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Fuzzy Optimization Model on Maintaining Inventory for Roadside Caterers of Fast Food

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ABSTRACT

This paper studies the maintenance of inventory for fast food sellers of roadside caterers using fuzzy set theory which helps in modeling any inventory problem for its ability to overcome vagueness and imprecision. We develop a fuzzy model applicable to single period inventory problem. Uncertain demand causes uncertain total cost function. In this paper we find an analytical method for determining the exact expected value of total cost function for a fuzzy single period inventory problem for roadside caterers. The optimum order quantity that minimizes the fuzzy total cost function is determined using the expected value of a fuzzy function based on credibility theory and hence closed form solutions to optimum order quantities & corresponding to total cost values are derived.

Keywords: Fuzzy demand, membership function, uncertainty, inventory problem.

1 Introduction:

Inventory management is an important aspect of all business undertakings as it is a function of directing the movement of goods through the entire manufacturing cycle from the requirement of raw materials to the inventory of finished goods so as to meet the objective of maximum customer service with minimum investment and efficient low cost plant operation with simultaneous balance between over-stocking and under-stocking.

In single period inventory problem, product orders are given before the selling period begins. There is no option for an additional order during the selling period or there will be a penalty cost for this re-order. The assumption of the single period inventory problem is that if any inventory remains at the end of the period, either a discount is used to sell it if it is non-perishable item or it is disposed of, if it is perishable item. Fast food items served hot by roadside caterer refers to single period inventory problem as it is valuable for a limited time. Customers are only interested in consuming freshly prepared hot cooked food item. The fast food vendor in a roadside eatery has to sell all of the prepared fast food items, then it becomes worthless the next day. So he ends up in a loss as the inventory in stock to prepare the food items are perishable.

In the literature survey, most of the extensions have been made in the probabilistic framework in which the uncertainty of demand is described by probability distribution. But in realistic terms, the probability distributions of the demands for products are difficult to acquire due to lack of information and data in which case the demands are specified approximately based on the experience and managerial subjective judgements. In such cases, the fuzzy set theory introduced by Zadeh is the best form that adapts all the uncertainty in the model. The fuzzy set modeling using possibility instead of probability theory, can represent linguistic data which cannot be easily modeled by other methods.

Our paper intends to find an analytical method for determining the exact expected value of total cost function for a single-period inventory problem under uncertainty. To determine the optimal order quantity that minimizes the fuzzy total cost function we use the expected value of a function of a fuzzy variable with a continuous membership function .

2 Preliminaries

In this paper the optimization of single period inventory problem of food items prepared for each day is analyzed. If there is less preparation of that particular foodie then there is possibility of less profit as the customers are lost and if there is more preparation and no consumption then it becomes obsolete and it cannot be used the next day as it is perishable item and there is loss again for the seller. Our goal is to find an analytical method to determine the exact expected value of total cost function for single period inventory problem under uncertainty. The source of the uncertainty in the analyzed problem results from the imprecise demand. The preliminary concepts about the fuzzy set theory and the credibility theory which will be useful for understanding the proposed model and the solution procedure are elucidated. Fuzzy logic provides solutions to complex problems through a similar approach as human reasoning. The major characteristic of fuzzy logic is its ability to eradicate ambiguity in human thinking, subjectivity and knowledge to the model.

Fuzzy Set

L.A.Zadeh invented fuzzy sets in mathematical terminology to represent ambiguity and vagueness as a generalization of crisp set. It is a class of objects with membership grades defined by a membership function. A fuzzy set can be mathematically represented as

$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) : x \in X \right\}$ whereX is the universal set and $\mu_{\tilde{A}}(x)$ is the membership function.

Fuzzy Number

It is a common fuzzy set whose membership function is piecewise continuous. If $\tilde{X}$ is a generalized fuzzy number (known as L-R type fuzzy number), whose membership function $\mu_{\tilde{X}}(x)$ satisfies the following conditions with $0 < w \le 1$and $-\infty < l < m < n < u < \infty$.

1) $\mu_{\tilde{X}}(x)$ is a continuous mapping from $\Re$, to the closed interval [0,1],
2) $\mu_{\tilde{X}}(x) = 0, -\infty < x < l$,
3) $\mu_{\tilde{X}}(x) = L(x)$, is strictly increasing on $[l, m]$,
4) $\mu_{\tilde{X}}(x) = w, m < x < n$,
5) $\mu_{\tilde{X}}(x) = R(x)$, is strictly decreasing on $[n, u]$,
6) $\mu_{\tilde{X}}(x) = 0, u < x < \infty$.

