International Research Journal of Mathematics, Engineering and IT
Vol. 3, Issue 1, Jan 2016 IF- 2.868 ISSN: (2349-0322)

Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

ON SBĜ – CONTINUOUS FUNCTIONS AND SBĜ - HOMEOMORPHISMS IN TOPOLOGICAL SPACES

K. Bala Deepa Arasi[1] and S.Navaneetha Krishnan[2]

[1]Assistant Professor of Mathematics, A.P.C.Mahalaxmi College for Women, Thoothukudi, TN, India.
[2] Associate Professor of Mathematics, V.O. Chidambaram College, Thoothukudi, TN, India.

ABSTRACT

In this paper, we define new class of functions namely sbĝ – continuous functions and sbĝ – open maps and we prove some of their basic properties. Also, we introduce a new class of sbĝ – homeomorphisms and we prove some of their relationship among other homeomorphisms.Throughout this paper f: $(X,\tau) \rightarrow (Y,\sigma)$ is a function from a topological space (X,τ) to a topological space (Y,σ).

Keywords: closed set, sbĝ-closed sets, sbĝ – continuous functions, sbĝ – irresolute functions, sbĝ – open maps, sbĝ – closed maps and sbĝ- homeomorphisms.

AMS Mathematics Subject Classification (2010): 54A05.

1. INTRODUCTION

In 1963, N.Levine[9] introduced semi-open sets in Topology and studied their properties. Andrijevic[2] introduced one such new version called b-open sets in 1996. N.Levine[10] introduced the concept of generalized closed sets and studied their properties in 1970. By considering the concept of g-closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. M.K.R.S.Veerakumar[21] defined ĝ-closed sets in 2003. Also, R.Subasree and M.Mariasingam[17] introduced bĝ-closed sets in 2013. We introduced sbĝ-closed sets[4]and studied their properties in 2015. K.Balachandran et al[7] introduced the concept of generalized continuous maps in Topological spaces.

These concepts motivate us to define a new version of maps namely sbĝ – continuous, sbĝ – open maps and sbĝ – homeomorphisms. Also, we prove some properties of these functions and establish the relationships between sbĝ – homeomorphisms and other homeomorphisms.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi-open set[9] if A $\subseteq$ Cl(Int(A)).
2. an α-open set[6] if A $\subseteq$ Int(Cl(Int(A))).
3. a b-open set[2] if A $\subseteq$ Cl(Int(A)) ∪ Int(Cl(A)).
4. a regular open[13] set if A = Int(Cl(A)).

The complement of a semi–open (resp.α–open, b–open, regular-open) set is called semi-closed (resp.α–closed, b–closed, regular-closed) set.

The intersection of all semi-closed (resp.α-closed, b-closed, regular-closed) sets of X containing A is called the semi-closure (resp.α-closure, b-closure, regular closure) of A and is denoted by sCl(A) (resp.αCl(A), bCl(A), rCl(A)). The family of all semi-open (resp. α-open, b-open, regular-open) subsets of a space X is denoted by SO(X) (resp. αO(X), bO(X), rO(X)).

Definition 2.2: A subset A of a topological space (X,τ) is called

1) a generalized closed set (briefly g-closed)[11] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
2) a sg-closed set[5] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.
3) a gs-closed set[3] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
4) a gb-closed set[1] if bCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
5) a rb-closed set[12] if rCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is b-open in X.
6) a g*b-closed set[23] if bCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
7) a ĝ-closed set[19] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.
8) a bĝ-closed set[17] if bCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ĝ-open in X.
9) a sbĝ -closed set[4] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is bĝ-open in X

Definition 2.3: A function f: (X,τ) → (Y,σ) is called a

1. continuous[21] if f^{-1}(V) is closed in X for every closed set V in Y.
2. α-continuous[6] if f^{-1}(V) is α-closed in X for every closed set V in Y.
3. b-continuous[6] if f^{-1}(V) is b-closed in X for every closed set V in Y.
4. regular continuous[12] if f^{-1}(V) is regular closed in X for every closed set V in Y.
5. sg-continuous[18] if f^{-1}(V) is sg-closed in X for every closed set V in Y.
6. gs-continuous[7] if f^{-1}(V) is gs-closed in X for every closed set V in Y.
7. gb-continuous[24] if f^{-1}(V) is gb-closed in X for every closed set V in Y.
8. g*b-continuous[24] if f^{-1}(V) is g*b-closed in X for every closed set V in Y.

