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Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

ANALYSIS OF OCCURRENCE OF DIGIT 1 IN PRIME NUMBERS TILL 1 TRILLION

Neeraj Anant Pande

Associate Professor, Department of Mathematics and Statistics,
Yeshwant Mahavidyalaya (College), Nanded – 431602, Maharashtra, India.

ABSTRACT

Prime numbers less than 1 trillion are examined in detail for occurrence of digit 1 in them. Multiple occurrences of 1's are explored. The first and last instances of occurrence of all possible repetitions of 1's in them are determined within blocks of increasing powers of 10 till one trillion.

Keywords : All occurrences, digit 1, prime numbers.

Mathematics Subject Classification 2010 : 11Y35, 11Y60, 11Y99.

1. Introduction

There have been lots of explorations about prime numbers, particularly regarding their distributions. Although we lack exact formula for this, many theoretical properties [1] of their distribution patterns are known. Rigorous analysis for initial ranges till one trillion is done from various points of views in [4].

This paper presents the analysis of occurrence of digit 1 within all primes in blocks of increasing power of 10 till 1 trillion: primes p with $1 < p < 10^n$, $1 \leq n \leq 12$.

2. Occurrence of Single Digit 1 in Prime Numbers

Many historic numeral systems are compared in [2]. The specialty of digit 1 is that it is invariably present in every number system; whatever be the base of that and symbol for that. The trend of occurrence of digit 0 [5], [6], [7] and digit 1 [11], [12], [13] in natural numbers is formulated. Similar analysis of digit 0 for primes is also done [8], [9], [10]. Here we examine all prime numbers p in the range $1 < p < 10^{12}$ for occurrence of digit 1.

Long execution on many computer systems of a program written in Java Programming Language fetched these results.

Table 1: Number of Prime Numbers in Various Ranges with Single 1 in Their Digits

Sr. No.	Range	Number of Primes with Single 1
1.	$1 – 10^1$	0
2.	$1 – 10^2$	7
3.	$1 – 10^3$	56
4.	$1 – 10^4$	446
5.	$1 – 10^5$	3,650
6.	$1 – 10^6$	30,883
7.	$1 – 10^7$	265,086
8.	$1 – 10^8$	2,297,167
9.	$1 – 10^9$	20,051,508
10.	$1 – 10^{10}$	176,128,174
11.	$1 – 10^{11}$	1,553,607,785
12.	$1 – 10^{12}$	13,750,086,182

The number of primes containing single 1 in first 10 ranges above is more compared to those containing single 0 in corresponding ranges, where after the trend has reversed for last two ranges [8].

3. OCCURRENCE OF MULTIPLE DIGITS 1'S IN PRIME NUMBERS

The detail analysis of all positive integers containing single, double, triple and higher number of digit 1's in them within the ranges $1\text{-}10^n$, $1 \leq n \leq 12$ is available [11]. In this work, counts of prime numbers in these ranges containing multiple number of digit 1's have been determined and are as follows :

Table 2: Number of Prime Numbers in Various Ranges with Multiple 1's in Their Digits

Sr. No.	Number Range <	Number of Prime Numbers with 2 1's	Number of Prime Numbers with 3 1's	Number of Prime Numbers with 4 1's
1.	10^2	1	0	0
2.	10^3	11	0	0
3.	10^4	104	9	0
4.	10^5	1,108	149	10
5.	10^6	11,031	1,960	186
6.	10^7	109,336	24,268	3,112
7.	10^8	1,080,356	282,281	45,270
8.	10^9	10,601,309	3,191,363	607,785
9.	10^{10}	103,285,509	35,159,258	7,735,560
10.	10^{11}	1,000,219,616	379,597,320	94,716,113
11.	10^{12}	9,637,543,057	4,030,945,058	1,124,202,510

Table 2: Continued …

Sr. No.	Number Range <	Number of Prime Numbers with 5 1's	Number of Prime Numbers with 6 1's	Number of Prime Numbers with 7 1's
1.	10^6	13	0	0
2.	10^7	221	7	0
3.	10^8	4,663	236	16
4.	10^9	76,342	6,146	328
5.	10^{10}	1,152,705	117,956	8,226
6.	10^{11}	16,359,194	2,002,908	172,207
7.	10^{12}	220,704,383	31,348,740	3,245,154

Table 2: Continued …

Sr. No.	Number Range <	Number of Primes with 8 1's	Number of Primes with 9 1's	Number of Primes with 10 1's	Number of Primes with 11 1's
1.	10^9	10	0	0	0
2.	10^{10}	391	11	0	0
3.	10^{11}	10,411	385	11	0
4.	10^{12}	243,182	12,799	424	13

This count of multiple 1's as digits occurring in primes in various ranges of 1 - 10^i is graphically plotted as follows, where vertical axis in on logarithmic scale.

