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THE SPLIT COMPLEMENT LINE DOMINATION IN GRAPHS

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ABSTRACT

Harary and Norman introduced the Line graph L(G). We introduced the Split complement line domonation number. In this paper, we extend the Split domination concept for the complement of line graph and defined Split complement line domination number for the graph G. Also, we studied its graph theoretical properties in terms of elements of G.

Keywords: Graph, Line graph, Domination number, Line domination number, Split line domination number, Split Complement line domination number.

AMS subject classification: 05C69, 05C76.

1.Introduction

The graph we mean G = (V, E) is a finite, simple, undirected, and connected graph with p vertices and q edges. Terms not defined here are used in the sense of Harary[1].

Line graph L(G) is the graph whose vertices correspond to the edge of G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent. This was introduced by Harary and Norman in [9].

A set $D \subseteq V(G)$ of a graph is a **dominating set** of G, if every vertex in V\D is adjacent to some vertices in D. This concept was introduced by Ore in [2].

A set $D \subseteq V(L(G))$ is said to be **line dominating** set of G, if every vertex not in D is adjacent to a vertex in D. The **line domination number** of G, is denoted by $\gamma_l(G)$ is the minimum cardinality of a line dominating set. This concept was introduced by Harary and Norman in [9].

A dominating set $D \subseteq V(G)$ is a **split dominating set**, if the induced subgraph <V\D> is diconnected. This concept was introduced by V.R.Kulli and B.Janakiram in[7].

A line dominating set $D \subseteq V\big(L(G)\big)$ is a **split line dominating set**, if the induced subgraph < V (L(G)) \ D > is disconnected. The minimum cardinality of such a set is called a **split line domination number** of G and is denoted by $\gamma_{sl}(G)$. This was introduced by M.H.Muddenbihal, U.A.Panfarosh, Anil R.Sedamkar[10].

In this paper, we introduced the split complement line domination in graphs. The bound of this parameter was studied and its exact values was obtained for some standard graph.

2.MAIN RESULTS

Definition 2.1

A dominating set D of a complement line graph $\overline{L(G)}$ is said to be a **split complement line dominating set** (SCLD-set), if the induced subgraph $< V\,\overline{(L(G))}\ \backslash D >$ is disconnected. The minimum cardinality of SCLD-set is called **split complement line domination number** of G and is denoted by $\gamma_{\overline{sl}}(G)$.

Example:

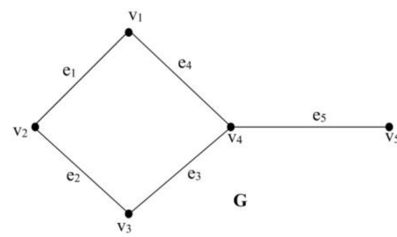


Figure 1

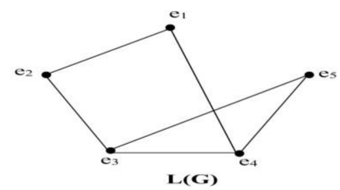


Figure 2

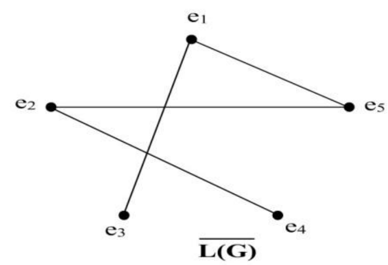


Figure 3

For the graph $\overline{L(G)}$ in Figure 3, the vertex set D = {e_1, e_2} is a $\boldsymbol{\gamma_{\overline{sl}}}$-set and hence $\boldsymbol{\gamma_{\overline{sl}}(G)}$= 2.

Theorem 2.2:

For the cycle graph C_n,

$$\gamma_{\overline{sl}}(C_n) = \begin{cases} 2 & ,if\ n = 4 \\ (n-3) & ,if\ n \geq 5 \end{cases}$$

Proof:

Let G be a cycle graph with atleast 4 vertices.

Let V (G) = {v_1, v_2,………, v_n} and E(G)={e_1, e_2,………,e_n} be the vertex and edge set of G.

Then V $(\overline{L(G)})$ = {e_1, e_2,………,e_n} is the vertex set of $\overline{L(G)}$, n≥ 4

case i: n = 4

The result is obvious.

case ii: n≥ 5

Then D = {e_1, e_2,………,e_{n-3}} is a SCLD-set of G, and hence

$\gamma_{\overline{sl}}(G) \le |D|$=n-3 ………..(1)

Conversely,

Suppose D is a $\gamma_{\overline{sl}}$-set of G. For the disconnectedness of < V $(\overline{L(G)})$\D>, it must contain atleast two vertices and hence D must contain atleast n - 2 vertices.

Hence, | D | ≥ n- 2 ≥ n - 3

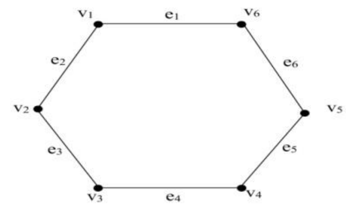
Therefore,

$\gamma_{\overline{sl}}(G) \ge$ n-3 …………(2)

From (1) and (2), we get

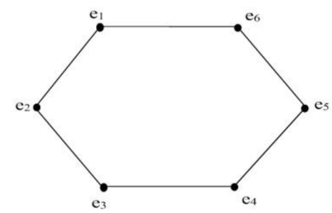
$$\gamma_{\overline{sl}}(G) = n-3 ,\ n \ge 5$$

Example:



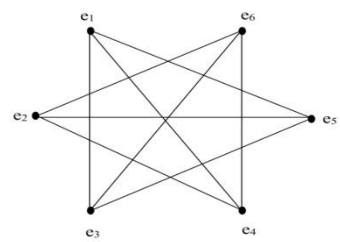
C6

Figure 4



L(C6)

Figure 5



$\overline{L(C_6)}$

Figure 6

For the graph $\overline{L(C_6)}$ in Figure 6, the vertex set D = {e_1, e_2, e_3} is a $\gamma_{\overline{sl}}$- set and hence $\gamma_{\overline{sl}}(C_6)$ = 3.

