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SKOLEM DIFFERENCE FIBONACCI MEAN LABELLING OF SOME SPECIAL CLASS OF TREES

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ABSTRACT

The concept of Skolem difference mean labelling was introduced by K. Murugan and A. Subramanian[2]. The concept of Fibonacci labelling was introduced by David W. Bange and Anthony E. Barkauskas[1] in the form Fibonacci graceful. This motivates us to introduce skolem difference Fibonacci mean labelling and is defined as follows: "A graph G with p vertices and q edges is said to have skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set $\{1,2,...,F_{p+q}\}$ *in such a way that the edge e = uv is labelled with* $\left|\frac{f(u)-f(v)}{2}\right|$ *if* $|f(u)-f(v)|$ *is even and* $\frac{|f(u)-f(v)|+1}{2}$ *if* $|f(u)-f(v)|$ *is odd and the resulting edge labels are distinct and are from* $\{F_1, F_2,...,F_q\}$*. A graph that admits **Skolem difference Fibonacci mean labelling** is called a Skolem difference Fibonacci mean graph". In this paper, we prove that caterpillar,* $S_{m,n}$*, olive tree,* $K_{1,n} \odot K_1$*, the graph obtained by identifying* v_{in} *with* $v_{(i+1)1}$ *of* $K_{1,n}$*, the graph obtained by joining two pendant vertices to each of the pendant vertices of* $K_{1,n}$ *of graphs are Skolem difference Fibonacci mean graphs.*

Keywords: Skolem difference mean labelling, Fibonacci labelling, Skolem difference Fibonacci mean labelling

1. Introduction

A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set $\{1,2,...,F_{p+q}\}$ in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from $\{F_1, F_2,...,F_q\}$. A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph. It was found that standard graphs [7], H- class of graphs [8], some special class of graphs [9] and path related graphs [10] are Skolem difference Fibonacci mean graphs.

2. PRELIMINARIES

In this section, some basic definitions and preliminary ideas are given which is useful for proving theorems.

2.1 Definition [3]:

A *caterpillar* is a tree with the property that the removal of its end points or pendant vertices (vertices of degree 1) results in a path.

2.2 Definition [3]:

$S_{m,n}$ denotes a star with n spokes in which each spoke is path of length m.

2.3 Definition [3]:

Let O_n be the olive tree having n paths of length 1, 2,..., n adjoined at one vertex v_0. Let v_0, v_{11}, v_{12},..., v_{1n}, v_{21}, v_{22},..., $v_{2(n-1)}$,..., v_{n1} be the vertices of O_n.

2.4 Definition [3]:

The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

3. Skolem difference Fibonacci mean labelling of some special class of trees

3.1 Theorem

The caterpillar $S(X_1, X_2, \dots, X_n)$ is skolem difference Fibonacci mean graph for all $n \geq 2$.

Proof:

Let G be the caterpillar $S(X_1, X_2, \dots, X_n)$.

Let V (G) = $\{v_i / 1 \leq i \leq n\} \cup \{u_{ij}/1 \leq i \leq n, 1 \leq j \leq x_i\}$

E (G) = $\{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_i u_{ij}/1 \leq i \leq n, 1 \leq j \leq x_i\}$

Then $|V(G)| = X_1 + X_2 + \cdots + X_n + n$ and

$|E(G)| = X_1 + X_2 + \cdots + X_n + n - 1$

Let f: V (G) → {1, 2,..., $F_{2(X_1+X_2+\cdots+X_n+n)-1}$ } be defined as follows

f (v_1) = 1

$f(v_i) = 2F_{\sum_{k=1}^{i-1} X_k+(i-1)} + f(v_{i-1})$, $2 \le i \le n$

$f(u_{1j}) = 2F_j + f(v_1)$, $1 \le j \le x_1$

$f(u_{ij}) = 2F_{\sum_{k=1}^{i-1} X_k+j+(i-1)} + f(v_i)$, $2 \le i \le n$, $1 \le j \le x_i$

$f^+(E) = \{ f(v_iv_{i+1})/1 \le i \le n-1\} \cup \{f(v_iu_{ij})/1 \le i \le n, 1 \le j \le x_i\}$

$= \{f(v_1v_2), f(v_2v_3),\ldots,f(v_{n-1}v_n)\} \cup \{f(v_1u_{11}), f(v_1u_{12}),\ldots,f(v_1u_{1x1}), f(v_2u_{21}), f(v_2u_{22}),\ldots,f(v_2u_{2x2}),\ldots,f(v_nu_{n1}), f(v_nu_{n2}),\ldots,f(v_nu_{nxn})\}$

$= \{\left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|,\ldots, \left|\frac{f(v_{n-1})-f(v_n)}{2}\right|\} \cup \{ \left|\frac{f(v_1)-f(u_{11})}{2}\right|, \left|\frac{f(v_1)-f(u_{12})}{2}\right|,\ldots, \left|\frac{f(v_1)-f(u_{1X_1})}{2}\right|, \left|\frac{f(v_2)-f(u_{21})}{2}\right|, \left|\frac{f(v_2)-f(u_{22})}{2}\right|,\ldots, \left|\frac{f(v_2)-f(u_{2X_2})}{2}\right|,\ldots, \left|\frac{f(v_n)-f(u_{n1})}{2}\right|, \left|\frac{f(vn)-f(u_{n2})}{2}\right|,\ldots, \left|\frac{f(vn)-f(u_{nX_n})}{2}\right|\}$

