International Research Journal of Mathematics, Engineering and IT
Vol. 4, Issue 4, April 2017 Impact Factor- 5.489
ISSN: (2349-0322)

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FOURIER SERIES INVOLVING I–FUNCTION OF ONE VARIABLE

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ABSTRACT

The object of this paper is to establish some new Fourier seriesinvolving I–function of one variable.

1. Introduction:

The I-function of one variable is defined by Saxena [4, p.366-375] and we will represent here in the following manner:

$$I^{m,n}_{p_i,q_i:r}\left[x\middle|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right] = \frac{1}{2\pi\omega}\int_L \theta(s)\, x^s ds \qquad (1)$$

where $\omega = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right]}$$

integral is convergent, when (R>0, S ≤ 0), where

$$R = \sum_{j=1}^{npi} \alpha_j - \sum_{j=n+1}^{m}\alpha_{ji} + \sum_{j=1}^{qi}\beta_j - \sum_{j=m+1}\beta_{ji}, \qquad (2)$$

$$S = \sum_{j=1}^{piqi}\alpha_{ji} - \sum_{j=1}\beta_{ji},$$

$|\arg x| < \frac{1}{2} R\pi$, $\forall\, i \in (1, 2, \ldots, r)$.

2. Result Required:

The following results are required in our present investigation:

From Macrobert [1, 2]:

$$\frac{\sqrt{\pi}\,\Gamma(2-s)}{2\Gamma(\frac{3}{2}-s)}(sin\theta)^{1-2s} = \sum_{r=0}^{\infty}\frac{(s)_r}{(2-s)_r}\sin(2r+1)\,\theta, \qquad (3)$$

where $0 < \theta \leq \pi$, Re $s \leq \frac{1}{2}$.

$$\frac{\sqrt{\pi}\,\Gamma(1-s)}{\Gamma(\frac{1}{2}-s)}\left(sin\frac{\theta}{2}\right)^{-2s} = 1 + 2\sum_{r=0}^{\infty}\frac{(s)_r}{(1-s)_r}\cos r\theta, \qquad (4)$$

where $0 < \theta \leq \pi$.

3. Main Result:

In this paper we will establish the following Fourier series:

$$\sum_{t=0}^{\infty} I^{m+1,n+1}_{p_i+2,q_i+2:r}\left[x\middle|^{(1-t,1),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}],(2+t,1)}_{\left(\frac{3}{2},1\right),[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,1)}\right]\sin(2t+1)\,\theta$$
$$= \frac{\sqrt{\pi}}{2}\,sin\theta.\,I^{m,n}_{p_i,q_i:r}\left[x/\sin^2\theta\middle|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right] \qquad (5)$$

provided that $0 < \theta \leq \pi$, $|\arg x| < \frac{1}{2}\pi R$, where Ris given in (2).

$$I^{m+1,n}_{p_i+1,q_i+2:r}\left[x\middle|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}],(1,1)}_{\left(\frac{1}{2},1\right),[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right]$$
$$+2\sum_{t=0}^{\infty} I^{m+1,n+1}_{p_i+2,q_i+2:r}\left[x\middle|^{(1-t,1),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}],(1+t,1)}_{\left(\frac{1}{2},1\right),[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,1)}\right]\cos r\theta$$
$$= \sqrt{\pi}I^{m,n}_{p_i,q_i:r}\left[x/\sin^2\frac{\theta}{2}\middle|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right], \qquad (6)$$

provided that $0 < \theta \leq \pi$, $|\arg x| < \frac{1}{2}\pi R$, where Ris given in (2).

Proof:

To prove (5), expressing the I-function on the left-hand side as Mellin-Barnes type integral (1), we have

$$\sum_{t=0}^{\infty}\frac{1}{2\pi\omega}\int_L \theta(s)\left[\frac{\Gamma(\frac{3}{2}-s)\Gamma(t+s)}{\Gamma(s)\Gamma(2+t-s)}\sin(2t+1)\,\theta\right]x^s ds$$

On changing the order of integration and summation which is easily seen to be justified, the above expression becomes

$$\frac{1}{2\pi\omega}\int_L \theta(s)\frac{\Gamma(\frac{3}{2}-s)}{\Gamma(2-s)}\left[\sum_{t=0}^{\infty}\frac{(s)_t}{(2-s)_t}\sin(2t+1)\,\theta\right]x^s ds.$$

and on using the relation (4), it takes the form

$$\frac{\sqrt{\pi}}{2}\,sin\theta.\frac{1}{2\pi\omega}\int_L \theta(s)\,(x/\sin^2\theta)^s ds.$$

which is just the expression on the right side of (5). (5) is the Fourier sine series for the I-function of one variable.

The Fourier cosine series (6) is proved in an analogous by using (4).

4. Special Cases:

On specializing the parameters in main results, we get following Fourier series in terms of H-function of one variable, which is a result given by Nigam [3, p. 53 (1.1) and (1.2)]:

$$\sum_{r=0}^{\infty} H_{p+2,q+2}^{m+1,n+1}\left[x\Big|_{\left(\frac{3}{2},1\right),\left(b_j,\beta_j\right)_{1,q},(1,1)}^{(1-r,1),\left(a_j,\alpha_j\right)_{1,p},(2+r,1)}\right]\sin(2r+1)\,\theta$$

$$=\frac{\sqrt{\pi}}{2}\,sin\theta.\,H_{p,q}^{m,n}\left[x/\sin^2\theta\Big|_{\left(b_j,\beta_j\right)_{1,q}}^{\left(a_j,\alpha_j\right)_{1,p}}\right] \qquad (7)$$

provided that 0 <θ≤π, |argx|<½πA, where A is given by $\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j\equiv A>0$.

$$H_{p+1,q+1}^{m+1,n}\left[x\Big|_{\left(\frac{1}{2},1\right),\left(b_j,\beta_j\right)_{1,q}}^{\left(a_j,\alpha_j\right)_{1,p},(1,1)}\right]+2\sum_{r=0}^{\infty}H_{p+2,q+2}^{m+1,n+1}\left[x\Big|_{\left(\frac{1}{2},1\right),\left(b_j,\beta_j\right)_{1,q},(1,1)}^{(1-r,1),\left(a_j,\alpha_j\right)_{1,p},(1+r,1)}\right]\cos r\theta$$

$$=\sqrt{\pi}H_{p,q}^{m,n}\left[x/\sin^2\frac{\theta}{2}\Big|_{\left(b_j,\beta_j\right)_{1,q}}^{\left(a_j,\alpha_j\right)_{1,p}}\right], \qquad (8)$$

provided that 0 <θ≤π, |argx|<½πA, where A is given by $\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j\equiv A>0$.

References

1. Macrobert, T. M.: Fourier series for E-function, Math. Zeitsclie, 75, 79-82 (1961).
2. Macrobert, T. M.: Infinite series for E-function, Math. Zeitsclie, 71, 143-154 (1959).
3. Nigam, H. N.: Fourier series for Fox's H-Functions, İstanbul University Science Faculty The Journal of Mathematics, Physics and Astronomy Vol. 34 (1969), 53-58.
4. Saxena, V. P.: Formal Solution of Certain New Pair of Dual Integral Equations Involving H-function, Proc. Nat. Acad. Sci. India, 52(A), III (1982).