Theorem 2.3:

For the path graph P_n,

$$\gamma_{\overline{sl}}(P_n) = \begin{cases} 2 & if\ n = 5 \\ (n-4) & if\ n \ge 6 \end{cases}$$

Proof:

Let G be a path graph P_n with atleast 5 vertices.

Let V (G) = {v_1, v_2,………, v_n} and E(G)={e_1, e_2,……….., e_{n-1}} be the vertex and edge set of G.

Then V $(\overline{L(G)})$ = {e_1, e_2,……….., e_{n-1}} is the vertex set of $\overline{L(G)}$, n ≥ 5

case i: n = 5

The result is obvious.

case ii: $n \geq 6$

In this case, D = {e_1, e_2,……….., e_{n-4}} is a SCLD-set of G, and hence

$\gamma_{\overline{sl}}(G) \leq |D| = n-4$ ………..(1)

Conversely,

Suppose D is a $\gamma_{\overline{sl}}$-set of G.

For the disconnectedness of <$V\overline{(L(G))}$\D>, it must contain atleast two vertices and hence D must contain atleast $n-2$ vertices.

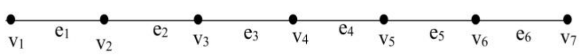
Hence, | D | ≥ n- 2 ≥ n - 4

Therefore,

$\gamma_{\overline{sl}}(G) \geq n-4$ …………(2)

The result follows from (1) and (2).

Example

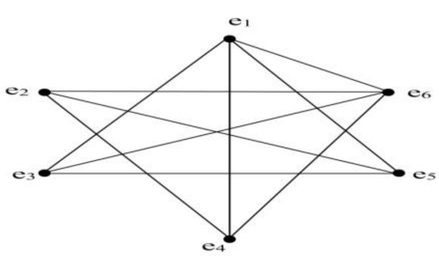


P_7

Figure 7

$L(P_7)$

Figure 8



$\overline{L(P_7)}$

Figure 9

For the graph $\overline{L(P_7)}$ in Figure 9, the vertex set D = {e_1, e_2, e_3} is a $\gamma_{\overline{sl}}$ -set and hence $\gamma_{\overline{sl}}(P_7) = 3$.

Theorem 2.4:

For the complete bipartite graph $K_{m,n}$,

$$\gamma_{\overline{sl}}(K_{m,n}) = \begin{cases} 2 & if\ m = n = 2 \\ n & if\ m = 2, n \neq 2 \\ m & if\ m \neq 2, n = 2 \\ (m-1)(n-1) & if m > 2, n > 2 \end{cases}$$

Proof:

Let G be the complete bipartite graph$K_{m,n}$, m, n ≥ 2.

Let V (G) = {u_1, u_2,........u_m, v_1,v_2,……,v_n}

And E(G) = {u_iv_j/ i=1 to m, j=1 to n}be the vertex and edge set of G.

Then V $\overline{(L(G))}$ = {u_iv_j/ i = 1 to m, j =1 to n} is the vertex set of $\overline{L(G)}$.

case i: m = n = 2

In this case, the set D = {u_1v_1, u_1v_2} is a SCLD-set with minimum cardinality.

Hence the result is obvious.

case ii: m = 2; n ≠ 2

In this case, the set D = {u_1v_j / j=1,2,…..,n} is a SCLD-set with minimum cardinality, and < V $\overline{(L(G))}$ \ D > is disconnected, which gives

| D | = n

Hence, $\gamma_{\overline{sl}}(G)$ = | D | = n

case iii: m ≠ 2, n = 2

In this case, the set D = {u_iv_1/ i=1,2,….,m} is a SCLD-set with minimum cardinality, and < V$\overline{(L(G))}$ \D > is disconnected, which gives

|D| = m

Hence, $\gamma_{\overline{sl}}(G)$ = | D | = m

case iv: m > 2, n > 2

In this case, the set $D = \{u_1v_j \,/\, j = 1 \text{ to } n\text{-}1\} \cup \{u_2v_j \,/\, j = 1 \text{ to } n\text{-}1\} \cup \ldots\ldots \cup \{u_{m-1}v_j \,/\, j = 1 \text{ to } n\text{-}1\}$ is a SCLD-set with minimum cardinality, and $< V(\overline{L(G)}) \setminus D >$ is disconnected, which gives

$|D| = (m-1)(n-1)$

Hence,

$$\gamma_{\overline{sl}}(G) = |D| = (m-1)(n-1)$$

Example

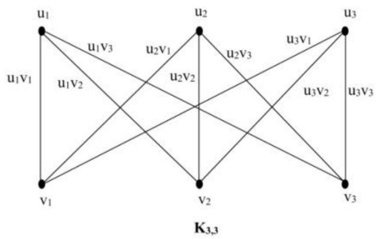


Figure 10

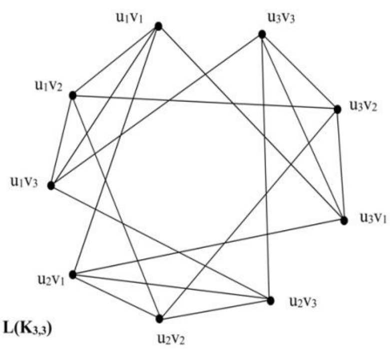


Figure 11

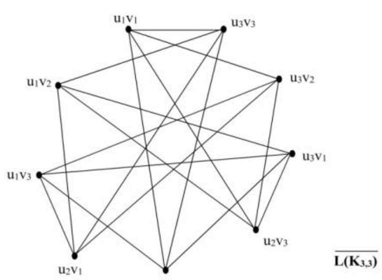


Figure 12

For the graph $\overline{L(K_{3,3})}$ in Figure 12, the vertex set $D = \{u_1v_1, u_1v_2, u_2v_1, u_2v_2\}$ is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(K_{3,3}) = 4$.

Theorem 2.5:

For the wheel graph W_n,

$$\gamma_{\overline{sl}}(W_n) = \begin{cases} 3 & if\ n = 3,4 \\ n-2 & if\ n \geq 5 \end{cases}$$

Proof:

Let G be a wheel graph W_n, with atleast 3 vertices.

Let $V(G) = \{u, v_1, v_2, \ldots, v_n\}$ and $E(G)=\{e_1, e_2, \ldots\ldots, e_{2n}\}$ be the vertex and edge set of G.

