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A STUDY OF STRENGTH-RELIABILITY FOR SHUSHILA DISTRIBUTED STRESS

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ABSTRACT

This paper is a study of stress-strength reliability of the distributionsof manufacturing items through establishing the relationship among their parameters, here, Stress follows Shushila distribution. The obtained results are further used to get the optimum cost when the cost function is linear in terms of parameters.

Key Words: Shushila distribution, Power function distribution, Stress-Strength Reliabilityand incomplete gamma function.

1 Introduction:

Reliability of any system become more significant as industries are introducing more and more complex mechanization and automation in the industrial process to meet the increasing demand of society. The science of reliability is concerned with evaluating the risks and their consequences. One of the statistical modelsfor evaluating the risk and their consequences is the stress-strength testing model. The probability model $P = P(X > Y)$ which represent the performance of an item of strength Y subject to a stress X, where X and Y are taken to benon-negative independent continuous random variables. The term stress-strength was first introduced by the Church and Harries (1970). A lot of works havebeendone in this direction by various researchers. For a brief review, one may refer to Downton (1873), Tong (1974),Kelly (1976),Sathe and Vande (1981), Chao (1982), Awad (1986), Chaturvedi and Surinder (1999), Alam and Roohi (2003), etc.

Shankar *etal.*(2013), proposedShushila distribution, which is the mixture of exponential and gamma distribution, in which Lindley distribution is a particular case. In this paper,Shushila distribution has been considered. We obtain strength-reliability of an item for Shushila distributed stress.

2 Strength reliability for finite strength:

An infinite stress distribution is justifiable in the sense that huge stress may tends to infinity but the strength of various devices/equipment's depends upon its subcomponents which may not be recorded as infinite lifetime. Here, the maximum value of the unreliability of items is obtained by $P\left(X>\theta\right)$.Alam and Roohi (2003) have termed it as probability of disaster.

It is assumed that the random variable X represent the stress that item faces, follows the Shushila distribution having probability density function (pdf)

$$f\left(x;\lambda,\sigma\right)=\frac{\sigma^2}{\lambda\left(\sigma+1\right)}\left(1+\frac{x}{\lambda}\right)e^{-\frac{\sigma x}{\lambda}},\ x>0,\lambda>0,\sigma>0 \ (2.1)$$

We obtain the strength reliability when the finite strength follows power function distribution having pdf.

$$g\left(y\right)=\left(\frac{a}{\theta}\right)\left(\frac{y}{\theta}\right)^{a-1},\ 0<y<\theta,\ a>0 \,(2.2)$$

where θ and a are scale and shape parameters respectively.

Theorem 2.1: If the random variable X and Y follows the Shushila distribution $\left(1.1\right)$ and power function distribution$\left(1.2\right)$, respectively, then $\alpha=P\left(X>\theta\right)$ is given by

$$\alpha\equiv P(X>\theta)=\frac{\sigma}{(\sigma+1)}\left[1+m+\frac{1}{\sigma}\right]e^{-m\sigma} \quad (2.3)$$

where $m=\frac{\theta}{\lambda}$

Proof: We know that

$$\alpha\equiv P(X>\theta)=\frac{\sigma^2}{\lambda(\sigma+1)}\int_{\theta}^{\infty}(1+\frac{x}{\lambda})e^{-\frac{\sigma x}{\lambda}}dx$$

$$=\frac{\sigma}{(\sigma+1)}\left[e^{-\frac{\sigma\theta}{\lambda}}+\frac{\theta}{\lambda}e^{-\frac{\sigma\theta}{\lambda}}+\frac{1}{\sigma}e^{-\frac{\sigma\theta}{\lambda}}\right]$$

or,

$$\alpha = \frac{\sigma}{(\sigma+1)}\left[1+m+\frac{1}{\sigma}\right]e^{-m\sigma} \quad (2.4)$$

where $m = \frac{\theta}{\lambda}$

Hence, the theorem follows.