$= \{\left|\frac{f(v_1)-2F_{X_1+1}-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2F_{X_1+X_2+2}-f(v_2)}{2}\right|,\ldots, \left|\frac{f(v_{n-1})-2F_{X_1+X_2+\cdots+X_{n-1}+n-1}-f(v_{n-1})}{2}\right|\} \cup \{\left|\frac{f(v_1)-2F_1-f(v_1)}{2}\right|, \left|\frac{f(v_1)-2F_2-f(v_1)}{2}\right|,\ldots, \left|\frac{f(v_1)-2F_{X_1}-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2F_{X_1+2}-f(v_2)}{2}\right|, \left|\frac{f(v_2)-2F_{X_1+3}-f(v_2)}{2}\right|,\ldots, \left|\frac{f(v_2)-2F_{X_1+X_2+1}-f(v_2)}{2}\right|,\ldots, \left|\frac{f(v_n)-2F_{X_1+X_2+\cdots+X_{n-1}+n}-f(v_n)}{2}\right|, \left|\frac{f(v_n)-2F_{X_1+X_2+\cdots+X_{n-1}+n+1}-f(v_n)}{2}\right|,\ldots, \left|\frac{f(v_n)-2F_{X_1+X_2+\cdots+X_{n-1}+X_n+n-1}-f(v_n)}{2}\right|\}$

$= \{F_{X_1+1}, F_{X_1+X_2+2},\ldots, F_{X_1+X_2+\cdots+X_{n-1}+n-1}, F_1, F_2,\ldots, F_{X_1}, F_{X_1+2}, F_{X_1+3}, \ldots, F_{X_1+X_2+1},\ldots, F_{X_1+X_2+\cdots+X_{n-1}+n}, F_{X_1+X_2+\cdots+X_{n-1}+n+1},\ldots, F_{X_1+X_2+\cdots+X_{n-1}+X_n+n-1}\}$

$= \{F_1, F_2,\ldots, F_{X_1}, F_{X_1+1}, F_{X_1+2}, F_{X_1+3}, \ldots, F_{X_1+X_2+1}, F_{X_1+X_2+2},\ldots, F_{X_1+X_2+\cdots+X_{n-1}+n-1}, F_{X_1+X_2+\cdots+X_{n-1}+n}, F_{X_1+X_2+\cdots+X_{n-1}+n+1},\ldots, F_{X_1+X_2+\cdots+X_{n-1}+X_n+n-1}\}$

$= \{F_1, F_2,\ldots, F_{X_1+X_2+\cdots+X_{n-1}+X_n+n-1}\}$

Thus, the induced edge labels are distinct.

Hence, the caterpillar $S(X_1, X_2, \ldots, X_n)$ is skolem difference Fibonacci mean graph for all $n \ge 2$.

3.2 Example:

The Skolem difference Fibonacci mean labelling of the caterpillar S (5,3,6) is

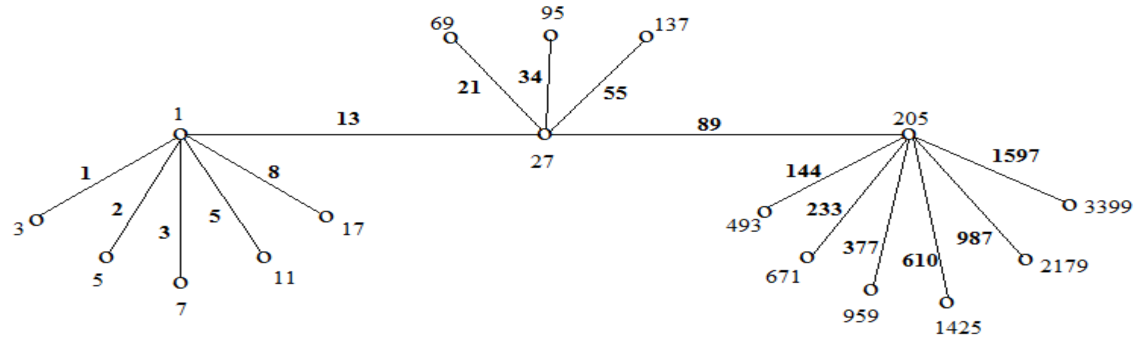

Figure1

3.3 Corollary

When $x_i = m$, $1 \leq i \leq n$, the graph $P_n \odot \overline{Km}$ is skolem difference Fibonacci mean graph for all $n \geq 2$ and $m \geq 1$.

3.4 Example:

The Skolem difference Fibonacci mean labelling of $P_3 \odot \overline{K4}$ is

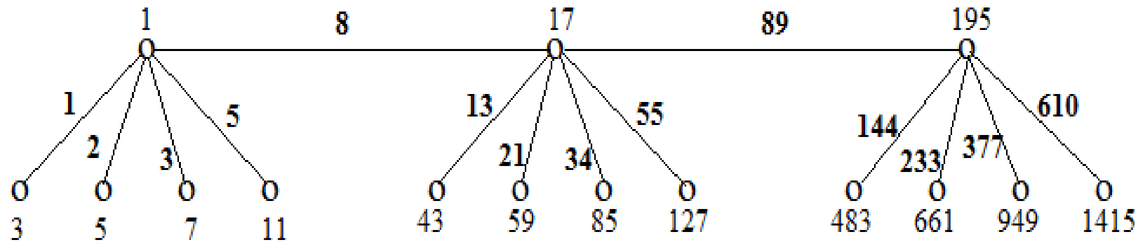

Figure 2

3.5 Corollary When m = 1, the graph $P_n \odot K_1$ is called a comb. The comb is skolem difference Fibonacci mean graph.