9. rb-continuous[12] if f^{-1}(V) is rb-closed in X for every closed set V in Y.
10. bĝ-continuous[19] if f^{-1}(V) is bĝ-closed in X for every closed set V in Y.

Definition 2.4: A function f: (X,τ) → (Y,σ) is called a

1. open map[21] if f(V) is open in (Y,σ) for every open set V in (X,τ).
2. α – open map[6] if f(V) is α – open in (Y,σ) for every open set V in (X,τ).
3. b – open map[6] if f(V) is b – open in (Y,σ) for every open set V in (X,τ).
4. regular open map[12] if f(V) is regular open in (Y,σ) for every open set V in (X,τ).
5. sg – open map[18] if f(V) is sg – open in (Y,σ) for every open set V in (X,τ).
6. gs – open map[7] if f(V) is gs – open in (Y,σ) for every open set V in (X,τ).
7. gb – open map[24] if f(V) is gb – open in (Y,σ) for every open set V in (X,τ).
8. g*b – open map[24] if f(V) is g*b – open in (Y,σ) for every open set V in (X,τ).
9. rb – open map[12] if f(V) is rb – open in (Y,σ) for every open set V in (X,τ).
10. bĝ - open map[19] if f(V) is bĝ – open in (Y,σ) for every open set V in (X,τ).

Definition 2.5: A bijection f: (X,τ) → (Y,σ) is called a

1. homeomorphism[21] if f is both continuous map and open map
2. b – homeomorphism[6] if f is both b – continuous map and b – open map
3. gs – homeomorphism[7] if f is both gs – continuous map and gs – open map
4. gb – homeomorphism[24] if f is both gb – continuous map and gb – open map
5. g*b – homeomorphism[24] if f is both g*b – continuous map and g*b – open map
6. bĝ – homeomorphism[9] if f is both bĝ – continuous map and bĝ – open map.

Definition 2.6: A function f: (X,τ) → (Y,σ) is said to be bĝ – irresolute function if the inverse image of every bĝ-closed set in (Y,σ) is bĝ-closed in (X,τ).

Remark 2.7: The family of all sbĝ – open subsets of a space X is denoted by sbĝ-O(X). The family of all sbĝ – closed subsets of a space X is denoted by sbĝ-C(X).

3. sbĝ-CONTINUOUS AND sbĝ – IRRESOLUTE FUNCTIONS

We introduce the following definitions.

Definition 3.1: A map f: (X,τ) → (Y,σ) is said to be sbĝ – continuous map if the inverse image of every closed set in (Y,σ) is sbĝ-closed in (X,τ).

Examplel 3.2: Let X = Y = {a,b,c}
τ = { X, ϕ, {a}, {b}, {a,b}, {a,c}} and σ = { Y, ϕ, {a}, {a,b}}
Define a map f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.

Then f is sbĝ-continuous, since the inverse images of closed sets {c},{b,c} in (Y,σ) are {c},{b,c} respectively which are sbĝ-closed in (X,τ).

Definition 3.3: A map f: (X,τ) → (Y,σ) is said to be sbĝ – irresolute map if the inverse image of every sbĝ-closed set in (Y,σ) is sbĝ-closed in (X,τ).

Examplel 3.4: Let X = Y = {a,b,c}
τ = { X, ϕ, {a}, {b}, {a,b}} and σ = { Y, ϕ, {a}, {b,c}}
Define a map f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
Then f is sbĝ-irresolute map, since the inverse images of sbĝ-closed sets {a},{b,c} in (Y,σ) are {a},{b,c} respectively which are sbĝ-closed sets in (X,τ).