Table 1: Numerical values for Probability of disaster $\alpha = P\left(X > \theta\right)$ for different values of m and σ

m	$P\left(X>\theta\right)$				
	$\sigma=0.5$	$\sigma=1$	$\sigma=1.5$	$\sigma=2$	$\sigma=2.5$
0.5	0.9086	0.7582	0.6141	0.4905	0.3888
1.0	0.8087	0.5518	0.3570	0.2256	0.1407
1.5	0.7085	0.3905	0.2003	0.0996	0.0487
2.0	0.6131	0.2707	0.1095	0.0427	0.0164
2.5	0.5253	0.1847	0.0588	0.0180	0.0054
3.0	0.4463	0.1245	0.0311	0.0074	0.0017
3.5	0.3765	0.0830	0.0163	0.0030	0.0005
4.0	0.3158	0.0549	0.0084	0.0012	0.0001
4.5	0.2635	0.0361	0.0043	0.00049	0.000054
5.0	0.2189	0.0236	0.0022	0.000197	0.000017

Alternatively we may also obtain the values of m for fixed values of σ at different tolerance level α from the equation.

$$\alpha = \frac{\sigma}{(\sigma+1)}\left[1+m+\frac{1}{\sigma}\right]e^{-m\sigma}$$

$$\alpha e^{m\sigma} = \frac{\sigma}{(\sigma+1)}\left[1+m+\frac{1}{\sigma}\right]$$

for $\sigma = 0.5$,we get the expression for

$$f\left(m\right) = \alpha e^{0.5m} - \frac{m}{3} - 1 \quad (2.5)$$

Here, the equation (2.5) is a nonlinear equation and hence solved by Newton Raphson method for different real values of m using Mathematica Software.

Table: 2 Values of m for tolerance levels α and for $\sigma = 0.5$

α	0.1	0.05	0.02	0.01	0.001	0.0001	0.00001
m	7.01639	8.71618	10.889	12.494	17.6763	22.7178	27.6756

Remarks:

1. Table 1 depicts the probability of disaster for Shushila distributed stress. It is interested to note that theprobability of disaster decreases for increasing values of m and decreases for the increasing values of α .
2. Table 2 shows the values of m for different values of α for fixed $\sigma = 0.5$.It is obvious that values of m increases as α decreases i.e. the ultimate strength capacity must increase if we wish to have a small tolerance level.

3 Stress and Strength-Reliability

For the stress strength model the probability $P = \Pr(Y > X)$ when the random variable X and Y follows pdfs (1.1) and (1.2), respectively is given by the following theorem.

Theorem 3.1: $P = \Pr(Y > X)$ is given by

$$P(Y > X) = \frac{\sigma}{(1+\sigma)}\left(1 - e^{-m\sigma}\right) + \frac{1}{(1+\sigma)}\left(1 - e^{-m\sigma} - m\sigma e^{-m\sigma}\right) - \frac{\sigma}{(1+\sigma)} \cdot \frac{1}{(m\sigma)^a} \gamma(a+1, m\sigma) - \frac{1}{(1+\sigma)} \cdot \frac{1}{(m\sigma)^a} \gamma(a+2, m\sigma) \qquad (3.1)$$

where $m = \frac{\theta}{\lambda}$ and $\gamma(a, m) = \int_0^m u^{a-1} e^{-u} du$, is incomplete gamma function.

Proof:

$$P(Y > X) = \int_0^{\theta} \int_{y=x}^{\theta} f(x;\lambda,\sigma) g(y;a,\theta) dy dx \quad (3.2)$$

Substituting $y = vx$ in equation (3.2), we get

$$P(Y > X) = \int_0^{\theta} \int_{y=vx}^{\frac{m\lambda}{x}} x f(x;\lambda,\sigma) g(vx;a,\theta) dv dx$$

$$= \int_0^{\theta} \int_{v=1}^{\frac{m\lambda}{x}} \frac{\sigma^2}{\lambda(1+\sigma)} \left(1 + \frac{x}{\lambda}\right) e^{-\frac{\sigma x}{\lambda}} \left(\frac{a}{\theta}\right)\left(\frac{vx}{\theta}\right)^{a-1} dv dx$$

$$= \frac{\sigma^2}{\lambda(1+\sigma)} \int_0^{\theta} \left(1 + \frac{x}{\lambda}\right) e^{-\frac{\sigma}{\lambda}x} \left(1 - \frac{x^a}{\theta^a}\right) dx$$

setting $m=\dfrac{\theta}{\lambda}$

$$=\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}\left(1+\frac{x}{\lambda}\right)e^{-\frac{\sigma}{\lambda}x}\left(1-\frac{x^a}{m^a\lambda^a}\right)dx$$

$$=\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}\left(1+\frac{x}{\lambda}\right)e^{-\frac{\sigma}{\lambda}x}dx-\int_0^{m\lambda}\frac{x^a}{m^a\lambda^a}\left(1+\frac{x}{\lambda}\right)dx$$