This type of generalized fuzzy number is denoted as $\tilde{X} = (l, m, n, u; w)_{LR}$. When $w = 1$, it can be simplified as $\tilde{X} = (l, m, n, u)_{LR}$.

Lattice of Fuzzy Numbers

The linear ordering of real numbers does not extend to fuzzy numbers. In order to define the lattice operations of fuzzy numbers corresponding to those of crisp numbers we have to define MIN and MAX for any two fuzzy numbers A and B as follows:

$$MIN\left(\tilde{A}, \tilde{B}\right)(Z) = \sup_{Z=\min(x,y)} \min\left[\tilde{A}(x), \tilde{B}(\mathrm{y})\right]$$

$$MAX\left(\tilde{A}, \tilde{B}\right)(Z) = \sup_{Z=\max(x,y)} \max\left[\tilde{A}(x), \tilde{B}(\mathrm{y})\right]$$

The symbol MIN and MAX are continuous operation and hence are different from the symbols of min and max of the crisp sets.

Credibility Theory

The credibility theory developed as a branch of mathematics for studying the behavior of fuzzy phenomena. The possibility theory was proposed by Zadeh[2], and developed by many researchers such as Dubois and Prade[18]. In the possibility theory, there are two measures including possibility and necessary measures. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. Possibility measure is thought as a parallel concept of probability measure. However, as many researchers mentioned before, these two measures have partial differences. Necessity measure is the dual of possibility measure. However, either possibility measure nor necessity measure has self-duality property. A self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu[14] introduced the concept of credibility measure in 2002. In this concept credibility measure resembles the similar properties by probability measure. Credibility measure plays the role of probability measure in fuzzy world.

Let $\left\{\xi_i, i=1,2,...n\right\}$ be a fuzzy variable with the membership function $\mu(x)$ and $r$be a real number, then the possibility and necessity measure of the fuzzy event $\left\{\xi_i \le r\right\}$ can be respectively represented as follows:

$$Pos\left\{\xi_i \le r\right\} = \operatorname*{supp}_{x\le r} \mu(x) \tag{1}$$

$$Nec\left\{\xi_i \le r\right\} = 1 - Pos\left\{\xi_i > r\right\} = 1 - \operatorname*{supp}_{x>r} \mu(x) \tag{2}$$

The credibility measure which is introduced by Liu is the average of possibility measure and necessity measure. The credibility measure of the fuzzy event $\left\{\xi_i \le r\right\}$ can be represented as follows:

$$Cr\left\{\xi_i \le r\right\} = \frac{1}{2}\left(Pos\left\{\xi_i \le r\right\} + Nec\left\{\xi_i \le r\right\}\right) \tag{4}$$

Similarly the credibility measure of the fuzzy event $\left\{\xi_i \ge r\right\}$ can be represented as follows:

$$Cr\left\{\xi_i \ge r\right\} = \frac{1}{2}\left(Pos\left\{\xi_i \ge r\right\} + Nec\left\{\xi_i \ge r\right\}\right)$$

Expected Value of a Function of a Fuzzy Variable

Several ranking methods for fuzzy numbers are used to find the expected value of a fuzzy number which was a complex process for researchers in previous years. Now the expected value is usually computed using simulation techniques or heuristic algorithms which result in errors. Hence, Xue et al. introduced an analytical method which uses the properties of the credibility measure, to calculate the expected value of a function of a fuzzy variable.

Let ξ_i be a fuzzy variable and r be a real number, the expected value of a fuzzy variable $E[\xi_i]$ can be calculated as follows:

$$E[\xi_i] = \int_0^{+\infty} Cr\{\xi_i \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi_i \leq r\}dr \tag{5}$$

provided that at least one of the two integrals is finite.