3.6 Example:

The Skolem difference Fibonacci mean labelling of $P_5 \odot K_1$ is

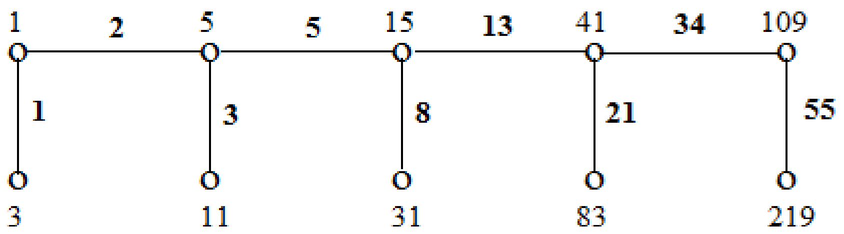

Figure 3

3.7 Definition:

The graph $P_{n-1}(1, 2,..., n)$ is a graph obtained from a path of vertices $v_1, v_2,...,v_n$ having the path length n-1 by joining i pendant vertices at each of its vertices.

3.8 Corollary

The graph $P_{n-1}(1, 2,..., n)$ is Skolem difference Fibonacci mean graph.

3.9 Example:

The Skolem difference Fibonacci mean labelling of the graph $P_4(1, 2, 3, 4, 5)$ is

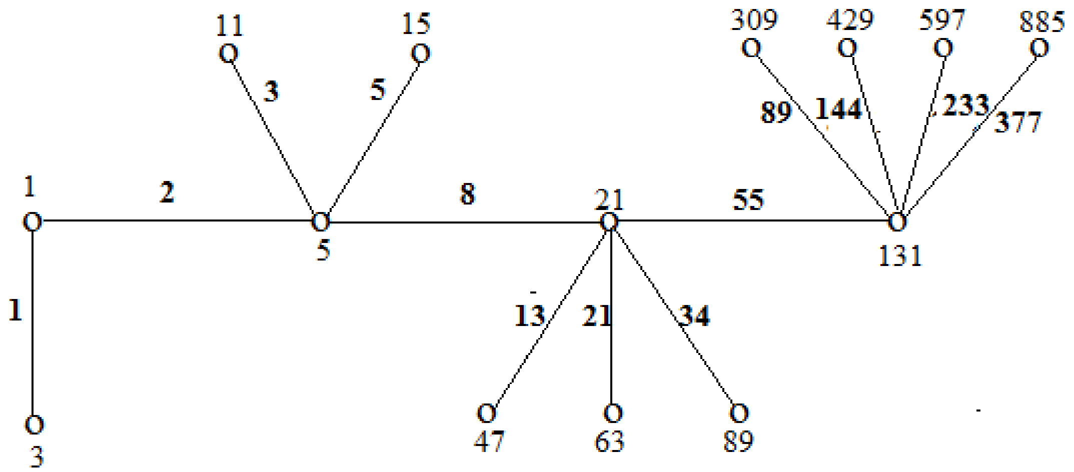

Figure 4

3.10 Corollary

Let the path P_n has Skolem difference Fibonacci mean labelling f. Then the twig graph G obtained from the path P_n by attaching exactly two pendant edges to each internal vertex of the path is also Skolem difference Fibonacci mean graph.

Proof:

Note that G $\cong S(X_1, X_2, ..., X_{n-2})$, where $X_i = 4$, $1 \leq i \leq n-2$.

Hence, G is Skolem difference Fibonacci mean graph.

3.11 Example:

The Skolem difference Fibonacci mean labelling of the graph twig of P_5 is

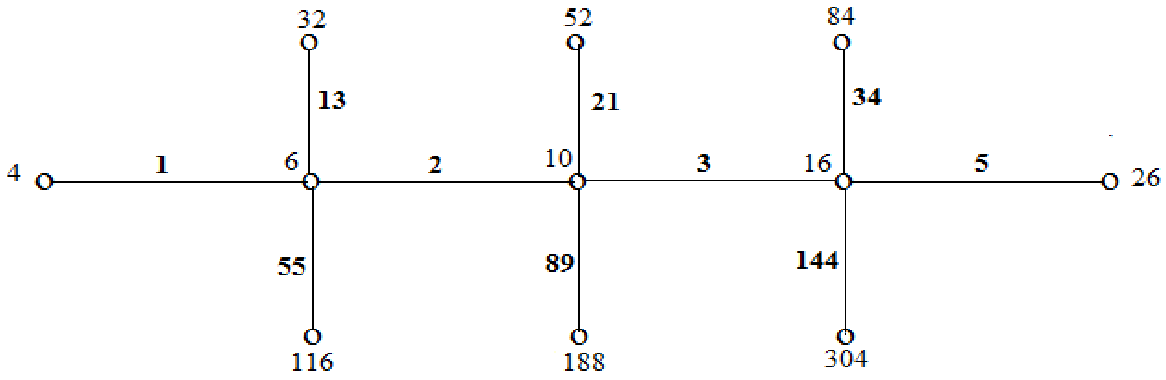

Figure 5

3.12 Theorem

$S_{m,n}$ is a skolem difference Fibonacci mean graph for all m, n ≥ 2.