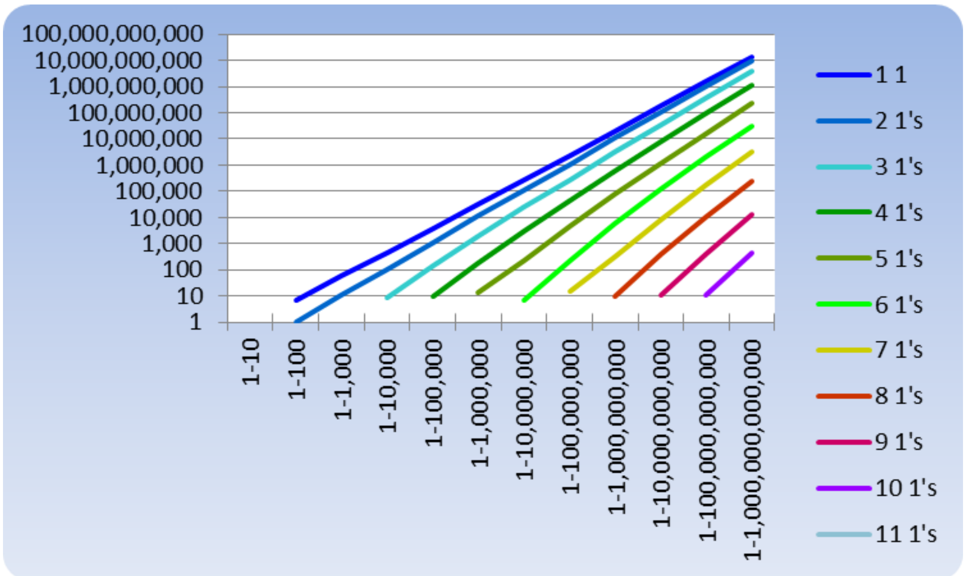


Figure 1: Number of Primes in Various Ranges with Multiple 1's in Their Digits

The percentage of primes containing multiple 1's calculated with respect to number of all such integers with those many 1's in corresponding ranges fluctuates.