Let ξ_i be a fuzzy variable with a continuous membership function $\mu_{\xi_i}(x)$, and $f:\Re \to \Re$ is a strictly monotonic function. If the Lebesgue integrals, $\int_0^{+\infty} Cr\{\xi_i \geq r\}dr$ and $\int_{-\infty}^{0} Cr\{\xi_i \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E\left[f(\xi_i)\right]$ can be calculated as follows:

$$E\left[f(\xi_i)\right] = \int_{-\infty}^{+\infty} f(r)dCr\{\xi_i \leq r\} \tag{6}$$

Let ξ_i, be a fuzzy variable with a continuous membership function $\mu(x)$ and $f:\Re \to \Re$ and $g:\Re \to \Re$ be two different strictly monotonic functions. If the expected values of ξ_i, $f(\xi_i)$ and $g(\xi_i)$ exist, the following properties can be defined for any numbers p and q,

$$E\left[pf(\xi_i)+q\right] = pE\left[f(\xi_i)\right]+q \tag{7}$$

$$E\left[f(\xi_i)+g(\xi_i)\right] = E\left[f(\xi_i)\right]+E\left[g(\xi_i)\right] \tag{8}$$

Let ξ_i be a fuzzy variable whose support is $[a,b]$ and $f:\Re \to \Re$ is a strictly monotonic function. If the Lebesgue integrals, $\int_0^{+\infty} Cr\{\xi_i \geq r\}dr$ and $\int_{-\infty}^{0} Cr\{\xi_i \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E\left[f(\xi_i)\right]$ can be calculated as follows:

$$E\left[f(\xi_i)\right] = \int_a^b f(r)dCr\{\xi_i \leq r\} \tag{9}$$

Methodology

A. Expected Value of a Function of a Fuzzy Variable

Consider a single-period inventory problem where the demand is subjectively believed to be imprecise and represented by a generalized fuzzy number $\left(\tilde{X}\right)$ with the following membership function given by (10) and Fig.1;

$$\mu_x(x) = \begin{cases} L(x), & l \le x \le m \\ 1, & m \le x \le n \\ R(x), & n \le x \le u \\ 0, & other \end{cases} \qquad (10)$$

Here [*m*,*n*] are the most likely values of fuzzy number $\bar{X}$, *l* and u are the lower and upper values; L(x) and R(x) are the left and right shape functions, respectively.

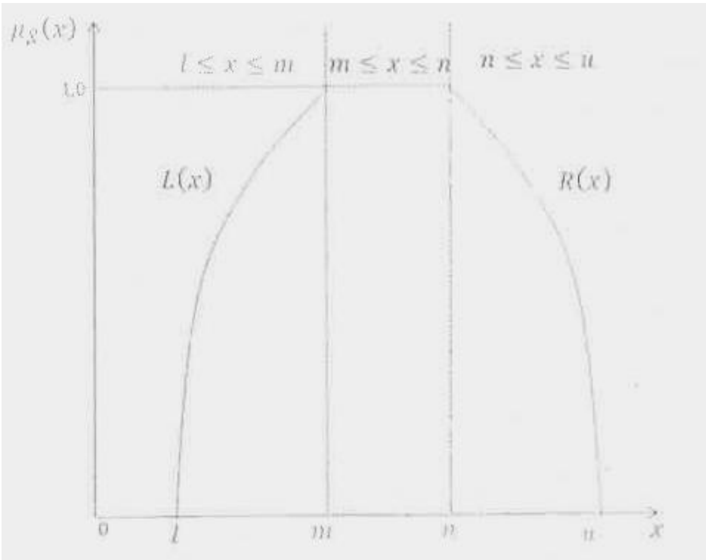

Fig. 1 Membership function of a generalized fuzzy number

Contrary a demand, inventory cost coefficients are known precisely. A product is produces (or produced) at a cost of *v* for a single–period. h is the unit inventory holding cost per unit remaining at the end of the period , ($h< 0$ represents have salvage value per remaining unit), normally the salvage value is less than the unit production cost, i.e. ($v>-h$) [12]. The selling price per unit is assumed to be equal to the unit inventory shortage cost which is represented by b in the model ($b>v$). We assume that there is no on-hand inventory at the beginning of the period.