Proposition 3.5:

a) Every continuous map is sbĝ - continuous.
b) Every α – continuous map is sbĝ – continuous.
c) Every regular continuous map is sbĝ – continuous.
d) Every rb – continuous map is sbĝ – continuous.

Proof:

a) Let f: (X,τ) → (Y,σ) be continuous. Let V be a closed set in (Y,σ). Since f is continuous, f^{-1}(V) is closed set in (X,τ). By Proposition 3.4 in [4],f^{-1}(V) is sbĝ-closed set in (X,τ). Therefore, f is sbĝ – continuous.
b) Let f: (X,τ) → (Y,σ) be α - continuous. Let V be a closed set in (Y,σ). Since f is α - continuous, f^{-1}(V) is α - closed set in (X,τ). By Proposition 3.7 in [4], f^{-1}(V) is sbĝ-closed set in (X,τ). Therefore, f is sbĝ – continuous.
c) Let f: (X,τ) → (Y,σ) be α - continuous. Let V be a closed set in (Y,σ). Since f is regular continuous, f^{-1}(V) is regular closed set in (X,τ). By Proposition 3.9 in [4], f^{-1}(V) is sbĝ-closed set in (X,τ). Therefore, f is sbĝ – continuous.
d) Let f: (X,τ) → (Y,σ) be rb - continuous. Let V be a closed set in (Y,σ). Since f is rb - continuous, f^{-1}(V) is rb - closed set in (X,τ). By Proposition 3.19 in [4],f^{-1}(V) is sbĝ-closed set in (X,τ). Therefore, f is sbĝ – continuous.

The following examples show that the converse of the above proposition need not be true.

Example 3.6:

a) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a}, {a,b}, {a,c}}. Let f: (X,τ) → (Y,σ) be the identity map. Since the inverse image of a closed set {b} in (Y,σ) is {b} which is sbĝ – closed but not closed in (X,τ), f is sbĝ – continuous but not continuous.
b) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {c}, {a,c}, {b,c}} and σ = {Y, ϕ, {a}, {b,c}}. Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = b. Since the inverse image of a closed set {a} in (Y,σ) is {b} which is sbĝ – closed but not α - closed in (X,τ), f is sbĝ – continuous but not α – continuous.

c) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {b}, {a,b}, {a,c}} and σ = {Y, ϕ, {b}}. Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c. Since the inverse image of a closed set {a,c} in (Y,σ) is {b,c} which is sbĝ – closed but not regular closed in (X,τ), f is sbĝ – continuous but not regular continuous.

d) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {b}, {a,b}} and σ = {Y, ϕ, {a}, {a,b}, {a,c}}. Define a function f:(X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c. Here, sbĝ – closed set in X are X, ϕ, {a}, {b}, {c}, {a,c}, {b,c}, and rb – closed set in X are X, ϕ, {a,c}, {b,c}. Since the inverse image of a closed set {b} in (Y,σ) is {b} which is sbĝ – closed but not rb - closed in (X,τ), f is sbĝ – continuous but not rb – continuous.

Proposition 3.7: `

a. Every sbĝ – continuous is b – continuous.
b. Every sbĝ – continuous is sg – continuous.
c. Every sbĝ – continuous is gs – continuous.
d. Every sbĝ – continuous is gb – continuous.
e. Every sbĝ – continuous is g*b – continuous.
f. Every sbĝ – continuous is bĝ – continuous.

Proof:

a) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.11 in [4],f^{-1}(V) is b - closed set in (X,τ). Therefore, f is b – continuous.

b) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.13 in [4], f^{-1}(V) is sg - closed set in (X,τ). Therefore, f is sg – continuous.

c) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.15 in [4], f^{-1}(V) is gs - closed set in (X,τ). Therefore, f is gs – continuous.

d) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.15 in [4], f^{-1}(V) is gs - closed set in (X,τ). Therefore, f is gs – continuous.

e) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.17 in [4], f^{-1}(V) is gb - closed set in (X,τ). Therefore, f is gb – continuous.

f) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.21 in [4], f^{-1}(V) is g*b - closed set in (X,τ). Therefore, f is g*b – continuous.

g) Let f: (X,τ) → (Y,σ) be sbĝ - continuous. Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, f^{-1}(V) is sbĝ - closed set in (X,τ). By Proposition 3.23 in [4], f^{-1}(V) is bĝ - closed set in (X,τ). Therefore, f is bĝ – continuous.