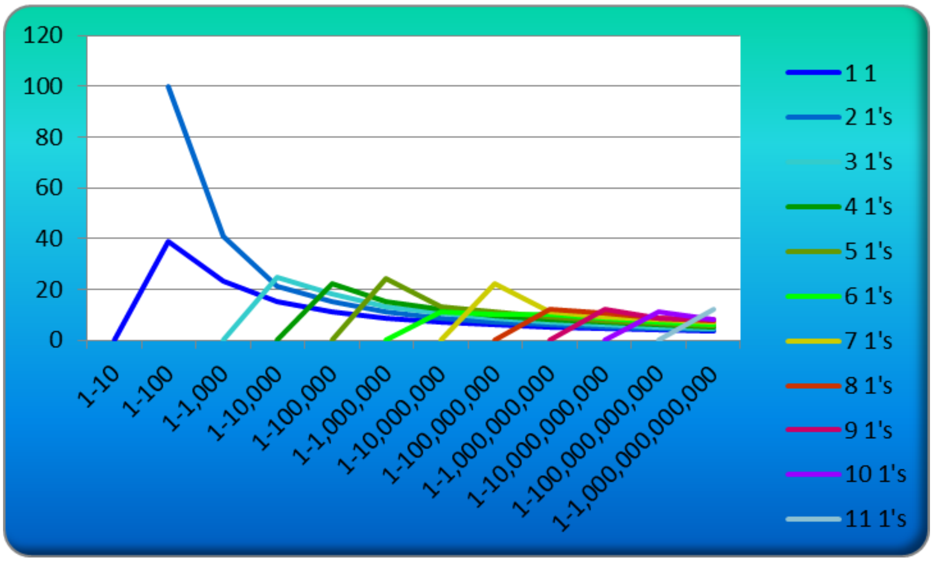


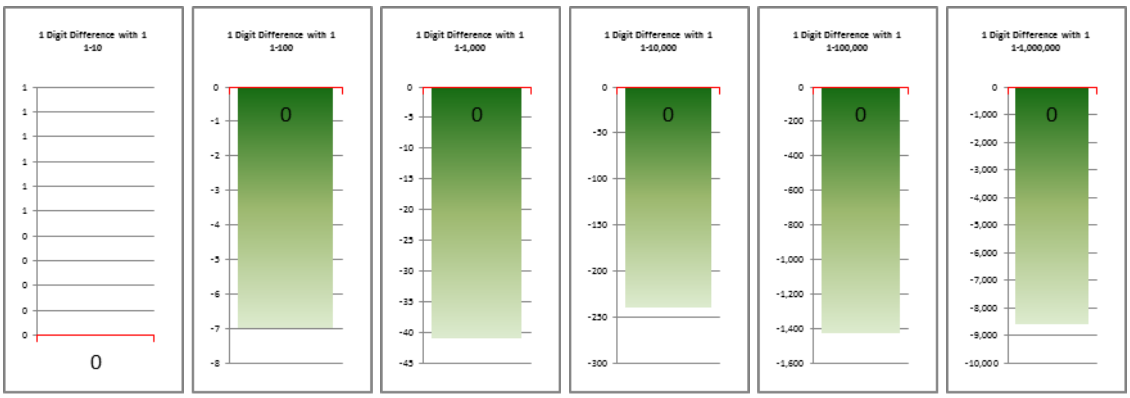
Figure 2: Percentage of Primes in Various Ranges with Multiple 1's in Their Digits With Respect To All Such Integers in Respective Ranges

The peak observed for 2 1's in the range 1 – 100 is obvious owing to the maximum percentage of 100; the only number 11 with 2 1's in this range is itself a prime. Recalling that 11 falls in the category of repunit primes, this can be generalized. Any number with all 1's in its digits is denoted by R_n, where n stands for the number of digits it has. Like 11 = R_2.

Remark : Whenever R_n happens to be (repunit) prime, the percentage of the primes with n number of 1's in digits calculated with base of total number of such integers in the range 1 – 10^n becomes maximum, viz., 100.

Just like 11, this is the case with all first 5 repunit primes R_2, R_{19}, R_{23}, R_{317}, and R_{1031}. Higher repunit primes are yet to be confirmed (R_{49081}, R_{86453}, R_{109297}, R_{270343} are potentially good candidates to be prime), but whatever those, all of them will exhibit above property.

Exclusive comparison of number of primes with different number of 0's in their digits [8] and equal number of 1's in them in our ranges follows :



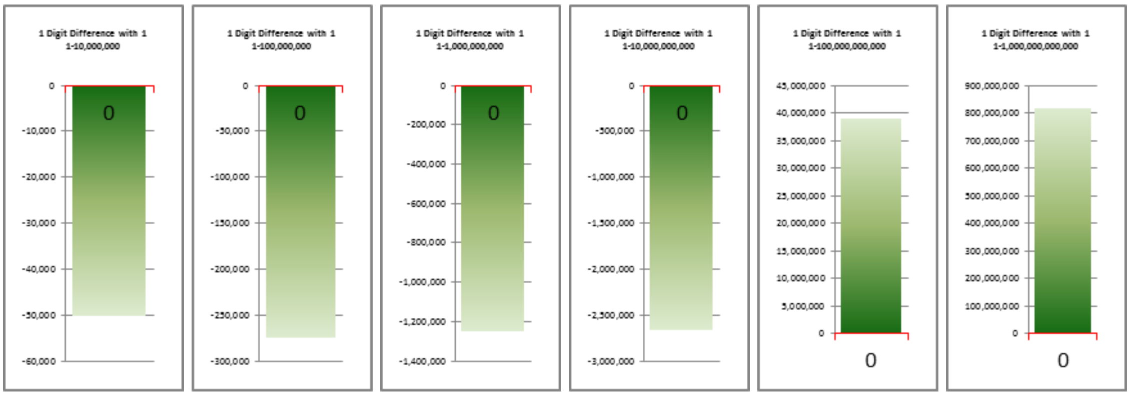


Figure 3: Differences of Number of Primes with single 0's & those with single 1's in Ranges of 1 - 10^n.

Except in the first range of 1 – 10, and last 2 ranges of 1 – 10^{11} and 1 – 10^{12}, the number of primes with single digit 0 are less than those with single digit 1 by amounts seen the graphs.