Then $V(\overline{L(G)}) = \{e_1, e_2, \ldots\ldots, e_{2n}\}$ is the vertex set of $\overline{L(G)}$, $n \geq 3$

case i: n = 3; 4

The result is obvious.

case ii: $n \geq 5$

In this case, $D = \{e_3, e_5, \ldots\ldots, e_{2n-3}\}$ is a SCLD-set of G, and hence

$\gamma_{\overline{sl}}(G) \leq |D| = n\text{-}2$ ……………(1)

Conversely,

Suppose D is a $\gamma_{\overline{sl}}$-set of G.

For the disconnectedness of $<V(\overline{L(G)}) \setminus D>$, it must contain atleast two vertices and hence D must contain atleast n−2 vertices.

Hence, $|D| \geq n\text{-}2$

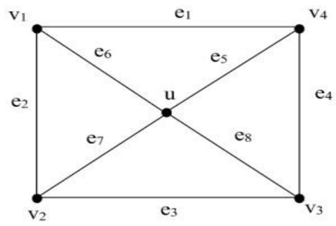
Therefore,

$\gamma_{\overline{sl}}(G) \geq n\text{-}2$ ……………(2)

From (1) and (2), we get

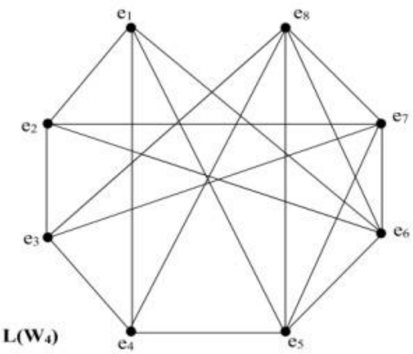
$$\gamma_{\overline{sl}}(G) = n\text{-}2, \ n \geq 5$$

Example



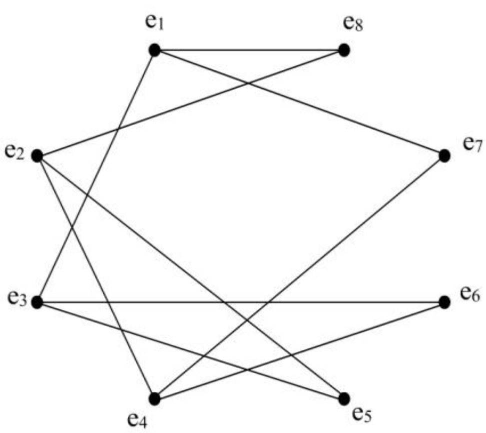
W_4

Figure 13



$L(W_4)$

Figure 14



$\overline{L(W_4)}$

Figure 15

For the graph $\overline{L(W_4)}$ in Figure 15, the vertex set D = {e_1, e_2, e_3} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(W_4) = 3$.

Theorem 2.6:

For the Bistar tree $B_{n,n}$,

$$\gamma_{\overline{sl}}(B_{n,n}) = n+1,\ n \geq 2$$

Proof:

Let G be the Bistar tree $B_{n,n}$, $n \geq 2$

Let V (G) = {v_1, v_2,….., $v_{2(n+1)}$} and E(G)={e_1, e_2,……, e_{2n+1}} be the vertex and edge set of G.

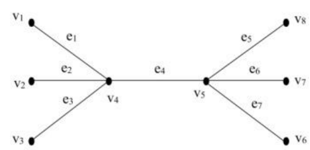
Then $V(\overline{L(G)})$ = {e_1, e_2,……, e_{2n+1}} is the vertex set of $\overline{L(G)}$.

Then the set D = {e_1,e_2,……,e_{n+1}} is a SCLD-set with minimum cardinality, and <V($\overline{L(G)}$) \ D > is disconnected, which gives

$$|D| = n + 1.$$

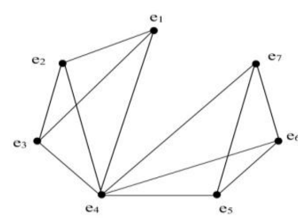
Hence, $\gamma_{\overline{sl}}(G) = |D| = n+1,\ n \geq 2$

Example:



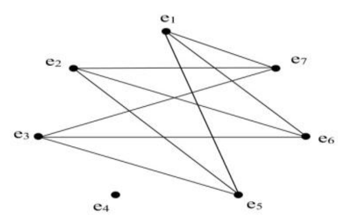
$B_{3,3}$

Figure 16



$L(B_{3,3})$

Figure 17



$\overline{L(B_{3,3})}$

Figure 18

For the graph $\overline{L(B_{3,3})}$ in Figure 18, the vertex set D = {e_1, e_2, e_3, e_4} is a $\gamma_{\overline{sl}}$ -set and hence $\gamma_{\overline{sl}}(B_{3,3}) = 4$.

Theorem 2.7:

For the Crown graph C_n^+,

$$\gamma_{\overline{sl}}\ (C_n^+) = \begin{cases} 3, & if\ n = 3 \\ 2n-5, & if\ n \geq 4 \end{cases}$$

Proof:

Let G be a Crown graph C_n^+, n ≥ 3.

Let V (G) = {v_1, v_2,….., v_{2n}} and E(G)={e_1,e_2,……,e_{2n}} be the vertex and edge set of G.

Then $V(\overline{L(G)})$ = {e_1,e_2,……,e_{2n}} is the vertex set of $\overline{L(G)}$, n ≥ 3.

case i: n = 3

In this case, the set D = {e_1, e_3, e_5} is a SCLD-set of G with minimum cardinality.

Then |D| = 3

Hence the result is obvious.

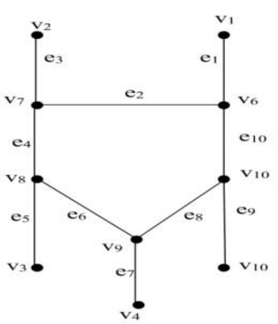
case ii: n ≥ 4

In this case, the set D={e_3,e_4,e_5,….., e_{2n-3}} is a SCLD-set of G with minimum cardinality, and <$V(\overline{L(G)})$\D> is disconnected, which gives

|D| = 2n-5.