Proof:

Let V ($S_{m,n}$) = {u, v_{ij} / 1 ≤ i ≤ m and 1 ≤ j ≤ n}

E ($S_{m,n}$) = {uv_{1j}/ 1 ≤ j ≤ m} ∪ {$v_{ij} v_{(i+1)j}$ /1 ≤ i ≤ m-1 and 1 ≤ j ≤ n}

Then $|V(S_{m,n})|$ = mn+1 and $|E(S_{m,n})|$ = mn

Let f: V → {1,2,...,F_{2mn+1}} be defined as follows

f (u) = 1

f (v_{1j}) = $2F_j$+1, 1 ≤ j ≤ n

f (v_{ij}) = $2F_{(i-1)n+j}$ +f($v_{(i-1)j}$), 2 ≤ i ≤ m and 1 ≤ j ≤ n

f^+ (E) = { f(uv_{1j})/ 1 ≤ j ≤ m}∪ {f($v_{ij} v_{(i+1)j}$) /1 ≤ i ≤ m-1 and 1 ≤ j ≤ n}

= {f(uv_{11}), f(uv_{12}),..., f(uv_{1n}), f($v_{11}v_{21}$), f($v_{21}v_{31}$),..., f($v_{(m-1)1}v_{m1}$), f($v_{12}v_{22}$),f($v_{22}v_{32}$),...,f($v_{(m-1)2}v_{m2}$),..., f($v_{1n}v_{2n}$), f($v_{2n}v_{3n}$),..., f($v_{(m-1)n} v_{mn}$)}

$$= \left\{\left|\frac{f(u)-f(v_{11})}{2}\right|, \left|\frac{f(u)-f(v_{12})}{2}\right|,\ldots, \left|\frac{f(u)-f(v_{1n})}{2}\right|, \left|\frac{f(v_{11})-f(v_{21})}{2}\right|, \left|\frac{f(v_{21})-f(v_{31})}{2}\right|,\ldots, \left|\frac{f(v_{(m-1)1})-f(v_{m1})}{2}\right|, \left|\frac{f(v_{12})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{32})}{2}\right|,\ldots, \left|\frac{f(v_{(m-1)2})-f(v_{m2})}{2}\right|,\ldots, \left|\frac{f(v_{1n})-f(v_{2n})}{2}\right|, \left|\frac{f(v_{2n})-f(v_{3n})}{2}\right|,\ldots, \left|\frac{f(v_{(m-1)n})-f(v_{mn})}{2}\right|\right\}$$

$= \{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, \ldots, \left|\frac{1-2F_n-1}{2}\right|, \left|\frac{f(v_{11})-2F_{n+1}-f(v_{11})}{2}\right|, \left|\frac{f(v_{21})-2F_{2n+1}-f(v_{21})}{2}\right|, \ldots,$
$\left|\frac{f(v_{(m-1)1})-2F_{(m-1)n+1}-f(v_{(m-1)1})}{2}\right|, \left|\frac{f(v_{12})-2F_{n+2}-f(v_{12})}{2}\right|, \left|\frac{f(v_{22})-2F_{2n+2}-f(v_{22})}{2}\right|, \ldots,$
$\left|\frac{f(v_{(m-1)2})-2F_{(m-1)n+2}-f(v_{(m-1)2})}{2}\right|, \ldots, \left|\frac{f(v_{1n})-2F_{2n}-f(v_{1n})}{2}\right|, \left|\frac{f(v_{2n})-2F_{3n}-f(v_{2n})}{2}\right|, \ldots,$
$\left|\frac{f(v_{(m-1)n})-2F_{(m-1)n+n}-f(v_{(m-1)n})}{2}\right|\}$

$= \{F_1, F_2, \ldots, F_n, F_{n+1}, F_{2n+1}, \ldots, F_{(m-1)n+1}, F_{n+2}, F_{2n+2}, \ldots, F_{(m-1)n+2}, \ldots, F_{2n}, F_{3n}, \ldots, F_{mn}\}$

$= \{F_1, F_2, \ldots, F_n, F_{n+1}, F_{n+2}, \ldots, F_{2n}, F_{2n+1}, F_{2n+2}, \ldots, F_{3n}, \ldots, F_{(m-1)n+1}, F_{(m-1)n+2}, \ldots, F_{mn}\}$

$= \{F_1, F_2, \ldots, F_{mn}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{mn}$.

Hence, $S_{m,n}$ is a skolem difference Fibonacci mean graph for all m, n ≥ 2.