$$=\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}e^{-\frac{\sigma}{\lambda}x}dx+\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}\frac{x}{\lambda}e^{-\frac{\sigma}{\lambda}x}dx-\frac{\sigma^2}{\lambda(1+\sigma)}\cdot\frac{1}{m^a\lambda^a}\int_0^{m\lambda}x^a e^{-\frac{\sigma}{\lambda}x}dx$$

$$-\frac{\sigma^2}{\lambda(1+\sigma)}\cdot\frac{1}{m^a\lambda^a}\int_0^{m\lambda}x^{a+1}e^{-\frac{\sigma}{\lambda}x}dx$$

$$P(Y>X)=I+II-III-IV$$

Now Integral is given by

$$I==\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}e^{-\frac{\sigma}{\lambda}x}dx$$

$$I=\frac{\sigma}{(\sigma+1)}\left[1-e^{-m\sigma}\right]$$

$$II=\frac{\sigma^2}{\lambda(1+\sigma)}\int_0^{m\lambda}\frac{x}{\lambda}e^{-\frac{\sigma}{\lambda}x}dx$$

$$II=\frac{1}{(1+\sigma)}\left(1-e^{-m\sigma}-m\lambda e^{-m\sigma}\right)$$

$$III=\frac{\sigma^2}{\lambda(1+\sigma)}\cdot\frac{1}{m^a\lambda^a}\int_0^{m\lambda}x^a e^{-\frac{\sigma}{\lambda}x}dx$$

Using Incomplete gamma function

$$\gamma(a,x)=\int_0^x t^{a-1}e^{-t}dt$$

$$III=\frac{\sigma^2}{\lambda(1+\sigma)}\cdot\frac{1}{m^a\lambda^a}\cdot\frac{\lambda^{a+1}}{\sigma^{a+1}}\int_0^{m\lambda}t^{a+1-1}e^{-t}dt$$

$$III = \frac{\sigma}{(1+\sigma)} \cdot \frac{1}{(m\sigma)^a} \gamma(a+1, m\sigma)$$

$$III = \frac{\sigma}{(1+\sigma)} \cdot \frac{1}{(m\sigma)^a} \gamma(a+1, m\sigma)$$

$$IV = \frac{\sigma^2}{\lambda(1+\sigma)} \cdot \frac{1}{m^a \lambda^a} \int_0^{m\lambda} x^{a+1} e^{-\frac{\sigma}{\lambda}x} dx$$

$$IV = \frac{1}{(1+\sigma)} \cdot \frac{1}{m^a \lambda^a} \cdot \left(\frac{\lambda}{\sigma}\right)^a \cdot \gamma(a+2, m\sigma)$$

so we get

$$P(Y > X) = I + II - III - IV$$

Therefore,

$$P(Y > X) = \frac{1}{(1+\sigma)}\left\{\sigma\left(1-e^{-m\sigma}\right) + \left(1-e^{-m\sigma} - m\sigma e^{-m\sigma}\right)\right\} - \frac{1}{(1+\sigma)} \frac{1}{(m\sigma)^a}\left\{\sigma\gamma(a+1, m\sigma) - \gamma(a+2, m\sigma)\right\} \quad (3.3)$$

Hence, the theorem follows.

Table: 3 Strength-Reliability of an item for selected values of m and a for $\sigma = 1.5$

$m \rightarrow$ $a \downarrow$	2	3	4	5	6
1.0	0.6528	0.9051	0.9773	0.9952	0.9990
1.5	0.6613	0.9072	0.9768	0.9948	0.9989
2.0	0.6806	0.9146	0.9781	0.9948	0.9989
2.5	0.7033	0.924	0.9804	0.9952	0.9989
3.0	0.7258	0.9334	0.9829	0.9957	0.9990
3.5	0.7466	0.9415	0.9852	0.9963	0.9991
4.0	0.7653	0.9483	0.9870	0.9967	0.9992
4.5	0.7817	0.9536	0.9884	0.9971	0.9993
5.0	0.7960	0.9577	0.9895	0.9973	0.9993
5.5	0.8086	0.9607	0.9902	0.9975	0.9994
6.0	0.8195	0.9630	0.9907	0.9976	0.9994

Remarks:

1. Table 3 shows that the strength reliability of an item is increasing when we increase the values of m accordingly. If we increase the parametric values of power function i.e. “a”, the strength reliability of the system also increases.

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