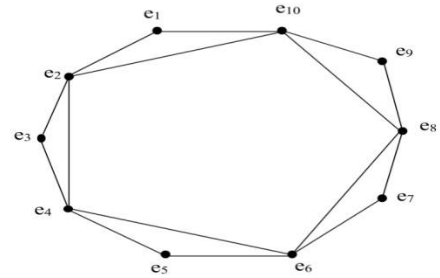
Hence, $\gamma_{\overline{sl}}$(G) = | D | = 2n-5, n ≥ 4

Example:



C5+

Figure 19



L(C5+)

Figure 20

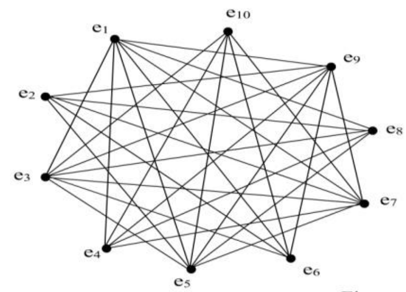


Figure 21

$\overline{L(C_5^+)}$

For the graph $\overline{L(C_5^+)}$ in Figure 21, the vertex set D={e_5,e_6,e_7,e_8,e_9} is a $\gamma_{\overline{sl}}$ -set and hence $\gamma_{\overline{sl}}(C_5^+)$=5.

Theorem 2.8:

For the Comb tree P_n^+,

$$\gamma_{\overline{sl}}(P_n^+) = \begin{cases} 2 & if\ n = 3 \\ 2n-6 & if\ n \geq 4 \end{cases}$$

Proof:

Let G be a Comb tree P_n^+, n ≥ 3

Let V (G) = {v_1,v_2,……,v_{2n}} and E(G)={e_1,e_2,…….,e_{2n-1}} be the vertex and edge set of G.

Then $V(\overline{L(G)})$={e_1,e_2,…….., e_{2n-1}} is the vertex set of $\overline{L(G)}$, n ≥ 3

case i: n = 3

In this case, the set D={e_1, e_2} is a SCLD-set of G with minimum cardinality.Then,

|D| = 2.

Hence the result is obvious.

case ii: $n \geq 4$

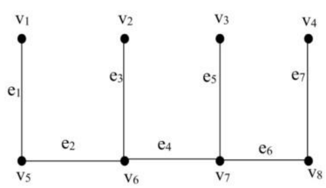
In this case, the set $D=\{e_1,e_2,\ldots\ldots,e_{2n-7}\} \cup \{e_{2n-1}\}$is a SCLD-set of G with minimum cardinality, and $<V(\overline{L(G)})\backslash D>$ is disconnected, which gives

$$|D| = 2n-7+1=2n-6.$$

Hence,

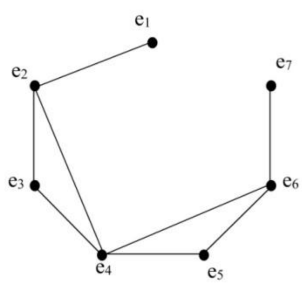
$$\gamma_{s\bar{l}}(G) = |D| = 2n-6, n \geq 4$$

Example:



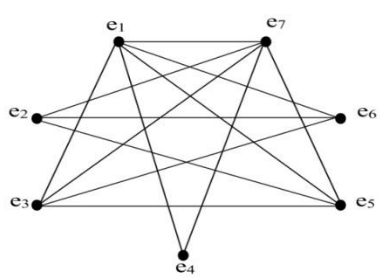
P_4^+

Figure 22



$L(P_4^+)$

Figure 23



$\overline{L(P_4^+)}$

Figure 24

For the graph $\overline{L(P_4^+)}$ in Figure 24, the vertex set $D = \{e_1,e_7\}$ is a $\gamma_{s\bar{l}}$-set and hence $\gamma_{s\bar{l}}(P_4^+)=2$.

Theorem 2.9:

For the Helm graph W_n^+,

$$\gamma_{s\bar{l}}(W_n^+) = \begin{cases} 3 & if\ n = 2 \\ 2n-3 & if\ n \geq 3 \end{cases}$$

Proof:

Let G be a Helm graph W_n^+, $n \geq 2$.

Let $V(G) =\{u,u_1,u_2,\ldots..,u_n,v_1,v_2,\ldots..,v_n\}$ and $E(G) =\{e_1,e_2,\ldots\ldots..,e_{3n+1}\}$ be the vertex and edge set of G.

Then $V(\overline{L(G)})= \{e_1,e_2,\ldots\ldots..,e_{3n+1}\}$ is the vertex set of $\overline{L(G)}$, $n \geq 2$

case i: $n = 2$

In this case, the set $D = \{e_1,e_2,e_3\}$ is a SCLD-set of G with minimum cardinality. Then, $|D| = 3$

Hence the result is obvious.

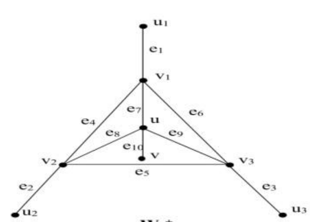
case ii: $n \geq 3$

In this case, the set $D =\{e_1,e_2,e_3,\ldots\ldots,e_{n-1}\} \cup \{e_{n+1},e_{n+2},\ldots\ldots,e_{2n-1}\}$is a SCLD-set of G with minimum cardinality, and $<V(\overline{L(G)})\backslash D>$ is disconnected, which gives

$$|D| = n - 1 + n - 2 = 2n - 3$$

Hence, $\gamma_{s\bar{l}}(G)= |D| = 2n – 3, n \geq 3$.

Example:



W_3^+

Figure 25

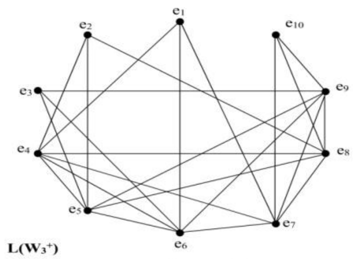


Figure 26

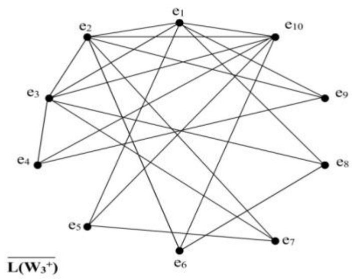


Figure 27

For the graph $\overline{L(W_3^+)}$ in Figure 27, the vertex set D = {e_1,e_2,e_4} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(W_3^+)$=3.