The following examples show that the converse of the above proposition need not be true.

Example 3.8:

a) Let X = Y = {a,b,c},
τ = {X, ϕ, {a}, {b,c}} and σ = {Y, ϕ, {a}, {a,b}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {a}, {b,c}},
b-C(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}
Since the inverse image of a closed set {c} in (Y,σ) is {c} which is b – closed but not sbĝ - closed in (X,τ), f is b – continuous but not sbĝ – continuous.

b) Let X = Y = {a,b,c},
τ = {X, ϕ, {c}, {a,b}} and σ = {Y, ϕ, {a}, {b}, {a,b}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {c}, {a,b}},
sg-C(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}
Since the inverse image of a closed set {b,c} in (Y,σ) is {b,c} which is sg – closed but not sbĝ - closed in (X,τ),f is sg – continuous but not sbĝ – continuous.

c) Let X = Y = {a,b,c},
τ = {X, ϕ, {b}} and σ = {Y, ϕ, {a}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {a}, {c}, {a,c}},
gs-C(X) = {X, ϕ, {a}, {c}, {a,b}, {a,c}, {b,c}}
Since the inverse image of a closed set {b,c} in (Y,σ) is {b,c} which is gs – closed but not sbĝ - closed in (X,τ), f is gs – continuous but not sbĝ – continuous.

d) Let X = Y = {a,b,c},
τ = {X, ϕ, {a}, {a,b}} and σ = {Y, ϕ, {a}, {b}, {a,b}, {a,c}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {b}, {c}, {b,c}},
gb-C(X) = {X, ϕ, {b}, {c}, {a,c}, {b,c}}
Since the inverse image of a closed set {a,c} in (Y,σ) is {a,c} which is gb – closed but not sbĝ - closed in (X,τ), f is gb – continuous but not sbĝ – continuous.

e) Let X = Y = {a,b,c},
τ = {X, ϕ, {a}, {b,c}} and σ = {Y, ϕ, {a}, {a,b}, {a,c}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {a}, {b,c}},
g*b-C(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}
Since the inverse image of a closed set {b} in (Y,σ) is {b} which is g*b – closed but not sbĝ - closed in (X,τ), f is g*b – continuous but not sbĝ – continuous.

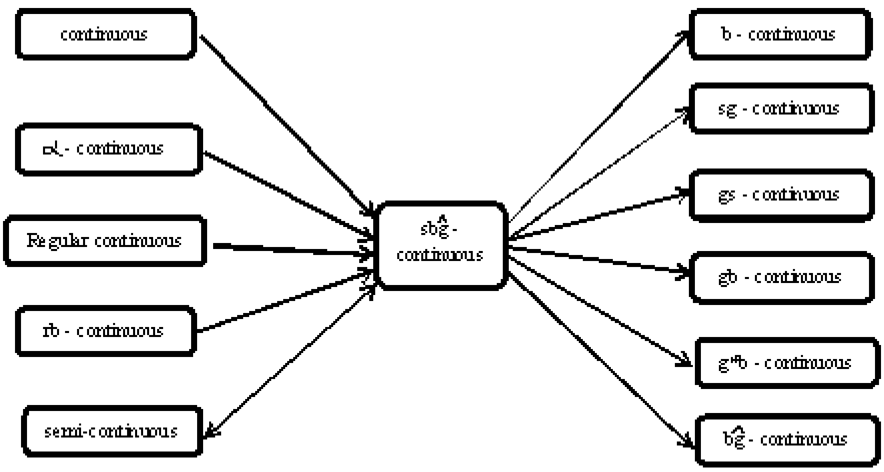
f) Let X = Y = {a,b,c},
τ = {X, ϕ, {c}, {a,b}} and σ = {Y, ϕ, {a}}.
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {c}, {a,b}},
bĝ-C(X) = {X, ϕ, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}
Since the inverse image of a closed set {b} in (Y,σ) is {b} which is bĝ – closed but not sbĝ - closed in (X,τ), f is bĝ – continuous but not sbĝ – continuous.