Uncertain demand will cause an uncertain total cost function. If we order Q unit, the total cost function $\tilde{T}\tilde{C}(Q)$ will be as in (11);

$$\tilde{T}\tilde{C}(Q)=\begin{cases}Qv+h(Q-\bar{X}) & \bar{X}\le Q\\ Qv+b(\bar{X}-Q) & \bar{X}\ge Q\end{cases} \tag{11}$$

Since the demand is a generalized fuzzy number ($\bar{X}$), the fuzzy total cost function $\tilde{T}\tilde{C}(Q)$ will also be a generalized fuzzy number with the same membership function of demand. The problem is to determine the optimal order quantity (Q*) that minimizes the fuzzy total cost. The membership function of demand the fuzzy total cost. The membership function of demand is represented as in fig. 1. According to this membership faction, it is obvious that the optimal order quantity (Q*) will remain between l and u, since (*b*>*v*>-*h*).

B. Analytical Model and Solution Methodology

We consider the above single-period inventory problem where the demand and total cost faction, which are formulated as in (11), are generalized fuzzy numbers. The fuzzy total cost function, is strictly decreasing in $l\le x\le Q$ and strictly increasing $Q\le x\le u$. In order to optimize the single-period inventory problem with fuzzy demand, we use the expected value method which is proposed by Xue et al. [20]. In this context, by using (6) and (8), the expected value of total cost function $E[\tilde{T}\tilde{C}(Q)]$ of fuzzy demand $\bar{X}$ will be formulated as in (12);

$$\begin{aligned}E[\tilde{T}\tilde{C}(Q)] &= \int_l^Q \left[Qv+h(Q-r)\right]dCr\left\{\bar{X}\le r\right\}+\\ &\int_Q^u \left[Qv+b(r-Q)\right]dCr\left\{\bar{X}\le r\right\} \\ &= Qv+hQ\int_Q^u dCr\left\{\bar{X}\le r\right\}-h\int_l^Q rdCr\left\{\bar{X}\le r\right\}\\ &+b\int_Q^u rdCr\left\{\bar{X}\le r\right\}-bQ\int_Q^u dCr\left\{\bar{X}\le r\right\}\end{aligned} \tag{12}$$

The optimal order quantity (*Q**), will be the value that minimizes the expected value of fuzzy total cost function. The find the minimizing value of *Q*, we set

$$\frac{dE\left[\tilde{T}\tilde{C}(\mathrm{Q})\right]}{dQ}=0 \tag{13}$$

Thus, the following equations are obtained,

$$\frac{dE\left[\tilde{T}\tilde{C}(\mathrm{Q})\right]}{dQ}=v+h\int_l^Q dCr\left\{\bar{X}\le r\right\}-b\int_Q^u dCr\left\{\bar{X}\le r\right\}$$

$$0=v+(h+b)\mathrm{C}r\left\{\bar{X}\le Q\right\}-b\,, \tag{14}$$

$$Cr\left\{\bar{X}\le Q\right\}=\frac{b-v}{h+b} \tag{15}$$

To find the optimum order quantity, we will determine the value of $cr\left\{\bar{X} \leq Q\right\}$. The credibility of fuzzy demand with the membership function as in (10) will be as in (16);

$$Cr\left\{\bar{X} \leq Q\right\} = \begin{cases} 0, & Q \leq l \\ \dfrac{L(Q)}{2}, & l \leq Q \leq m \\ \dfrac{1}{2}, & m \leq Q \leq n \\ 1 - \dfrac{R(Q)}{2}, & n \leq Q \leq u \\ 1, & Q \geq u \end{cases} \tag{16}$$

There are three cases to be analyzed for the value of Q in discussing the credibility value of fuzzy demand; $cr\left\{\bar{X} \leq Q\right\} : l \leq Q \leq m, m \leq Q \leq n \;\; and \;\; n \leq Q \leq u$

Case l: $l \leq Q \leq m$

$$Cr\left\{\bar{X} \leq Q\right\} = \frac{b-v}{h+b} = \frac{L(Q)}{2} \tag{17}$$

Thus, the following equations are obtained,

$$L(Q) = \frac{2(b-v)}{h+b} \, and \, Q^* = L^{-1}\left(\frac{2(b-v)}{h+b}\right) \tag{18}$$

Here, the value of $2(b-v)/(h+b)$ must lie between [0,1], thus we can say that under the conditions of $b \geq v$ and $b - v \leq h + v, Q^*$ will lie in [l,m]. Moreover, the second derivative of $E\left[\tilde{T}\tilde{C}(\mathrm{Q})\right]$ with respect to Q is

$$\frac{d^2 E\left[\tilde{T}\tilde{C}(Q)\right]}{dQ} = \frac{(h+b)}{2} L'(Q) \tag{19}$$