3.13 Example:

The Skolem difference Fibonacci mean labelling of the graph $S_{3,4}$ is

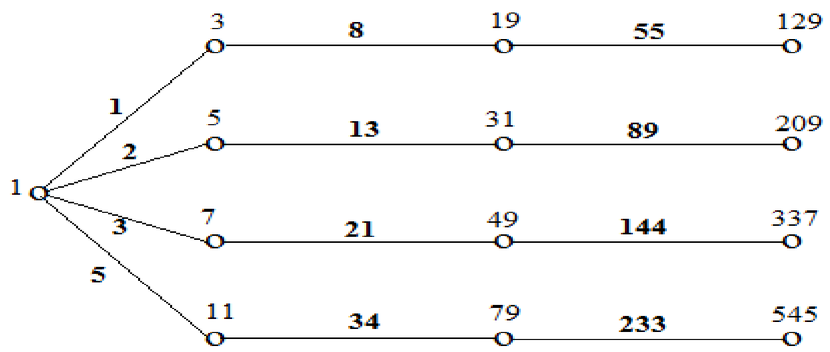

Figure 6

3.14 Theorem

The olive tree O_n is a skolem difference Fibonacci mean graph.

Proof:

Let O_n be the olive tree having n paths of length 1, 2,..., n adjoined at one vertex v_0. Let $v_0, v_{11}, v_{12}, \ldots, v_{1n}, v_{21}, v_{22}, \ldots, v_{2(n-1)}, \ldots, v_{n1}$ be the vertices of O_n.

Let E $(O_n) = \{v_0v_{i1} / 1 \leq i \leq n\} \cup \{v_{ij}\, v_{i(j+1)} / 1 \leq i \leq n-1 \text{ and } 1 \leq j \leq (n-i)\}$

Then $|V(O_n)| = \frac{n(n+1)}{2} + 1$ and $|E(O_n)| = \frac{n(n+1)}{2}$

Let f: V (G) → {1,2,...,$F_{n(n+1)+1}$} be defined as follows

f (v_0) = 1

f (v_{i1}) = $2F_i+1$, $1 \le i \le n$

f (v_{ij}) = $2F_{(j-1)n-\frac{(j-1)(j-2)}{2}+i}$ + f ($v_{i(j-1)}$), $2 \le j \le n$ and $1 \le i \le n-(j-1)$

$f^+(E)$ = {f (v_0v_{i1})/ $1 \le i \le n$} ∪ {f($v_{ij}\, v_{i(j+1)}$)/ $1 \le i \le n-1$ and $1 \le j \le (n-i)$}

= {f(v_0v_{11}), f(v_0v_{21}),..., f(v_0v_{n1})} ∪ {f($v_{11}v_{12}$), f($v_{12}v_{13}$),..., f($v_{1(n-1)}v_{1n}$), f($v_{21}v_{22}$), f($v_{22}v_{23}$),..., f($v_{2(n-2)}v_{2(n-1)}$),..., f($v_{(n-1)1}v_{(n-1)2}$)}

$f^+(E) = \{\left|\frac{f(v_0)-f(v_{11})}{2}\right|, \left|\frac{f(v_0)-f(v_{21})}{2}\right|,..., \left|\frac{f(v_0)-f(v_{n1})}{2}\right|\} \cup \{\left|\frac{f(v_{11})-f(v_{12})}{2}\right|,$
$\left|\frac{f(v_{12})-f(v_{13})}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-f(v_{1n})}{2}\right|, \left|\frac{f(v_{21})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{23})}{2}\right|,...,$
$\left|\frac{f(v_{2(n-2)})-f(v_{2(n-1)})}{2}\right|,..., \left|\frac{f(v_{(n-1)1})-f(v_{(n-1)2})}{2}\right|\}$

$= \{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|,..., \left|\frac{1-2F_n-1}{2}\right|\} \cup \{\left|\frac{f(v_{11})-2F_{n+1}-f(v_{11})}{2}\right|,$
$\left|\frac{f(v_{12})-2F_{2n}-f(v_{12})}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-2F_{n(n-1)-\frac{(n-1)(n-2)}{2}+1}-f(v_{1(n-1)})}{2}\right|,$
$\left|\frac{f(v_{21})-2F_{n+2}-f(v_{21})}{2}\right|, \left|\frac{f(v_{22})-2F_{2n+1}-f(v_{22})}{2}\right|,..., \left|\frac{f(v_{2(n-2)})-2F_{n(n-2)-\frac{(n-2)(n-3)}{2}+2}-f(v_{2(n-2)})}{2}\right|,...,$
$\left|\frac{f(v_{(n-1)1})-2F_{n+(n-1)}-f(v_{(n-1)1})}{2}\right|\}$

$= \{F_1, F_2,..., F_n\} \cup \{F_{n+1}, F_{2n},..., F_{\frac{n(n+1)}{2}}, F_{n+2}, F_{2n+1},..., F_{\frac{n(n+1)}{2}-1},..., F_{2n-1}\}$

$= \{F_1, F_2,..., F_n, F_{n+1}, F_{n+2},..., F_{2n-1}, F_{2n}, F_{2n+1},..., F_{\frac{n(n+1)}{2}-1}, F_{\frac{n(n+1)}{2}}\}$

$= \{F_1, F_2,..., F_{\frac{n(n+1)}{2}}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2,..., F_{\frac{n(n+1)}{2}}$.

Hence, the olive tree O_n is a skolem difference Fibonacci mean graph.

