\theta} = \frac{2\pi}{sin\theta}$, $C_{2\theta} = \frac{\pi}{tan\theta}$ $\quad 0 < \alpha < \frac{\pi}{2}$, $0 < \theta < \frac{\pi}{2}$. ---(2.1.2)

2.2 The Test Function

An infinitely differentiable complex valued smooth function $\emptyset(x,y,u,v)$ on R^n belongs to $E(R^n)$, if for each compact set $I \subset S_{a,b}$, $J \subset S_{c,d}$ where

$S_{a,b} = \{x,y: x,y\epsilon R^n, |x| \le a, |y| \le b, a > 0, b > 0\}$

$S_{c,d} = \{u,v: t,q\epsilon R^n, |t| \le c, |q| \le d, c > 0, d > 0\}$

$\gamma_{E,m,n,k,l}[\emptyset(x,y,u,v)] = \sup_{\substack{x,y\epsilon I\\ t,q\epsilon J}} \left|D_{x,y,t,q}^{m,n,k,l}\emptyset(x,y,u,v)\right| < \infty$ ---(2.2.3)

Thus $E(R^n)$ will denote the space of all $\emptyset(x,y,u,v)\epsilon E(R^n)$ with compact support contained in $S_{a,b} \cap S_{c,d}$. Note that the space E is complete and therefore a Frechet space. Moreover, we say that $f(x,y,u,v)$ is a fractional Fourier-Mellin transformable if it is a member of E.

2.3 Distributional Two Dimensional Fractional Fourier-Mellin Transform (FRFMT)

The two dimensional distributional Fractional Fourier -Mellin transform of $f(x,y,u,v)\epsilon E^*(R^n)$ can be defined by

$FRFMT\{f(x,y,u,v)\} = F_{\alpha,\theta}(p,q,r,s) = \langle f(x,y,u,v), K_{\alpha,\theta}(x,y,u,v,p,q,r,s)\rangle$---(4)

where, $K_{\alpha,\theta}(x,y,u,v,p,q,r,s)$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}$$

$$= C_{1\alpha}e^{i[(x^2+y^2+p^2+q^2)C_{2\alpha}-2(xp+yq)C_{3\alpha}}u^{C_{1\theta}ir-1}v^{C_{1\theta}is-1}e^{C_{2\theta}i[r^2+s^2+log^2u+log^2v]}$$

where, $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{cot\alpha}{2}, C_{2\alpha} = \frac{cosec\alpha}{2}, C_{1\theta} = \frac{2\pi}{sin\theta}$, $C_{2\theta} = \frac{\pi}{tan\theta}$, $0 < \alpha < \frac{\pi}{2}$,
$0 < \theta < \frac{\pi}{2}$. ---(2.3.1)

Right hand side of equation (4) has a meaning as the application of $f(x,y,t,q)\epsilon E^*(R^n)$to $K_{\alpha,\theta}(x,y,u,v,p,q,r,s)\epsilon E$.

It can be extended to the complex space as an entire function given by

$$FRFMT\{f(x,y,u,v)\} = F_{\alpha,\theta}(p',q',r',s')$$
$$= \langle f(x,y,u,v), K_{\alpha,\theta}(x,y,u,v,p',q',r',s')\rangle \text{---}(2.3.2)$$

The right hand side is meaningful because for each $p',q',r',s' \epsilon C^n$, $K_{\alpha,\theta}(x,y,u,v,p',q',r',s')\epsilon E$ as a function of x,y,u,v.

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3.1 Result

$$[FRFMT(1)](p,q,r,s) = \sqrt{\frac{1-icot\alpha}{2\pi}}\frac{e^{\pi i}\pi}{cot\alpha cot\theta}e^{\frac{i(1+2cos^2\alpha)}{2sin\,2\alpha}(p^2+q^2)}e^{2\pi i\frac{(1+cos^2\theta)}{sin\,2\theta}(r^2+s^2)}$$

Proof-

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2sin\alpha}[(x^2+p^2+y^2+q^2)cos\alpha-2(xp+yq)]}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}(1)dxdydudv$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2sin\alpha}[(x^2+p^2+y^2+q^2)cos\alpha-2(xp+yq)]}dxdy$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(1)\; u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}dudv$$