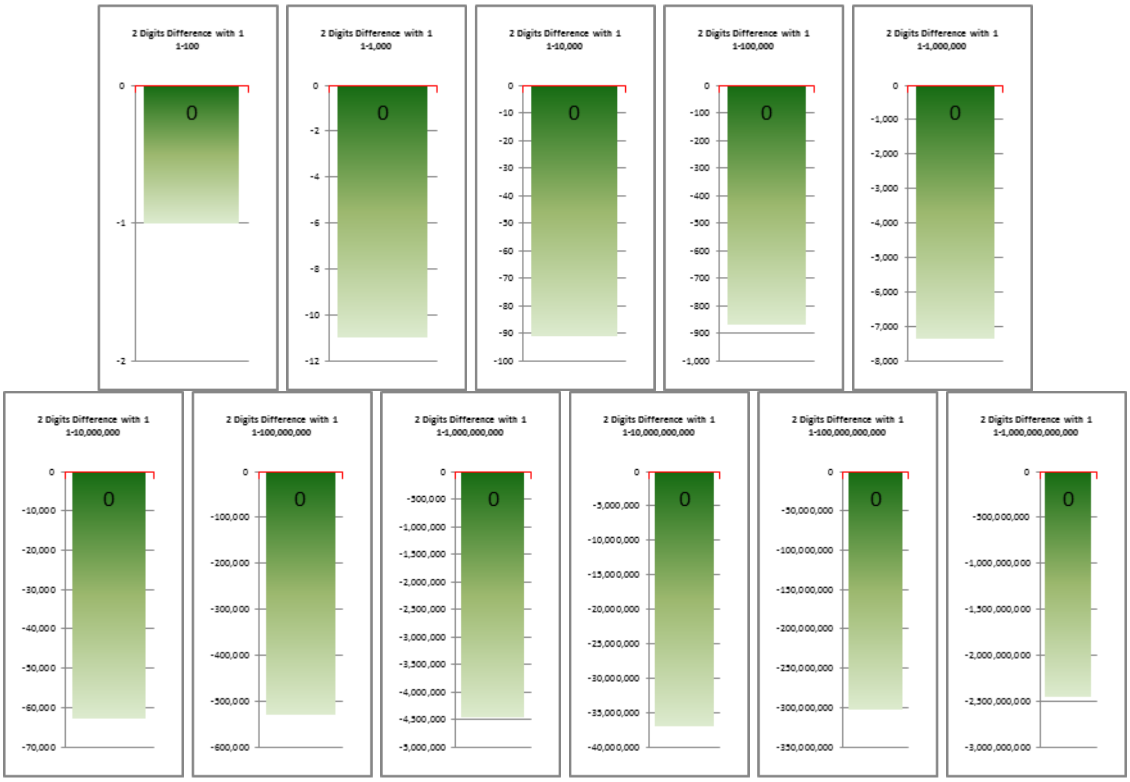


Figure 4: Differences of Number of Primes with 2 0's & those with 2 1's in Ranges of 1 - 10^n.

For two digits, in each range under consideration, primes with 2 0's are less than those with 2 1's.

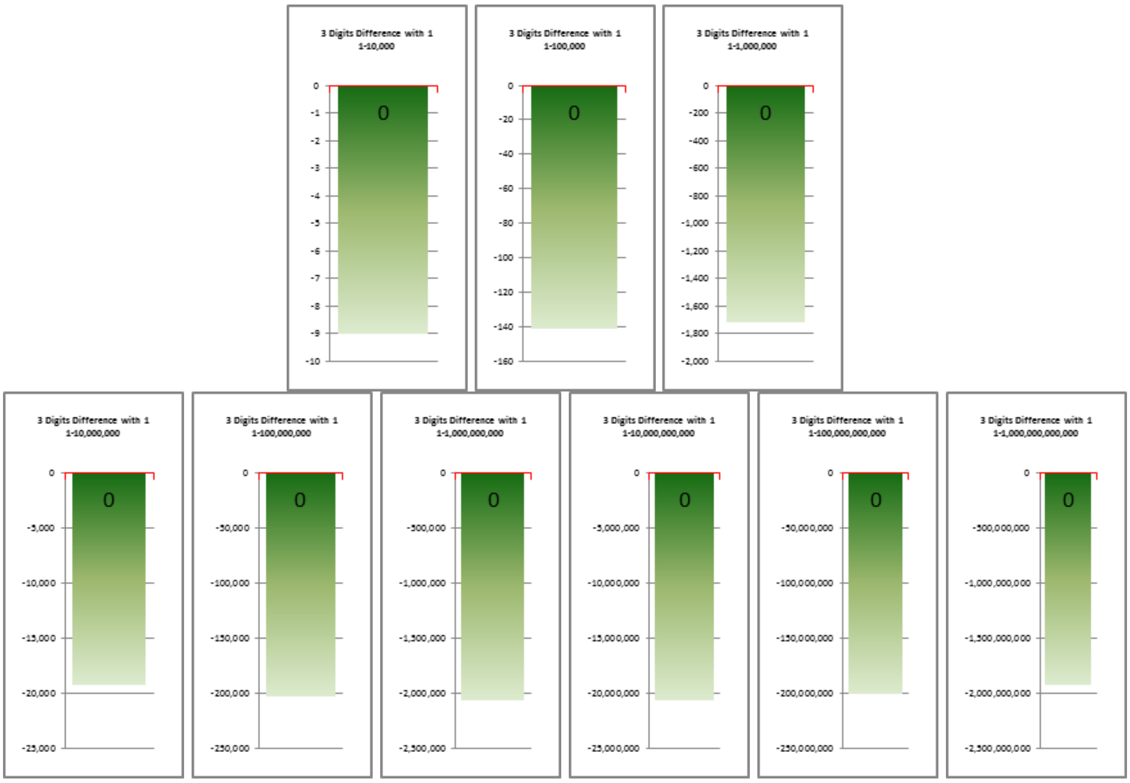
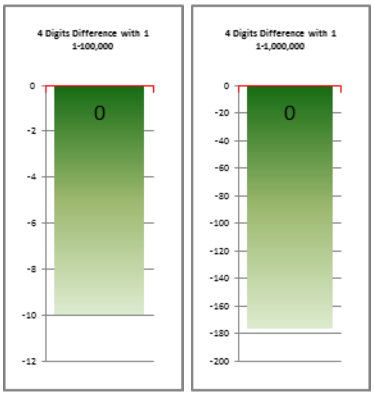