Proposition 3.9: If f: (X,τ) → (Y,σ) is a semi – continuous map if and only if f is a sbĝ – continuous map.

Proof: Let V be a closed set in (Y,σ). Since f is semi-continuous, then f^{-1}(V) is semi-closed in (X,τ). By proposition 3.6 in [4],f^{-1}(V) is sbĝ- closed in (X,τ). Hence, f is sbĝ – continuous.

Conversely, Let V be a closed set in (Y,σ). Since f is sbĝ - continuous, then f^{-1}(V) is sbĝ-closed in (X,τ). By proposition 3.6 in [4], f^{-1}(V) is semi - closed in (X,τ). Hence, f is semi – continuous.

Remark 3.10: The following diagram shows the relationships of sbĝ-continuous functions with other known existing functions. A → B represents A implies B but not conversely.



Proposition 3.11: Every sbĝ – irresolute is sbĝ – continuous.

Proof: Let f: (X,τ) → (Y,σ) be sbĝ – irresolute. Let V be closed in (Y,σ). Since every closed set is sbĝ-closed, V is sbĝ-closed in (Y,σ). Since f is sbĝ – irresolute, f^{-1}(V) is a sbĝ-closed set in (X,τ). Hence, f is sbĝ – continuous.

The following example shows that the converse of the above proposition need not be true.

Example 3.12: Let X = Y = {a,b,c},
τ = {X, ϕ, {a}, {b}, {a,b}, {a,c}} and σ = {Y, ϕ, {a},{b}, {a,b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {b}, {c}, {a,c}, {b,c}},
sbĝ-C(Y) = {X, ϕ, {a}, {b}, {c}, {a,c}, {b,c}},
C(X) = {X, ϕ, {c}, {a,c}, {b,c}}
Here, f is sbĝ – continuous. But, inverse image of a sbĝ-closed set {a} in Y is {a}, which is not sbĝ-closed set in X. Hence, f is not sbĝ – irresolute.

Proposition 3.13: Every sbĝ – continuous is bĝ – irresolute.

Proof: Let f: (X,τ) → (Y,σ) be sbĝ – continuous. Let V be closed in (Y,σ). Since every closed set is bĝ-closed, V is bĝ-closed in (Y,σ). Since f is sbĝ – continuous, f^{-1}(V) is a sbĝ-closed set in (X,τ). By Proposition 3.23 in [4], f^{-1}(V) is bĝ-closed set in (X,τ). Hence, f is bĝ – irresolute.

The following example shows that the converse of the above proposition need not be true.

Example 3.14: Let X = Y = {a,b,c},
τ = {X, ϕ, {a,c}} and σ = {Y, ϕ, {a},{a,b}, {a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ-C(X) = {X, ϕ, {b}},
bĝ-C(X) = {X, ϕ, {a}, {b}, {c}, {a,c}, {b,c}},
bĝ-C(Y) = {X, ϕ, {b}, {c}, {b,c}}
C(Y) = {X, ϕ, {b}, {c}, {b,c}}
Here, f is bĝ – irresolute. But, inverse image of a closed set {b,c} in Y is {b,c} which is not sbĝ-closed set in X. Therefore, f is not sbĝ – continuous.

Proposition 3.15: Let f: (X,τ) → (Y,σ) be a sbĝ – continuous map. If (X,τ) is $T_{sb\hat{g}}$ – space then f is continuous.

Proof: Let f: (X,τ) → (Y,σ) be sbĝ – continuous. Let V be a closed set in (Y,σ). Since f is sbĝ-continuous, f^{-1}(V) is sbĝ-closed set in (X,τ). Since (X,τ) is $T_{sb\hat{g}}$ – space, f^{-1}(V) is closed set in (X,τ). Therefore, f is continuous.