Putting $logu = m,\; logv = n, u = e^m,\;\; v = e^n, du = e^m dm,\;\; dv = e^n dn$

$[FRFMT(1)](p,q,r,s)$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(x^2+y^2)cot\alpha-i(xp+yq)cosec\alpha}\,dxdy$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(e^m)^{\frac{2\pi ir}{sin\theta}-1}(e^n)^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[m^2+n^2]}e^m e^n dmdn$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}e^{i\left(\frac{cot\alpha}{2}\right)x^2+i(-pcosec\alpha\;)x}\,dx\int_{-\infty}^{\infty}e^{i\left(\frac{cot\alpha}{2}\right)y^2+i(-qcosec\alpha\;)y}\,dy$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i\left(\frac{\pi}{tan\theta}\right)m^2+i\left(\frac{2\pi r}{sin\theta}\right)m}e^{i\left(\frac{\pi}{tan\theta}\right)n^2+i\left(\frac{2\pi s}{sin\theta}\right)n}dn$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}e^{iAx^2+iBx}\,dx\int_{-\infty}^{\infty}e^{iAy^2+iCy}\,dy$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{iDm^2+iEm}e^{iDn^2+iFn}dn$$

$A = \frac{cot\alpha}{2}, B = -pcosec\alpha, C = -qcosec\alpha, D = \frac{\pi}{tan\theta}, E = \frac{2\pi r}{sin\theta}, F = \frac{2\pi s}{sin\theta}$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\,e^{\frac{\pi i}{tan\theta}[r^2+s^2]}\frac{e^{\pi i}\pi^2}{AD}e^{\frac{i}{4}\left(\frac{B^2+C^2}{A}+\frac{E^2+F^2}{D}\right)}$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\frac{e^{\pi i}\pi}{cot\alpha cot\theta}e^{\frac{2i(1+cos^2\alpha)}{2sin\,2\alpha}(p^2+q^2)}e^{2\pi i\frac{(1+cos^2\theta)}{sin\,2\theta}(r^2+s^2)}$$

8.3.2 Result

$$FRFMT\{\delta(x-a),\delta(y-b),\delta(u-c),\delta(v-d)\}(p,q,r,s)$$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2sin\alpha}[(a^2+p^2+b^2+q^2)cos\alpha-2(ap+bq)]}\,c^{\frac{2\pi ir}{sin\theta}-1}\,d^{\frac{2\pi is}{sin\theta}-1}\,e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2c+log^2d]}$$

8.3.3 Result

$$FRFMT\{e^{i(ax+by)}u^{ic}v^{id}\}(p,q,r,s)$$

$$=\frac{\sqrt{2\pi(1-icot\alpha)}}{cot\alpha cot\theta}expi\left\{\begin{matrix}\pi+\left(\frac{cos^2\alpha+1}{sin2\alpha}\right)(p^2+q^2)+\frac{sec\alpha}{2}\{sin\alpha[(a^2+b^2)-2(ap+bq)]\}\\+2\pi\left[\frac{cos^2\theta+1}{sin2\theta}(r^2+s^2)\right]+\frac{sec\theta}{2}\left[2(rc+sd)+\frac{sin\theta}{2\pi}(c^2+d^2)\right]\end{matrix}\right\}$$

Proof: Consider

$$FRFMT\{e^{i(ax+by)}u^{ic}v^{id}\}(p,q,r,s)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\{e^{i(ax+by)}u^{ic}v^{id}\}e^{\frac{i}{2}[(x^2+y^2+p^2+q^2)cot\alpha-2(xp+yq)cosec\alpha]}$$

$$(u^{ia}v^{ib})u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}(r^2+s^2+log^2u+log^2v)}dxdydudv$$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2}(p^2+q^2)cot\alpha}\,e^{\frac{\pi i}{tan\theta}(r^2+s^2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i(ax+by)}\,e^{\frac{i}{2}[(x^2+y^2)cot\alpha-2(xp+yq)cosec\alpha]}dxdy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u^{ic}v^{id}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}(log^2u+log^2v)}dudv$$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i(ax+by)}\,e^{\frac{i}{2}[(x^2cot\alpha+2xpcosec\alpha+2ax)}\,e^{\frac{i}{2}[(y^2cot\alpha+2yqcosec\alpha+2by)}$$