Theorem 2.10:

For the graph K_n^+,

$$\gamma_{\overline{sl}}(K_n^+)=\begin{cases} 3, & if\ n=3 \\ \frac{n(n-3)}{2}+1, & if\ n\geq 4 \end{cases}$$

Proof:

Let G be a K_n^+ graph, n ≥ 3.

Let V (G) = {v_1, v_2,……,v_{2n}} and E(G)={e_1,e_2,……,$e_{\frac{n(n+1)}{2}}$} be the vertex and edge set of G.

Then V($\overline{L(G)}$)= {e_1, e_2,……, $e_{\frac{n(n+1)}{2}}$} is the vertex set of $\overline{L(G)}$, n ≥ 3

case i: n = 3

In this case, the set D = {e_1,e_3,e_6} is a SCLD-set of G with minimum cardinality.

Then, $|D|$ = 3

Hence the result is obvious.

case ii: n ≥ 4

In this case, the set D= {e_1} ∪ {e_{2n+1},e_{2n+2},……, $e_{\frac{n(n+1)}{2}}$} is a SCLD-set of G with minimum cardinality, and <V($\overline{L(G)}$)\D> is disconnected, which gives |D| = $\frac{n(n-3)}{2}+1$

Hence, $\gamma_{\overline{sl}}$(G)= |D| = $\frac{n(n-3)}{2}+1$, n ≥ 4.

Example:

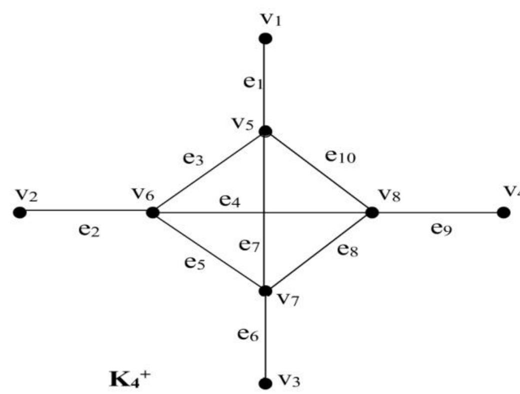


Figure 28

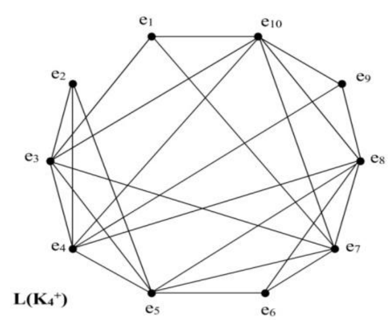


Figure 29

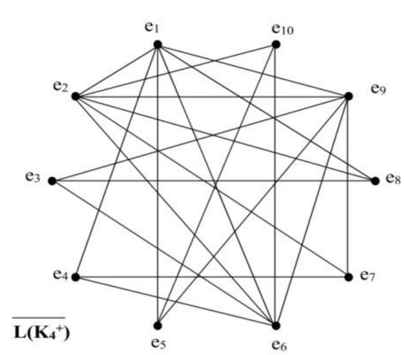


Figure 30

For the graph $\overline{L(K_4^+)}$ in Figure 30, the vertex set D = {e_1,e_9,e_{10}} is a $\gamma_{\bar{sl}}$-set and hence $\gamma_{\bar{sl}}(K_4^+)$=3.

Theorem 2.11:

For the Book graph B_n,

$$\gamma_{\bar{sl}}(B_n)=\begin{cases} 2, & if\ n=1 \\ n, & if\ n\geq 2 \end{cases}$$

Proof:

Let G be a Book graph B_n, n ≥ 1.

Let V (G) = {u,v,v_1,v_2,……..,v_{2n}} and E(G)={e,e_1,e_2,……., e_{3n}} be the vertex and edge set of G.

Then V($\overline{L(G)}$)= {e,e_1,e_2,……., e_{3n}} is the vertex set of $\overline{L(G)}$, n ≥ 1

case i: n = 1

In this case, the set D = {e,e_1} is a SCLD-set of G with minimum cardinality.

Then, |D| = 2

Hence the result is obvious.

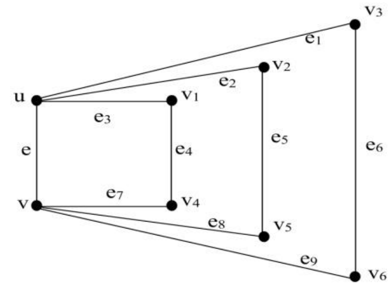
case ii: n ≥ 2

In this case, the set D={e_{n+1},e_{n+2},……., e_{2n}} is a SCLD-set of G with minimum cardinality, and <V($\overline{L(G)}$)\D> is disconnected, which gives

|D|= n

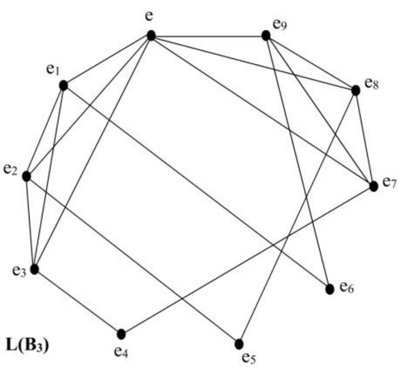
Hence, $\gamma_{\bar{sl}}(G)$= |D| = n, n ≥ 2.

Example:



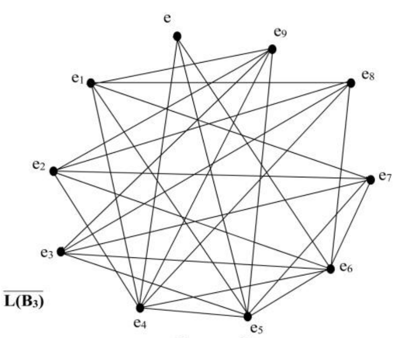
B_3

Figure 31



L(B_3)

Figure 32



$\overline{L(B_3)}$

Figure 33

For the graph $\overline{L(B_3)}$ in Figure 33, the vertex set D={e_4,e_5,e_6} is a $\gamma_{\bar{sl}}$-set and hence $\gamma_{\bar{sl}}(B_3)$=3.