Proposition 3.16: Let f: (X,τ) → (Y,σ) be a sbĝ – continuous map. If (X,τ) is $T^{\alpha}_{sb\hat{g}}$ – space then f is α - continuous.

Proof: Let f: (X,τ) → (Y,σ) be sbĝ – continuous. Let V be a closed set in (Y,σ). Since f is sbĝ-continuous, f^{-1}(V) is sbĝ-closed set in (X,τ). Since (X,τ) is $T^{\alpha}_{sb\hat{g}}$ – space, f^{-1}(V) is α-closed set in (X,τ). Therefore, f is α - continuous.

4. sbĝ – OPEN MAPS AND sbĝ – CLOSED MAPS

We introduce the following definitions.

Definition 4.1: Let X and Y be two topological spaces. A map f: (X,τ) → (Y,σ) is called sbĝ – open map if for each open set V of X, f(V) is sbĝ – open set in Y.

Definition 4.2: Let X and Y be two topological sapces. A map f: (X,τ) → (Y,σ) is called sbĝ – closed map if for each closed set V of X, f(V) is sbĝ – closed set in Y.

Example 4.3: Let X = Y = {a,b,c}
τ = { X, ϕ, {a,c}} and σ = { Y, ϕ, {b}}
Define a map f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c.
Then f is sbĝ – open map, since the image of a open set {a,c} in (X,τ) is {b,c} which is sbĝ-open set in (Y,σ).

Proposition 4.4:

a) Every open map is sbĝ – open map.
b) Every α – open map is sbĝ – open map.
c) Every regular open map is sbĝ – open map.
d) Every rb – open map is sbĝ – open map.

Proof:

a) Let f: (X,τ) → (Y,σ) be an open map and V be an open set in X. Since f is an open map f(V) is an open set in Y. By Proposition 3.4 in [4], f(V) is a sbĝ – open set in (Y,σ).Therefore, f is sbĝ – open map.

b) Let f: (X,τ) → (Y,σ) be an α - open map and V be an open set in X. Since f is an α - open map, f(V) is an α - open set in Y. By Proposition 3.7 in [4], f(V) is a sbĝ – open set in (Y,σ). Therefore, f is sbĝ – open map.

c) Let f: (X,τ) → (Y,σ) be regular open map and V be an open set in X. Since f is regular open map, f(V) is an regular open set in Y. By Proposition 3.9 in [4], f(V) is a sbĝ – open set in (Y,σ). Therefore, f is sbĝ – open map.

d) Let f: (X,τ) → (Y,σ) be an rb - open map and V be an open set in X. Since f is an rb - open map, f(V) is an rb - open set in Y. By Proposition 3.19 in [4], f(V) is a sbĝ – open set in (Y,σ). Therefore, f is sbĝ – open map.

The following examples show that the converse of the above proposition need not be true.

Example 4.5:

a) Let X = Y = {a,b,c}, τ = {X, ϕ, {a,c}}, σ = {Y, ϕ, {a}, {b}, {a,b}}
   Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
   O(X) = {X,ϕ,{a,c}}
   O(Y) = {Y, ϕ, {a}, {b}, {a,b}}
   sbĝ-O(Y) = {Y,ϕ,{a},{b},{a,b},{a,c},{b,c}}

Since the image of an open set {a,c} in (X,τ) is {a,c} which is sbĝ – open set in (Y,σ) but not open set in (Y,σ), f is sbĝ – open map, but not open map.

b) Let X = Y = {a,b,c}, τ = {X, ϕ, {c}, {a,b}}, σ = {Y, ϕ, {a}, {c}, {a,c}, {b,c}}
   Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = c, f(c) = c
   O(X) = {X,ϕ,{c},{a,b}}
   α-O(Y) = {Y, ϕ, {a}, {c}, {b,c}}
   sbĝ-O(Y) = {Y,ϕ,{a},{c},{a,c},{b,c}}