Figure 5: Differences of Number of Primes with 3 0's & those with 3 1's in Ranges of 1 - 10^n.

For three digits also, in each of our range, primes with 3 0's are less than those with 3 1's.



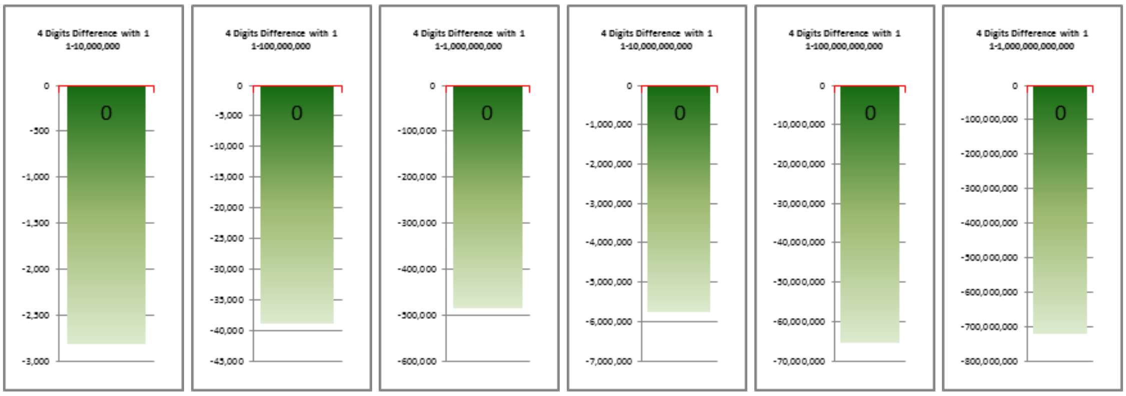


Figure 6: Differences of Number of Primes with 4 0's & those with 4 1's in Ranges of 1 - 10^n.

Here too in each range, primes with 4 0's are less than those with 4 1's.