Theorem 2.12:

For the Friendship graph $C_3^{(m)}$,

$$\gamma_{\bar{sl}}\left(C_3^{(m)}\right)=\begin{cases} 2, & if\ m=2 \\ m-1, & if\ m\geq 3 \end{cases}$$

Proof:

Let G be a Friendship graph $C_3^{(m)}$, m≥2

Let V (G) = {u,v_1,v_2,……..,v_{3m}} and E(G)={e_1,e_2,……, e_{3m}} be the vertex and edge set of G.

Then V($\overline{L(G)}$)= {e_1,e_2,……, e_{3m}} is the vertex set of ($\overline{L(G)}$, m ≥ 2.

case i: m = 2

In this case, the set D = {e_1,e_2} is a SCLD-set of G with minimum cardinality.

Then,

$$|D|= 2$$

Hence the result is obvious.

case ii: m ≥ 3

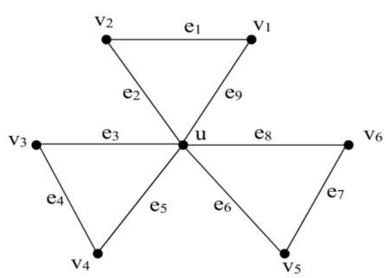
In this case, the set D={e_1,e_4,e_7,……,e_{3m-5}} is a SCLD-set of G with minimum cardinality, and <V($\overline{L(G)}$)\D> is disconnected, which gives

$$|D|= m-1$$

Hence,

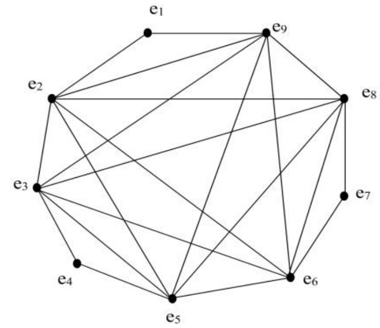
$$\gamma_{\overline{sl}}(G) = |D|= m – 1, m \geq 3.$$

Example:



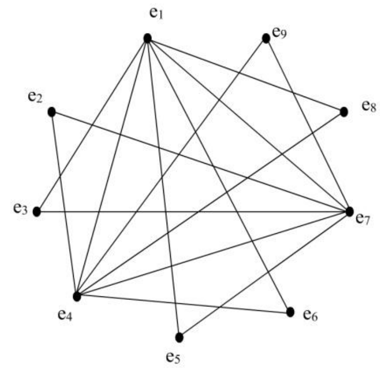
C_3^3

Figure 34



$L(C_3^3)$

Figure 35



$\overline{L(C_3^3)}$

Figure 36

For the graph $\overline{L(C_3^3)}$ in Figure 36, the vertex set D={e_1,e_4} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(C_3^3)$=2.

Theorem 2.13:

For the Triangular snake graph mC_3,

$$\gamma_{\overline{sl}}(mC_3)=\begin{cases} 2, & if\ m = 2 \\ 3m-7, & if\ m \geq 3 \end{cases}$$

Proof:

Let G be a Triangular snake graph mC_3, m≥2.

Let V (G) ={v_1,v_2,……..,v_{2m+1}} and E(G)={e_1,e_2,……., e_{3m}} be the vertex and edge set of G.

Then $V(\overline{L(G)})$={$e_1,e_2,\ldots\ldots,e_{3m}$} is the vertex set of $\overline{L(G)}$, m ≥ 2.

case i: m = 2

In this case, the set D={e_1,e_4} is a SCLD-set of G with minimum cardinality. Then

|D|= 2

Hence the result is obvious.

case ii: m ≥ 3

In this case, the set D={$e_1,e_2,e_3,\ldots\ldots,e_{2m-5}$}∪ {$e_{2m},e_{2m+1},\ldots\ldots,e_{3(m-1)}$} is a SCLD-set of G with minimum cardinality, and <$V(\overline{L(G)})$\D> is disconnected, which gives

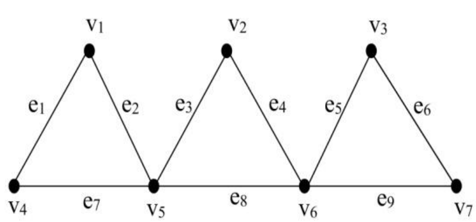
|D| = 2m - 5 + m - 2

= 3m - 7

Hence,

$\gamma_{\overline{sl}}(G)$ = |D|= 3m – 7, m ≥ 3.

Example:



$3C_3$

Figure 37

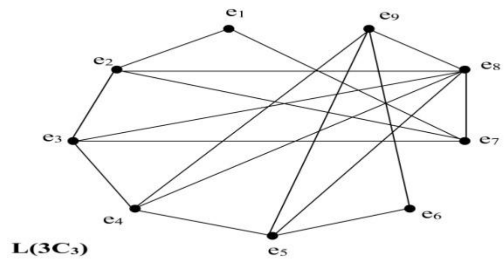


Figure 38

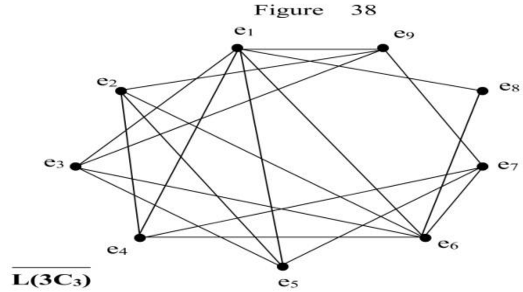


Figure 39

For the graph $\overline{L(3C_3)}$ in Figure 39, the vertex set D={e_1,e_6} is a $\gamma_{\overline{sl}}$-set and hence$\gamma_{\overline{sl}}(3C_3)$=2.

Theorem 2.14:

For the Dragon graph $C_m@P_n$,

$\gamma_{\overline{sl}}$ ($C_m@P_n$) = m + n - 4, m ≥ 3, n ≥ 3

Proof:

Let G be a Dragon graph $C_m@P_n$, m,n≥3.

Let V (G) ={$v_1,v_2,\ldots\ldots, v_{m+n}$} and E(G)={$e_1,e_2,\ldots\ldots, e_{m+n}$} be the vertex and edge set of G.

Then $V(\overline{L(G)})$ ={$e_1,e_2,\ldots\ldots, e_{m+n}$} is the vertex set of $\overline{L(G)}$.