Since the image of an open set {a,b} in (X,τ) is {a,c} which is sbĝ – open set in (Y,σ) but not an α - open set in (Y,σ), f is sbĝ – open map, but not an α - open map.

c) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {a,b}}, σ = {Y, ϕ, {a}, {b}, {a,b}, {a,c}}
   Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
   O(X) = {X,ϕ,{a},{a,b}}
   rO(Y) = {Y, ϕ, {b}, {a,c}}
   sbĝ-O(Y) = {Y,ϕ,{a},{b},{a,b},{a,c}}

Since the image of open sets {a},{a,b} in (X,τ) are {a},{a,b} which are sbĝ – open sets in (Y,σ) but not regular open sets in (Y,σ), f is sbĝ – open map, but not regular open map.

d) Let X = Y = {a,b,c}, τ = {X, ϕ,{a,c}}, σ = {Y, ϕ, {a},{a,b}, {a,c}}
   Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
   O(X) = {X,ϕ,{a,c}}
   rb-O(Y) = {Y, ϕ, {a}, {a,b}}
   sbĝ-O(Y) = {Y,ϕ,{a},{a,b},{a,c}}

Since the image of an open set {a,c} in (X,τ) is {a,c} which is sbĝ – open set in (Y,σ) but not rb - open set in (Y,σ), f is sbĝ – open map, but not rb - open map.

Proposition 4.6:

a. Every sbĝ- open map is b – open map
b. Every sbĝ – open map is sg – open map
c. Every sbĝ – open map is gs – open map
d. Every sbĝ – open map is gb – open map
e. Every sbĝ – open map is g*b – open map
f. Every sbĝ – open map is bĝ – open map.

Proof:

a. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.11 in [4], f(V) is b-open set in (Y,σ). Therefore, f is b – open map.
b. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.13 in [4], f(V) is sg-open set in (Y,σ). Therefore, f is sg – open map.
c. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.15 in [4], f(V) is gs - open set in (Y,σ). Therefore, f is gs – open map.
d. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.17 in [4], f(V) is gb - open set in (Y,σ). Therefore, f is gb – open map.
e. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.21 in [4], f(V) is g*b - open set in (Y,σ). Therefore, f is g*b – open map.
f. Let f: (X,τ) → (Y,σ) be a sbĝ – open map and V be an open set in X. Since f is sbĝ – open map, f(V) is sbĝ – open set in Y. By Proposition 3.23 in [4], f(V) is bĝ - open set in (Y,σ). Therefore, f is bĝ – open map.

The following examples show that the converse of the above proposition need not be true.

Example 4.7:

a. Let X = Y = {a,b,c}, τ = {X, ϕ,{a},{b},{a,b},{a,c}}, σ = {Y, ϕ, {a},{b,c}}
   Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c

$O(X) = \{X, \phi, \{a\},\{b\},\{a,b\},\{a,c\}\}$
$b\text{-}O(Y) = \{Y, \phi, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{a\},\{b,c\}\}$
Since the image of open sets $\{b\},\{a,b\},\{a,c\}$ in (X,τ) are $\{b\},\{a,b\},\{a,c\}$ respectively which are b – open sets in (Y,σ) but not $sb\hat{g}$ - open sets in (Y,σ), f is b – open map, but not $sb\hat{g}$ - open map.

b. Let $X = Y = \{a,b,c\}$, $\tau = \{X, \phi, \{b\}\}$, $\sigma = \{Y, \phi, \{c\},\{a,b\}\}$
Define a function $f: (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$
$O(X) = \{X, \phi, \{b\}\}$
$sg\text{-}O(Y) = \{Y, \phi, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{c\},\{a,b\}\}$
Since the image of an open set $\{b\}$ in (X,τ) is $\{b\}$ which is sg – open set in (Y,σ) but not $sb\hat{g}$ - open set in (Y,σ), f is sg – open map, but not $sb\hat{g}$ - open map.