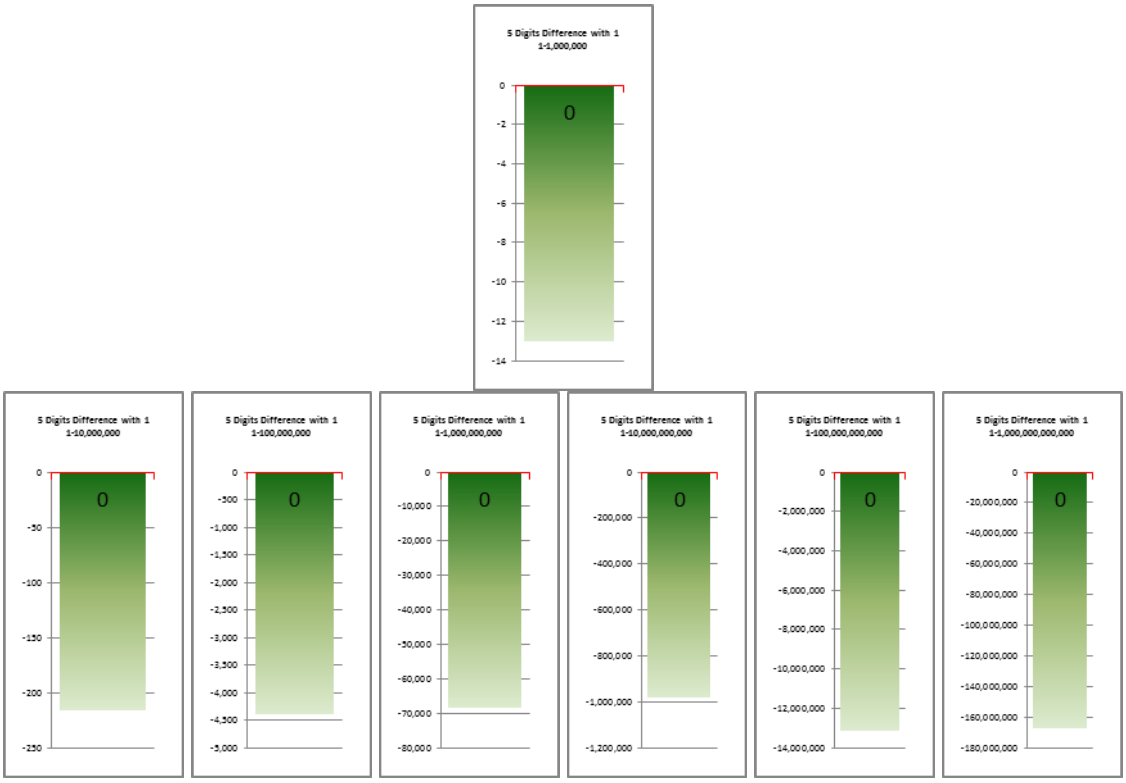


Figure 7: Differences of Number of Primes with 5 0's & those with 5 1's in Ranges of 1 - 10^n.

Again, primes with 5 0's are less than those with 5 1's.

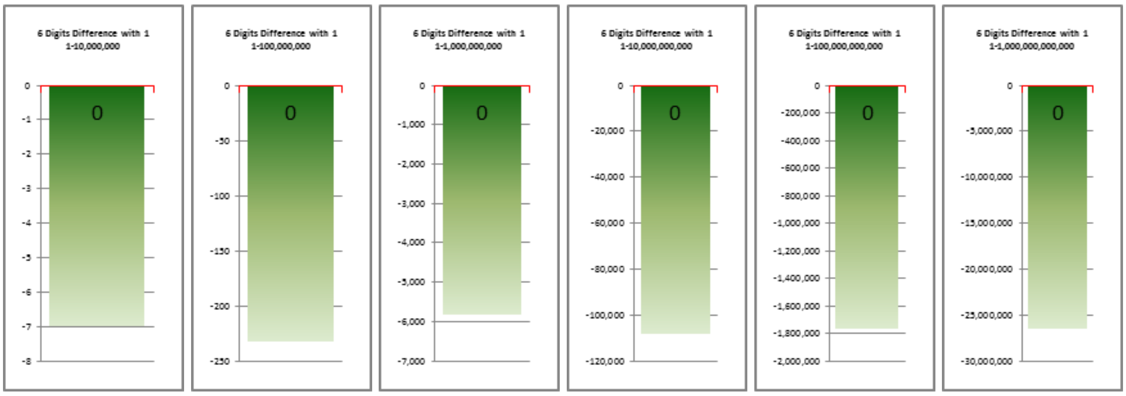


Figure 8: Differences of Number of Primes with 6 0's & those with 6 1's in Ranges of 1 - 10^n.

Trend continues & primes with 6 0's are less than those with 6 1's in our ranges.

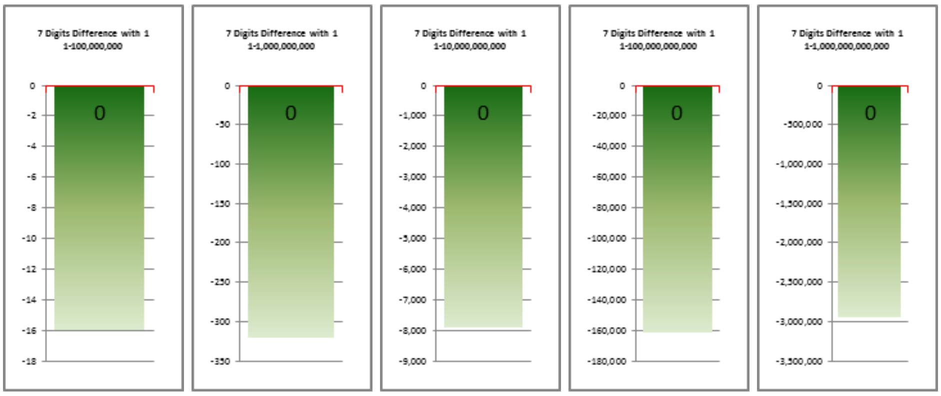


Figure 9: Differences of Number of Primes with 7 0's & those with 7 1's in Ranges of 1 - 10^n.