When m,n ≥ 3

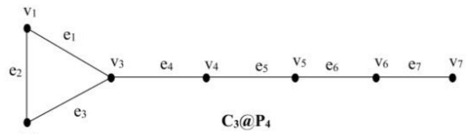
The set D = {$e_1,e_2,e_3,\ldots\ldots, e_{m-3}$}∪ {$e_{m+2},e_{m+3},\ldots\ldots, e_{m+n}$} is a SCLD-set of G with minimum cardinality, and <$V(\overline{L(G)})$\D> is disconnected, which gives

|D| = m - 3 + n + 1

= m + n - 4

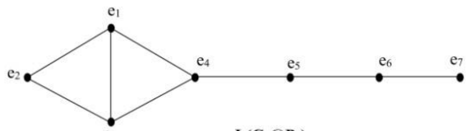
Hence, $\gamma_{\overline{sl}}(G)$ = |D|= m + n– 4, m,n ≥ 3

Example:



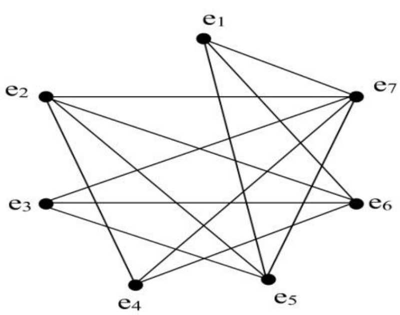
$C_3@P_4$

Figure 40



$L(C_3@P_4)$

Figure 41



$\overline{L(C_3@P_4)}$

Figure 42

For the graph $\overline{L(C_3@P_4)}$ in Figure 42, the vertex set D = {e_5,e_6,e_7} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(C_3@P_4)$=3.

Theorem 2.15:

For the Quadrilateral snake graph mC_4,

$$\gamma_{\overline{sl}}(mC_4)=\begin{cases} 2, & if\ m=1 \\ 3, & if\ m=2 \\ 4m-7, & if\ m\geq 3 \end{cases}$$

Proof:

Let G be a Quadrilateral snake graph mC_4, m ≥ 1.

Let V (G) ={v_1,v_2,……, v_{3m+1}} and E(G)={e_1,e_2,……, e_{4m}} be the vertex and edge set of G.

Then $V(\overline{L(G)})$={e_1,e_2,……..,e_{4m}}is the vertex set of $\overline{L(G)}$, m ≥ 1

case i: m = 1

In this case, the set D ={e_1,e_2} is a SCLD-set of G with minimum cardinality.

Then |D| = 2.

Hence the result is obvious.

case ii: m = 2

In this case, the set D ={e1,e2,e3} is a SCLD-set of G with minimum cardinality.

Then |D| = 3

Hence the result is obvious.

case iii: m ≥ 3

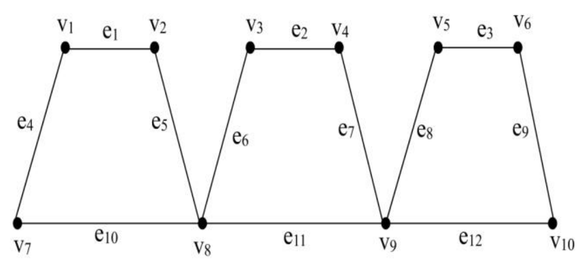
In this case, the set D = {e1,e_2,e_3,…….,e_{m+1}}∪{e_{m+6},e_{m+7},…..,e_{3m}}∪ {e_{3m+4},e_{3m+5},…….,e_{4m}} is a SCLD-set of G with minimum cardinality, and <$V(\overline{L(G)})$\D> is disconnected, which gives

|D| = m + 1 + 2m - 5 + m - 3

= 4m - 7

Hence, $\gamma_{\overline{sl}}(G)$ = |D|= 4m– 7, m ≥ 3.

Example:



$3C_4$

Figure 43

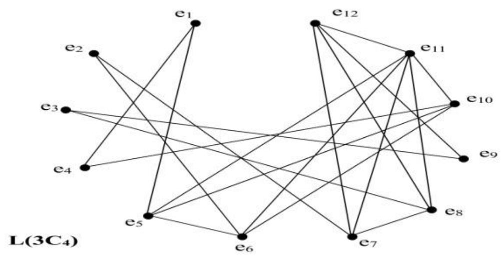


Figure 44

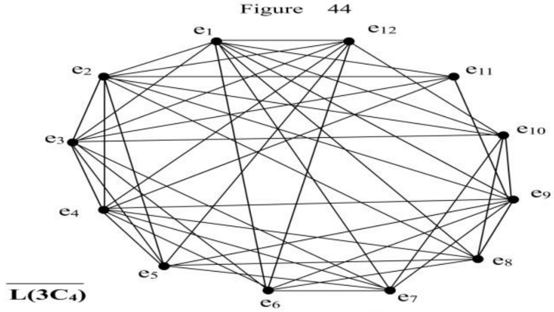


Figure 45

For the graph $\overline{L(3C_4)}$ Figure 45, the vertex set D = {e_1,e_2,e_3, e_4,e_9} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(3C_4)$=5.

Theorem 2.16:

For the graph $K^+_{m,,n}$,

$$\gamma_{\overline{sl}}\left(K^+_{m,,n}\right)=\begin{cases} 0, & if\ m = n = 1 \\ 2, & if\ m = 1, n = 2 \\ 2, & if\ m = 2, n = 1 \\ mn - 1, & otherwise \end{cases}$$

Proof:

Let G be a $K^+_{m,,n}$ graph, m,n ≥ 1.

Let V (G) = {u_1,u_2,….,u_{2m},v_1,v_2,….. v_{2n}} and E(G) ={e_1,e_2,……,e_{m+n},u_iv_j/i=1tom, j=1ton} be the vertex and edge set of G.

Then $V(\overline{L(G)})$ ={e_1,e_2,……,e_{m+n},u_iv_j /i=1tom, j=1ton} is the vertex set of $\overline{L(G)}$.

case i: m = n = 1

In this case, the SCLD does not exist.

case ii: m = 1; n = 2

In this case, the set D = {e_2,e_3} is a SCLD-set of G with minimum cardinality.