3.15 Example:

Skolem difference Fibonacci mean labelling of the graph O_4 is

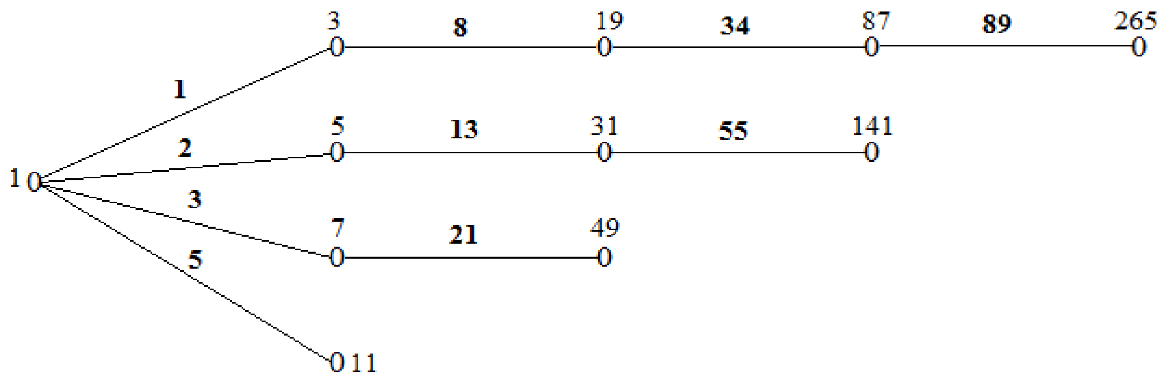

Figure 7

3.16 Theorem

The graph $K_{1,n} \odot K_1$ is skolem difference Fibonacci mean graph for all $n \geq 1$.

Proof:

Let $V(K_{1,n} \odot K_1) = \{u, u_i, v, v_i \,/\, 1 \leq i \leq n\}$

$E(K_{1,n} \odot K_1) = \{uv, uu_i, u_iv_i \,/ 1 \leq i \leq n\}$

Then $|V(K_{1,n} \odot K_1)| = 2n+2$ and $|E(K_{1,n} \odot K_1)| = 2n+1$

Let $f: V \to \{1,2,...,F_{4n+3}\}$ be defined as follows

$f(u) = 1$

$f(u_i) = 2F_i+1,\ 1 \leq i \leq n$

$f(v) = 2F_{2n+1}+f(u)$

$f(v_i) = 2F_{n+i}+f(u_i),\ 1 \leq i \leq n$

$f^+(E) = \{ f(uv), f(uu_i), f(u_iv_i) \,/ 1 \leq i \leq n\}$

$= \{f(uv), f(uu_1), f(uu_2), ..., f(uu_n), f(u_1v_1), f(u_2v_2), ..., f(u_nv_n)\}$

$= \{\left|\frac{f(u)-f(v)}{2}\right|, \left|\frac{f(u)-f(u_1)}{2}\right|, \left|\frac{f(u)-f(u_2)}{2}\right|, ..., \left|\frac{f(u)-f(u_n)}{2}\right|, \left|\frac{f(u_1)-f(v_1)}{2}\right|, \left|\frac{f(u2)-f(v_2)}{2}\right|, ..., \left|\frac{f(u_n)-f(v_n)}{2}\right|\}$

$= \{\left|\frac{f(u)-2F_{2n+1}-f(u)}{2}\right|, \left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, ..., \left|\frac{1-2F_n-1}{2}\right|, \left|\frac{f(u_1)-2F_{n+1}-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2F_{n+2}-f(u_2)}{2}\right|, ..., \left|\frac{f(u_n)-2F_{2n}-f(u_n)}{2}\right|\}$

$= \{F_{2n+1}, F_1, F_2, ..., F_n, F_{n+1}, F_{n+2}, ..., F_{2n}\}$

$= \{F_1, F_2, ..., F_{2n+1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{2n+1}$.

Hence, the graph $K_{1,n} \odot K_1$ is Skolem difference Fibonacci mean graph for all $n \geq 1$.

3.17 Example:

The Skolem difference Fibonacci mean labelling of the graph $K_{1,4} \odot K_1$ is

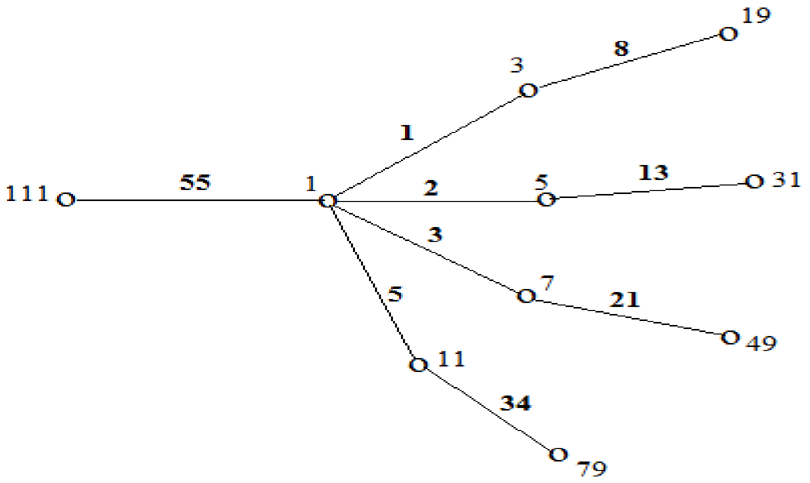

Figure 8

3.18 Theorem

Let $G_i = K_{1,n}$ for $1 \leq i \leq m$ with vertex set $V(G) = \{v_i, v_{ij} / 1 \leq i \leq m$ and $1 \leq j \leq n\}$. Let G be the graph obtained by identifying v_{in} with $v_{(i+1)1}$ for $1 \leq i \leq m-1$ then G is Skolem difference Fibonacci mean graph for all n and m.