The case of number of primes with 7 0's and 7 1's is not an exception.

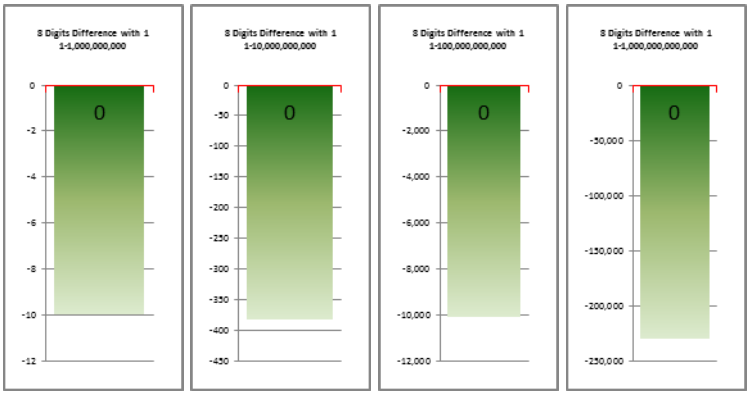


Figure 10: Differences of Number of Primes with 8 0's & those with 8 1's in Ranges of 1 - 10^n.

The primes with 8 0's are also lesser in number than those with 8 1's for our ranges.

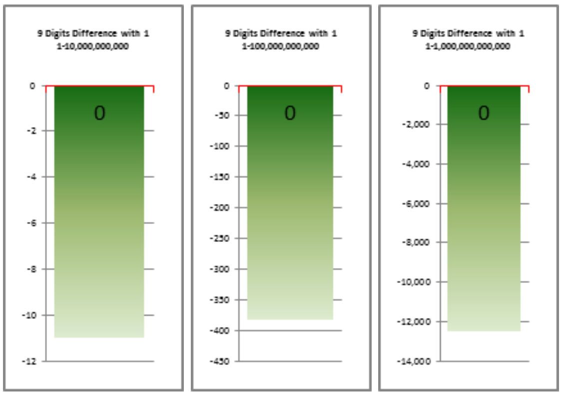


Figure 11: Differences of Number of Primes with 9 0's & those with 9 1's in Ranges of 1 - 10^n.

For the case of 9 repetitions of digits, primes with 1's dominate those with 0's.

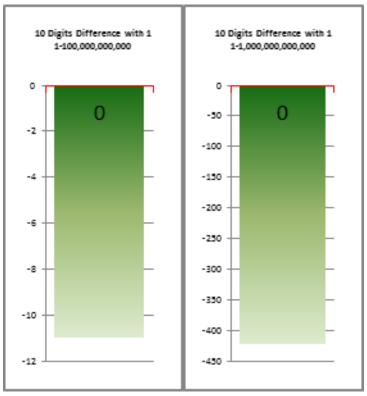


Figure 12: Differences of Number of Primes with 10 0's & those with 10 1's in Ranges of 1 - 10^n.

For 10 digit repetitions also, there are more primes with 1's than 0's in our ranges.

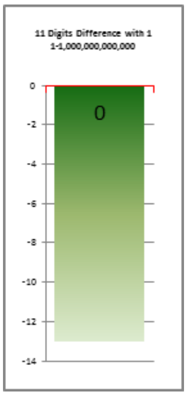


Figure 13: Differences of Number of Primes with 11 0's & those with 11 1's in Ranges of 1 - 10^n.

Finally, in the race of counts of the maximum 11 repetitions of a digit, primes with those many 0's are defeated with equal 1's.

4. FIRST OCCURRENCE OF DIGIT 1 IN PRIME NUMBERS

We know that the first natural number containing 1 digit 1 is the very first natural number 1 itself! For sufficiently large ranges, for 2 1's, the first example is 11, for 3 it is 111 and so on. It has been formulated as

Formula 1 [11] : If n and r are natural numbers, then the first occurrence of r number of 1's in numbers in range $1 \leq m < 10^n$ is

$$f = \begin{cases} - & \text{, if } r > n \\ \sum_{j=0}^{r-1}\left(1 \times 10^j\right), & \text{if } r \leq n \end{cases}.$$

Here we are concentrating on prime numbers. The first occurrence of r number of 1's in prime numbers in these range $1 \leq m < 10^n$ couldn't yet be fit into a formula. We have determined these by intensive calculations.