Since, L(Q) is an increasing function in [l,m], we find that $L'(Q) > 0$. The values of h and b are positive, so that we obtain $\dfrac{d^2 E\left[\tilde{T}\tilde{C}(Q)\right]}{dQ} > 0$ which implies that Q^* is the optimum value which minimizes $E\left[\tilde{T}\tilde{C}(Q)\right]$. The expected fuzzy total cost value for the optimum order quantity will be as below;

$$E\left[\tilde{T}\tilde{C}(Q^*)\right] = -h \int_{l}^{L^{-1}\left(\frac{2(b-v)}{h+b}\right)} rd\left(\frac{L(r)}{2}\right)$$

$$+b \int_{L^{-1}\left(\frac{2(b-v)}{h+b}\right)}^{m} rd\left(\frac{L(r)}{2}\right) \qquad (20)$$

$$+b\int_{n}^{u} rd\left(1-\frac{R(r)}{2}\right)$$

Case 2:$m \le Q \le n$

$$Cr\left\{\bar{X} \le Q\right\} = \frac{b-v}{h+b} = \frac{1}{2} \qquad (21)$$

Optimum order quantity (Q^*) is the value that the first derivative of the expected value of fuzzy total cost equals to zero, in this case,

$$\frac{dE\left[\tilde{T}\tilde{C}(Q)\right]}{dQ} = v+(h+b)Cr(X \le Q\} - b$$

$$0=v+(h+b)1/2 - b \qquad (22)$$

If there exits b-v = h+v case, then $E\left[\tilde{T}\tilde{C}(Q)\right]$ will be minimized by $Q \in [m,n]$. There by under the condition of b-v=h+v, we can say that $Q^* \in [m,n]$. The expected fuzzy total cost value for the optimum order quantity will be as in (23);

$$E\left[\tilde{T}\tilde{C}(Q^*)\right] = -h\int_{r}^{m} rd\left(\frac{L(r)}{2}\right) + b\int_{n}^{u} rd\left(1-\frac{\mathrm{R}(r)}{2}\right) \qquad (23)$$

Case 3: $n \le Q \le u$

$$Cr\left\{\bar{X} \le Q\right\} = \frac{b-v}{h+b} = 1-\frac{R(Q)}{2} \qquad (24)$$

and the following equations are obtained,

$$R(Q) = \frac{2(\mathrm{h}+v)}{h+b} \text{ and } Q^* = R^{-1}\left(\frac{2(h+v)}{h+b}\right) \qquad (25)$$

Since the value of $2(h+v)/(h+b)$ must be in the range of 0 and 1, under the conditions of $v \ge -h$ and b-$v \ge h+v$, we can say that the value of Q^* will lie in $[n,u]$. additionally, the second derivative of $E\left[\tilde{T}\tilde{C}(Q)\right]$ with respect to Q will be,

$$\frac{d^2E\left[\tilde{T}\tilde{C}(Q)\right]}{dQ} = -\frac{(\mathrm{h}+b)}{2}R'(Q) \qquad (26)$$

Since R(Q) is a decreasing function in the range of l and m, with $L'(Q) > 0$ and the values of h and be are positive, we obtain $\frac{d^2E\left[\tilde{T}\tilde{C}(Q)\right]}{dQ} > 0$ which implies that Q^* is the optimum value which minimize $E\left[\tilde{T}\tilde{C}(Q)\right]$. The excepted fuzzy total cost value for the optimum order quantity will be as in (27);

$$E\left[\tilde{T}\tilde{C}(Q^*)\right] = -h\int_l^m rd\left(\frac{L(r)}{2}\right) - h\int_n^{R^{-1}\left(\frac{2(h+v}{h+b}\right)} rd\left(1-\frac{R(r)}{2}\right) + b\int_{R^{-1}\left(\frac{2(h+v)}{h+b}\right)}^{u} rd\left(1-\frac{R(r)}{2}\right) \quad (27)$$

In the literature, trapezoidal fuzzy numbers are used commonly in the applications. Let the demand for the single-period inventory problem to be represented by a trapezoidal fuzzy number with the following membership function,

$$\mu_{\tilde{X}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ 1, & m \le x \le n \\ \frac{u-x}{u-n}, & n \le x \le u \\ 0, & other \end{cases} \quad (28)$$