$$dxdye^{\frac{\pi i}{tan\theta}(r^2+s^2)}\int_{0}^{\infty}\int_{0}^{\infty}u^{\frac{2\pi ir}{sin\theta}+ic-1}v^{\frac{2\pi is}{sin\theta}+id-1}e^{\frac{\pi i}{tan\theta}(log^2u+log^2v)}dudv$$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i\left[\left(\frac{cot\alpha}{2}\right)x^2+(a-pcosec\alpha)x\right]}e^{i\left[\left(\frac{cot\alpha}{2}\right)y^2+(b-qcosec\alpha)y\right]}$$

$$dxdye^{\frac{\pi i}{tan\theta}(r^2+s^2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u^{\frac{2\pi ir}{sin\theta}+ic}u^{-1}v^{\frac{2\pi is}{sin\theta}+id}v^{-1}e^{\frac{\pi i}{tan\theta}(log^2u+log^2v)}dudv$$

Putting $log\,u=m,\quad log\,v=n, u=e^m, v=e^n,\ du=e^m dm,\ dv=e^n dn$

$$=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i\left(\frac{cot\alpha}{2}\right)x^2+i(a-pcosec\alpha)x}\,e^{i\left(\frac{cot\alpha}{2}\right)y^2+i(b-qcosec\alpha)y}dxdy$$

$$e^{\frac{\pi i}{tan\theta}(r^2+s^2)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(e^m)^{\frac{2\pi ir}{sin\theta}+ic}e^{-m}(e^n)^{\frac{2\pi is}{sin\theta}+id}e^{-n}e^{\frac{\pi i}{tan\theta}(m^2+n^2)}e^me^ndmdn$$

$$A = \frac{cot\alpha}{2}, \quad B = (a - pcosec\alpha), \ \ C = (b - qcosec\alpha), \quad D = \frac{\pi}{tan\theta}$$

$$E = \frac{2\pi r}{sin\theta} + c, \quad F = \frac{2\pi s}{sin\theta} + d$$

$$= \frac{\sqrt{2\pi(1 - icot\alpha)}}{cot\alpha cot\theta} e^{\pi i} e^{\frac{i}{2}\left[\frac{cos\alpha}{sin\alpha} + \frac{1}{cos\,\alpha sin\alpha}\right](p^2+q^2)} e^{\frac{i}{2}\left\{\frac{sin\alpha}{cos\alpha}\left[(a^2+b^2) - \frac{2sin\alpha}{cos\alpha sin\alpha}(ap+bq)\right]\right\}}$$

$$e^{i\left[\frac{\pi cos\theta}{sin\theta} + \frac{\pi}{cos\theta sin\theta}(r^2+s^2)\right]} e^{\frac{i}{2}\left[\frac{2}{cos\theta}(rc+sd) + \frac{sin\theta}{2\pi cos\theta}(c^2+d^2)\right]}$$

$$= \frac{\sqrt{2\pi(1 - icot\alpha)}}{cot\alpha cot\theta} expi \left\{ \begin{array}{c} \pi + \left(\frac{cos^2\alpha + 1}{sin2\alpha}\right)(p^2+q^2) + \frac{sec\alpha}{2}\{sin\alpha[(a^2+b^2) - 2(ap+bq)]\} \\ +2\pi\left[\frac{cos^2\theta + 1}{sin2\theta}(r^2+s^2)\right] + \frac{sec\theta}{2}\left[2(rc+sd) + \frac{sin\theta}{2\pi}(c^2+d^2)\right] \end{array} \right\}$$

8.3.5 Result

$$FRFMT\{e^{i(ax^2+by^2)}u^{ic}v^{id}\}(p,q,r,s) = \frac{tan\theta sin\alpha\sqrt{2\pi(1 - icot\alpha)}}{[(2asin\alpha + cos\alpha)(2bsin\alpha + cos\alpha)]^{\frac{1}{2}}}$$

$$exp\left\{ i \left[\begin{array}{c} \pi + \frac{cot\alpha}{2}(p^2+q^2) + \frac{2\pi}{sin2\theta}[cos^2\theta + 1](r^2+s^2) \\ +\frac{1}{2}\left(\frac{p^2}{cot\alpha + 2a} + \frac{q^2}{cot\alpha + 2b}\right)cosec^2\alpha + sec\theta(rc+sd) + \frac{tan\theta}{4\pi}(c^2+d^2) \end{array} \right] \right\}$$