Proof:

Let $V(G) = \{v_i, v_{ij} / 1 \leq i \leq m$ and $1 \leq j \leq n\}$ and

$E(G) = \{v_i v_{ij} / 1 \leq i \leq m$ and $1 \leq j \leq n\}$ and

$v_{in} = v_{(i+1)1}$ for $1 \leq i \leq m-1$

Then $|V(G)| = mn+1$ and $|E(G)| = mn$

Let $f: V \rightarrow \{1,2,\ldots,F_{2mn+1}\}$ be defined as follows

$f(v_1) = 1$

$f(v_i) = 2F_{(i-1)n+1} + f(v_{(i-1)n}),\ 2 \leq i \leq m$

$f(v_{1j}) = 2F_j+1,\ 1 \leq j \leq n$

$f(v_{i1}) = f(v_{(i-1)n}),\ 2 \leq i \leq m$

$f(v_{ij}) = 2F_{(i-1)n+j} + f(v_i),\ 2 \leq i \leq m$ and $2 \leq j \leq n$

$f(v_{1n}) = f(v_{21}), f(v_{2n}) = f(v_{31}), \ldots, f(v_{(m-1)n}) = f(v_{m1})$

$f^{+}(E) = \{ f(v_i\, v_{ij}) / 1 \le i \le m \text{ and } 1 \le j \le n\}$

$= \{f(v_1v_{11}), f(v_1v_{12}),\dots, f(v_1v_{1n}), f(v_2v_{21}), f(v_2v_{22}),\dots, f(v_2v_{2n}),\dots, f(v_mv_{m1}), f(v_mv_{m2}),\dots, f(v_mv_{mn})\}$

$= \{\left|\frac{f(v_1)-f(v_{11})}{2}\right|, \left|\frac{f(v_1)-f(v_{12})}{2}\right|,\dots, \left|\frac{f(v_1)-f(v_{1n})}{2}\right|, \left|\frac{f(v_2)-f(v_{21})}{2}\right|, \left|\frac{f(v_2)-f(v_{22})}{2}\right|,\dots, \left|\frac{f(v_2)-f(v_{2n})}{2}\right|,\dots, \left|\frac{f(v_m)-f(v_{m1})}{2}\right|, \left|\frac{f(v_m)-f(v_{m2})}{2}\right|,\dots, \left|\frac{f(v_m)-f(v_{mn})}{2}\right|\}$

$= \{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, \dots, \left|\frac{1-2F_n-1}{2}\right|, \left|\frac{2F_{n+1}+f(v_{1n})-f(v_{1n})}{2}\right|, \left|\frac{f(v_2)-2F_{n+2}-f(v_2)}{2}\right|,\dots, \left|\frac{f(v_2)-2F_{2n}-f(v_2)}{2}\right|, \left|\frac{2F_{(m-1)n+1}+f(v_{(m-1)n})-f(v_{(m-1)n})}{2}\right|, \left|\frac{f(v_m)-2F_{(m-1)n+2}-f(v_m)}{2}\right|,\dots, \left|\frac{f(v_m)-2F_{(m-1)n+n}-f(v_m)}{2}\right|\}$

$= \{F_1, F_2,\dots, F_n, F_{n+1}, F_{n+2}, \dots, F_{2n},\dots, F_{(m-1)n+1}, F_{(m-1)n+2},\dots, F_{(m-1)n+n}\}$

$= \{F_1, F_2,\dots, F_n, F_{n+1}, F_{n+2}, \dots, F_{2n},\dots, F_{(m-1)n+1}, F_{(m-1)n+2},\dots, F_{mn}\}$

$= \{F_1, F_2,\dots, F_{mn}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2,\dots, F_{mn}$.

Hence, f is a Skolem difference Fibonacci mean labelling of the graph G.

3.19 Example:

Skolem difference Fibonacci mean labelling of the graph obtained by identifying v_{in} with $v_{(i+1)1}$ for $1 \le i \le m-1$ for $K_{1,6}$ is