In this case, optimum order quantity will be calculated as in (29);

$$Q^* = \begin{cases} 1+\left[\frac{2(b-v)}{h+b}\right](m-l) & b-v \le h+v \\ [m,n] & b-v = h+v \\ u-\left[\frac{2(h+v)}{h+b}\right](u-n) & b-v \ge h+v \end{cases} \quad (29)$$

The closed-form solutions for single-period inventorymodeldemand,for multi item case is given by the above equations under fuzzy demand. Optimum order quantity and the corresponding optimum cost values for singe-period inventory problem can be calculated easily by using the proposed approach. Moreover the closed-form solutions give the opportunity to analyze the effects of model parameters on optimum order quantity and optimum cost value.

Illustration

Suppose the roadside the caterer gets a regular demand for his food product in the form of the regular customers of the nearby commercial outlets in his vicinity.Let the demand be represented as a trapezoidal fuzzy number as follows:-

$\tilde{X}$ = (30, 50,100,150)

The member of fuzzy demand is given below :-

$$\mu_{\tilde{X}}(x) = \begin{cases} \frac{x-30}{20}, & 30 \le x \le 50 \\ 1, & 50 \le x \le 100 \\ \frac{150-x}{50}, & 100 \le x \le 150 \\ 0 & otherwise \end{cases}$$

Suppose the unit product cost is v= ₹ 20 for a single period and inventory holding cost and shortage cost values are respectively h = ₹5 and b =₹ 30.Since the following equationholds 30-20 ≤ 5+20,the optimum order quantity will be in the interval [30,50].In this case Q^* can be calculated using (29)and we get Q^*=40 and Corresponding cost value for Q^* =40 can be calculated using (20) and we get

$$E[\tilde{T}\tilde{C}(Q^*)] = ₹\ 14{,}373.312$$

If we decrease the unit production cost to v = ₹10 then the second case will remain valid (b-v = h+ v), and the value in the interval [50,100] will be optimal, that is $Q^* \in$ [50,100] and the corresponding cost will be equal to₹ 7173.317 .We can inference from the above calculations that the optimum order quantity and the corresponding cost values change according to the unit production cost values. As the unit cost values decrease, corresponding optimum cost values also decrease. We also observe that by decreasing the demand fuzziness the optimum order quantity increase and the total cost values decreases. The reason behind this,is that the less demand uncertainty causes less inventory overage and underage cost which lead to less total cost.

Conclusion

The proposed single-period inventory problem deals with finding the product's order quantity which minimizes the expected cost of seller with random demand. However, in real world, sometimes the probability distribution of the demand for products is difficult to acquire due to lack of information and historical data. This study focuses on possibilitic situations, where the demand causes an uncertain total cost function.

The paper has proposed an analytical method to obtain the exact expected value of total cost function which is composed of inventory holding, inventory shortage and unit production costs for a singe-period inventory problem under uncertainty. It has determined optimum order quantity that minimizes the fuzzy total cost function, the expected value of a fuzzy function based on the credibility theory. By this method, closed-from solutions to the optimum order quantities and corresponding total cost values are derived. The advantages of the closed-from solutions obtained are those that they eliminate the need for enumeration over alternative values and give the opportunity order quantity and optimum cost value. The proposed methodology, used for optimization based on the credibility theory can be applied to the solution of other complex real world problems where this complexity arises from uncertainty in the form of vagueness.

In this enumerated model when we consider the unit inventory shortage cost as the cost of missed profit, the value of b-v represents the unit profit gained from selling one unit of the item. On the other hand, the value of $h+v$represents the loss incurred for each unsold item when the unit production cost is$v$. When the unit profit gained from selling one unit is smaller than the loss incurred for each item left unsold, the inventory policy of the management should consider reducing the leftover unit conservatively. In contrast, if the profit gained is larger than the loss incurred, then the inventory policy should be aggressive to meet the possible demand. When the profit equals the loss incurred, the order quantity should be equal to the most likely demand.

This model has analyzed for multi product case develop from the case of single type of product enumerated by Behret and Kahraman and the solution procedure can be applied as a further research for the single period inventory problems of uncertainty besides imprecise demand.

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