Proof: Consider

$$FRFMT\{e^{i(ax+by)}u^{ic}v^{id}\}(p,q,r,s)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{\frac{1 - icot\alpha}{2\pi}} \{e^{i(ax^2+by^2)}u^{ic}v^{id}\} e^{\frac{i}{2}[(x^2+y^2+p^2+q^2)cot\alpha - 2(xp+yq)cosec\alpha\ ]}$$

$$(u^{ia}v^{ib}) u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}(r^2+s^2+log^2u+log^2v)} dxdydudv$$

$$= \sqrt{\frac{1 - icot\alpha}{2\pi}} e^{\frac{i}{2}(p^2+q^2)cot\alpha} e^{\frac{\pi i}{tan\theta}(r^2+s^2)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(ax^2+by^2)} e^{\frac{i}{2}[(x^2+y^2)cot\alpha - 2(xp+yq)cosec\alpha\ ]} dxdy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u^{ic}v^{id} u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}(log^2u+log^2v)} dudv$$

Putting $log\,u = m, \quad log\,v = n$

$$u = e^m, \qquad v = e^n$$

$$du = e^m dm, \ dv = e^n dn$$

$$= \sqrt{\frac{1 - icot\alpha}{2\pi}} e^{\frac{i}{2}(p^2+q^2)cot\alpha} \int_{-\infty}^{\infty} e^{iAx^2+iBx}\,dx \int_{-\infty}^{\infty} e^{iCy^2+iDy}\,dy\ e^{\frac{\pi i}{tan\theta}(r^2+s^2)} \int_{-\infty}^{\infty} (e)^{iEm^2+iFm}\,dm$$

$$\int_{-\infty}^{\infty} (e)^{iFn^2+iGn}\,dn,$$

$$A = \frac{cot\alpha}{2} + a, \quad B = -pcosec\alpha, \ \ C = \frac{cot\alpha}{2} + b, \ \ D = -qcosec\alpha,$$

$$E = \frac{\pi}{tan\theta}, \qquad F = \frac{2\pi r}{sin\theta} + c, \qquad G = \frac{2\pi s}{sin\theta} + d$$

$$= \sqrt{\frac{1 - icot\alpha}{2\pi}} e^{\frac{i}{2}(p^2+q^2)cot\alpha} e^{\frac{\pi i}{tan\theta}(r^2+s^2)} \frac{e^{\pi i}\pi^2}{\sqrt{AC}\, E} e^{i\left[\frac{B^2}{A}+\frac{D^2}{C}+\frac{F^2+G^2}{E}\right]}$$

$$= \frac{e^{\pi i}\, tan\theta sin\alpha\sqrt{2\pi(1 - icot\alpha)}}{[(2asin\alpha + cos\alpha)(2bsin\alpha + cos\alpha)]^{\frac{1}{2}}} e^{\frac{i}{2}(p^2+q^2)cot\alpha}\, e^{\pi i\left[\frac{1}{tan\theta}+\frac{1}{sin\theta cos\theta}\right](r^2+s^2)}$$

$$e^{\frac{i}{4}\left\{2\left(\frac{p^2}{cot\alpha\ +2a}+\frac{q^2}{cot\alpha\ +2b}\right)cosec\ ^2\alpha+4sec\theta\ (rc+sd)+\frac{tan\theta}{\pi}(c^2+d^2)\right\}}$$

$$= \frac{tan\theta sin\alpha\sqrt{2\pi(1 - icot\alpha)}}{[(2asin\alpha + cos\alpha)(2bsin\alpha + cos\alpha)]^{\frac{1}{2}}}$$

$$exp\left\{i\left[\begin{matrix}\pi + \frac{cot\alpha}{2}(p^2 + q^2) + \frac{2\pi}{sin2\theta}[cos^2\theta + 1](r^2 + s^2) \\ + \frac{1}{2}\left(\frac{p^2}{cot\alpha + 2a} + \frac{q^2}{cot\alpha + 2b}\right)cosec^2\alpha + sec\theta(rc + sd) + \frac{tan\theta}{4\pi}(c^2 + d^2)\end{matrix}\right]\right\}$$

Conclusion : In this paper we have calculated two dimensional fractional Fourier-Mellin transform.

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