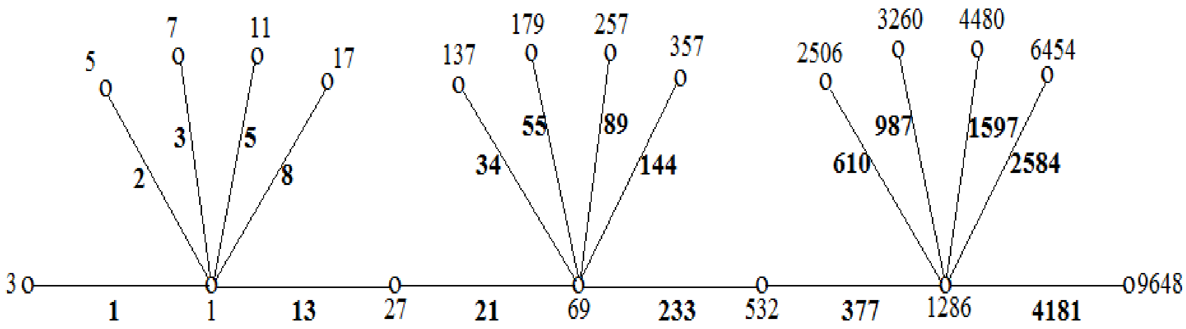

Figure 9

3.20 Theorem

Let G be the graph obtained by joining two pendant vertices to each of the pendant vertices of $K_{1,n}$. Then G is Skolem difference Fibonacci mean graph for all $n \ge 1$.

Proof:

Let V (G) = {u, u_i, u_i^1, u_i^{11}/ 1 ≤ i ≤ n}

E (G) = {uu_i, $u_iu_i^1$, $u_iu_i^{11}$/1 ≤ i ≤ n}

Let f: V (G) → {1,2,...,F_{6n+1}} be defined as follows

f (u) = 1

f (u_i) = $2F_i$+1, 1 ≤ i ≤ n

f (u_i^1) = $2F_{n+(2i-1)}$+f(u_i), 1 ≤ i ≤ n

f (u_i^{11}) = $2F_{n+2i}$+f(u_i), 1 ≤ i ≤ n

Let f^+ be the induced edge labelling of f. Then

f^+ (E) = { f(uu_i), f($u_iu_i^1$), f($u_iu_i^{11}$)/1 ≤ i ≤ n}

= {f(uu_1), f(uu_2),..., f(uu_n), f($u_1u_1^1$), f($u_2u_2^1$),..., f($u_nu_n^1$), f($u_1u_1^{11}$), f($u_2u_2^{11}$), ...,f($u_nu_n^{11}$) }

$= \{\left|\frac{f(u)-f(u_1)}{2}\right|, \left|\frac{f(u)-f(u_2)}{2}\right|,..., \left|\frac{f(u)-f(u_n)}{2}\right|, \left|\frac{f(u_1)-f(u_1{}^1)}{2}\right|, \left|\frac{f(u_2)-f(u_2{}^1)}{2}\right|,..., \left|\frac{f(u_n)-f(u_n{}^1)}{2}\right|,$
$\left|\frac{f(u_1)-f(u_1{}^{11})}{2}\right|, \left|\frac{f(u_2)-f(u_2{}^{11})}{2}\right|,..., \left|\frac{f(u_n)-f(u_n{}^{11})}{2}\right| \}$

$= \{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, ..., \left|\frac{1-2F_n-1}{2}\right|, \left|\frac{f(u_1)-2F_{n+1}-f(u_1)}{2}\right|,$
$\left|\frac{f(u_2)-2F_{n+3}-f(u_2)}{2}\right|,..., \left|\frac{f(u_n)-2F_{3n-1}-f(u_n)}{2}\right|, \left|\frac{f(u_1)-2F_{n+2}-f(u_1)}{2}\right|,$
$\left|\frac{f(u_2)-2F_{n+4}-f(u_2)}{2}\right|,..., \left|\frac{f(u_n)-2F_{3n}-f(u_n)}{2}\right| \}$

= {F_1, F_2, ..., F_n, F_{n+1}, F_{n+3}, ..., F_{3n-1}, F_{n+2}, F_{n+4}, ..., F_{3n}}

= {F_1, F_2, ..., F_{3n}}

Thus, the induced edge labels are distinct and are F_1, F_2,..., F_{3n}.

Hence, the graph is Skolem difference Fibonacci mean graph for all n ≥ 1.

3.21 Example:

The Skolem difference Fibonacci mean labelling of the graph obtained from $K_{1,3}$ is

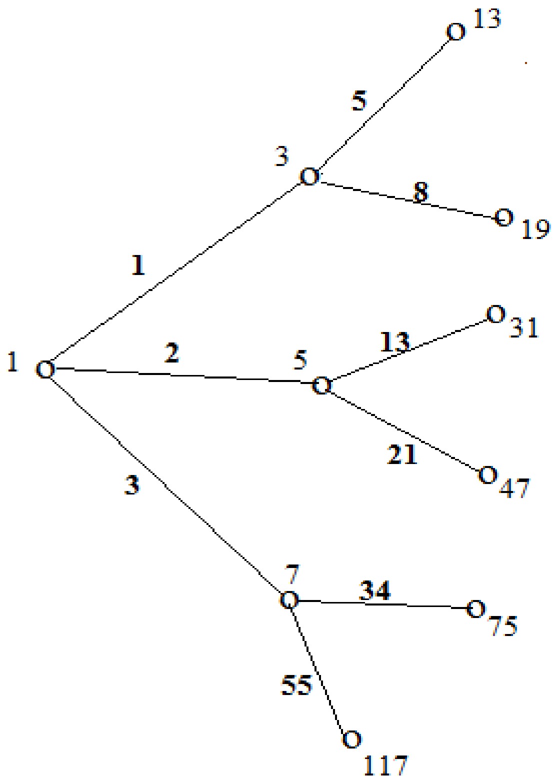

Figure 10

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