Table 3: First Prime Numbers in Various Ranges with Multiple 1's in Their Digits

Sr. No.	Range	First Prime Number in Range with						
		1 1	2 1's	3 1's	4 1's	5 1's	6 1's	7 1's
1.	$1 - 10^1$	-	-	-	-	-	-	-
2.	$1 - 10^2$	13	11	-	-	-	-	-
3.	$1 - 10^3$	13	11	-	-	-	-	-
4.	$1 - 10^4$	13	11	1,117	-	-	-	-
5.	$1 - 10^5$	13	11	1,117	10,111	-	-	-
6.	$1 - 10^6$	13	11	1,117	10,111	101,111	-	-
7.	$1 - 10^7$	13	11	1,117	10,111	101,111	1,111,151	-
8.	$1 - 10^8$	13	11	1,117	10,111	101,111	1,111,151	11,110,111
9.	$1 - 10^9$	13	11	1,117	10,111	101,111	1,111,151	11,110,111
10.	$1 - 10^{10}$	13	11	1,117	10,111	101,111	1,111,151	11,110,111
11.	$1 - 10^{11}$	13	11	1,117	10,111	101,111	1,111,151	11,110,111
12.	$1 - 10^{12}$	13	11	1,117	10,111	101,111	1,111,151	11,110,111

Table 3: Continued …

Sr. No.	Range	First Prime Number in Range with			
		8 1's	9 1's	10 1's	11 1's
1.	$1 - 10^1$	-	-	-	-
2.	$1 - 10^2$	-	-	-	-
3.	$1 - 10^3$	-	-	-	-
4.	$1 - 10^4$	-	-	-	-
5.	$1 - 10^5$	-	-	-	-
6.	$1 - 10^6$	-	-	-	-

Sr. No.	Range	First Prime Number in Range with			
		8 1's	9 1's	10 1's	11 1's
7.	$1-10^7$	-	-	-	-
8.	$1-10^8$	-	-	-	-
9.	$1-10^9$	101,111,111	-	-	-
10.	$1-10^{10}$	101,111,111	1,111,111,121	-	-
11.	$1-10^{11}$	101,111,111	1,111,111,121	11,111,111,113	-
12.	$1-10^{12}$	101,111,111	1,111,111,121	11,111,111,113	101,111,111,111

5. LAST OCCURRENCE OF DIGIT 1 IN PRIME NUMBERS

The last occurrence of r number of 1's within integers in ranges $1-10^n$, $1 \le n \le 12$, has been determined.

Formula 2 [11] : If n and r are natural numbers, then the last occurrence of r number of 1's in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & ,\text{if } r > n \\ \sum_{j=0}^{r-1}\left(1\times 10^j\right) + \begin{cases} 0 & ,\text{if } r = n \\ \sum_{j=r}^{n-1}\left(9\times 10^j\right) & ,\text{if } r < n \end{cases} \end{cases}.$$

As again primes don't fit in any such formula, the last prime numbers with r number of 1's in them in these ranges $1-10^n$, $1 \le n \le 12$, have been rigorously determined.

Table 4: Last Prime Numbers in Various Ranges with Multiple 1's in Their Digits

Sr. No.	Number of 1's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	-	71	991	9,941	99,991	999,961	9,999,991	99,999,971
2.	2	-	11	911	9,811	99,611	999,611	9,999,511	99,999,611
3.	3	-	-	-	8,111	95,111	998,111	9,991,811	99,998,111
4.	4	-	-	-	-	16,111	971,111	9,911,171	99,941,111
5.	5	-	-	-	-	-	911,111	9,511,111	99,151,111
6.	6	-	-	-	-	-	-	1,171,111	94,111,111
7.	7	-	-	-	-	-	-	-	71,111,111
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 4: Continued …

Sr. No.	Number of 1's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,761	9,999,999,881	99,999,999,871
2.	2	999,999,191	9,999,999,511	99,999,999,119

Sr. No.	Number of 1's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
3.	3	999,995,111	9,999,994,111	99,999,991,411
4.	4	999,911,411	9,999,931,111	99,999,991,111
5.	5	999,911,111	9,999,181,111	99,999,611,111
6.	6	998,111,111	9,991,151,111	99,997,111,111
7.	7	991,111,111	9,991,111,111	99,961,111,111
8.	8	131,111,111	9,511,111,111	99,811,111,111
9.	9	-	1,711,111,111	95,111,111,111
10.	10	-	-	31,111,111,111
11.	11	-	-	-

Remark : The maximum number of 1's in any prime number in the range 1 – 10^n is at most n; and when for some n if it is actually n, that prime number is nothing but just the repunit prime.

The numbers in various sections of this paper give rise to new integer sequences and deserve separate treatment.

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