Then, |D| = 2

Hence the result is obvious.

case iii: m = 2; n = 1

In this case, the set D = {e_1,e_2} is a SCLD-set of G with minimum cardinality.

Then, |D| = 2

Hence the result is obvious.

case iv: otherwise

In this case, the set D={e_1,e_2,e_3,……,e_{n-1}} ∪ {u_1v_j/ j=1ton-1} ∪ {u_2v_j/j = 1ton – 1} ∪ … ∪ {$u_{m-1}v_j$/j=1ton−1}∪{e_{n+1},e_{n+2},…,e_{m+n-1}} is a SCLD- set of G with minimum cardinality, and <$V(\overline{L(G)})$\D> is disconnected, which gives

|D| = n - 1 + (m - 1)(n - 1) + (m - 1)

= n - 1 + mn - m - n + 1 + m - 1

= mn - 1

Hence, $\gamma_{\overline{sl}}$(G) = |D| = mn - 1.

Example:

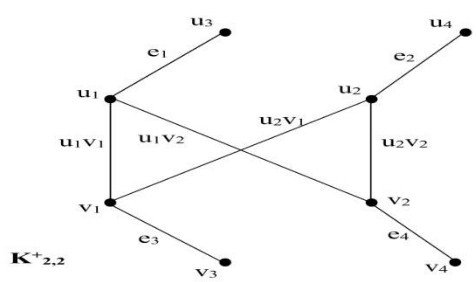


Figure 46

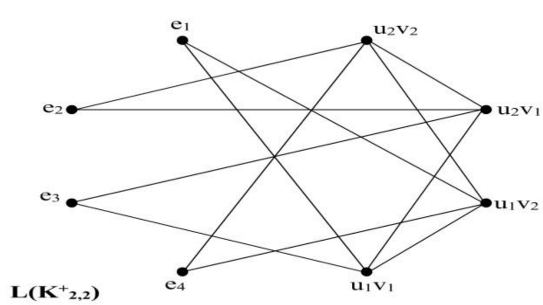


Figure 47

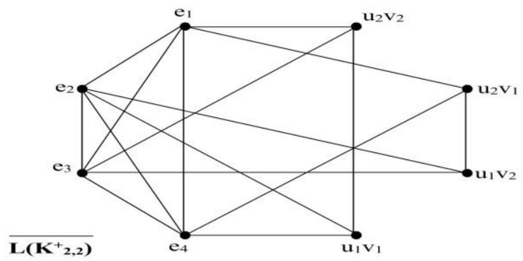


Figure 48

For the graph $\overline{L(K^{+}_{2,2})}$ in Figure 48, the vertex set D = {e_1,e_3,u_1v_1} is a $\gamma_{\overline{sl}}$-set and hence $\gamma_{\overline{sl}}(K^{+}_{2,2})$=3.

3. BOUNDS

Theorem 3.1:

For any graph G, $\gamma_{\overline{l}}$ (G) ≤ $\gamma_{\overline{sl}}$ (G)

Proof:

As, split complement line dominating set is necessarily a complement line dominating set, we have

$$\gamma_{\overline{l}}(G) \leq \gamma_{\overline{sl}}(G)$$

Theorem 3.2:

For any graph G, $\gamma_{\overline{sl}}$(G)= $\gamma_{\overline{l}}$ (G), If $\overline{L(G)}$ contains the set of end vertices.

proof:

Let v ∈ V ($\overline{L(G)}$) be an end vertex and there exists a support vertex u ∈ N(v).

Further let D be a split complement line dominating set of G.

Suppose u ∈ D, then D is a $\gamma_{\overline{sl}}$-set of G.

Suppose u ∉ D,then v ∈ D and hence (D−{v})∪{u} forms a minimal $\gamma_{\overline{sl}}$-set of G. Repeating this process for all end vertices in $\overline{L(G)}$, we obtain a $\gamma_{\overline{sl}}$-set of G containing all the end vertices and hence,

$$\gamma_{\overline{sl}}(G) = \gamma_{\overline{l}}(G)$$

Theorem 3.3

For any graph G, $\gamma_{\overline{sl}}$(G) ≤ q-$\delta(\overline{L(G)})$+1

Proof:

Let V be a vertex set of $\overline{L(G)}$ with minimum degree ≥ 2 implies there exist two vertices v_1 and v_2 adjacent to v.

Consider the vertex set, D = {V\N(v)} ∪ {v_1,v_2}, clearly v and the vertices N(v) are dominated by v1 and v2.

So, D is a vertex set of $\overline{L(G)}$. Also, V\D=N(v)\{v_1 ,v_2} which is disconnected.

Therefore,

$$\gamma_{\overline{sl}}(G) \leq |D| = q - (\delta(\overline{L(G)}) + 1) + 2$$
$$= q - \delta(\overline{L(G)}) + 1$$

Theorem 3.4

For any graph G, $\gamma_{\overline{sl}}$(G)+ γ_{sl}(G) ≤ q+3.

Proof:

By theorem 3.3, we obtain

γ_{sl}(G) ≤ q -δ(L(G)) + 1 and

$\gamma_{\overline{sl}}$(G) ≤ q - $\delta(\overline{L(G)})$+ 1 which implies,

$\gamma_{\overline{sl}}$(G)+ γ_{sl}(G) ≤ 2q-(δ(L(G))+ $\delta(\overline{L(G)})$+2 …………..……(1)

Since,

δ(L(G))+ $\delta(\overline{L(G)}$ ≤ d(e_i)+d($\overline{e_i}$), ∀e_i ∈ $L(G)$ ……………..(2)

where d(e_i) and d($\overline{e_i}$) are respectively denotes degree of the vertex e_i in L(G) and $\overline{L(G)}$.

From the line graph, we have

d(e_i)+d($\overline{e_i}$) = q - 1 ………(3)

Then the result follows from equation (1), (2) and (3).

CONCLUSION

In this paper we introduced split complement line domination number for some standard graphs like Cycle, Path, Complete bipartite, Wheel, Banana, Crown graph, Comb tree, Helm graph, K_n^+ graph, $K_{m,n}^+$ graph, Book graph, Friendship graph, Triangular snake graph, Dragon graph and Quadrilateral snake graph. Also found its bounds and studied the relationship with other domination parameters.

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