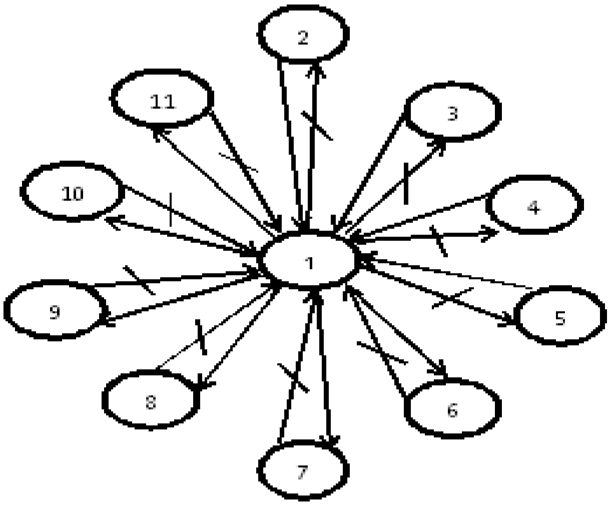
c. Let $X = Y = \{a,b,c\}$, $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{a,c\}\}$
Define a function $f: (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$
$O(X) = \{X, \phi, \{a\}\}$
$gs\text{-}O(Y) = \{Y, \phi, \{a\},\{c\},\{a,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{a,c\}\}$
Since the image of an open set $\{a\}$ in (X,τ) is $\{a\}$ which is gs – open set in (Y,σ) but not $sb\hat{g}$ - open set in (Y,σ), f is gs – open map, but not $sb\hat{g}$ - open map.

d. Let $X = Y = \{a,b,c\}$, $\tau = \{X, \phi, \{a\},\{b\},\{a,b\}\}$, $\sigma = \{Y, \phi, \{a\}\}$
Define a function $f: (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$
$O(X) = \{X, \phi, \{a\},\{b\},\{a,b\}\}$
$gb\text{-}O(Y) = \{Y, \phi, \{a\},\{b\},\{c\},\{a,b\},\{a,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{a\},\{a,b\},\{a,c\}\}$
Since the image of an open set $\{b\}$ in (X,τ) is $\{b\}$ which is gb – open set in (Y,σ) but not $sb\hat{g}$ - open set in (Y,σ), f is gb – open map, but not $sb\hat{g}$ - open map.

e. Let $X = Y = \{a,b,c\}$, $\tau = \{X, \phi, \{a\},\{b\},\{a,b\},\{a,c\}\}$, $\sigma = \{Y, \phi, \{a\},\{a,b\}\}$
Define a function $f: (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$
$O(X) = \{X, \phi, \{a\},\{b\},\{a,b\},\{a,c\}\}$
$g^*b\text{-}O(Y) = \{Y, \phi, \{a\},\{b\},\{a,b\},\{a,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{a\},\{a,b\},\{a,c\}\}$
Since the image of an open set $\{b\}$ in (X,τ) is $\{b\}$ which is g*b – open set in (Y,σ) but not $sb\hat{g}$ - open set in (Y,σ), f is g*b – open map, but not $sb\hat{g}$ - open map.

f. Let $X = Y = \{a,b,c\}$, $\tau = \{X, \phi, \{a\},\{a,b\},\{a,c\}\}$, $\sigma = \{Y, \phi, \{b\}\}$
Define a function $f: (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$
$O(X) = \{X, \phi, \{a\},\{a,b\},\{a,c\}\}$
$b\hat{g}\text{-}O(Y) = \{Y, \phi, \{a\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$
$sb\hat{g}\text{-}O(Y) = \{Y,\phi,\{a\},\{c\},\{a,c\}\}$
Since the image of an open set $\{a,b\}$ in (X,τ) is $\{a,b\}$ which is $b\hat{g}$ – open set in (Y,σ) but not $sb\hat{g}$ - open set in (Y,σ), f is $b\hat{g}$ – open map, but not $sb\hat{g}$ - open map.

Remark 4.8: The following diagram shows the relationships of $sb\hat{g}$ – open map with other known existing open maps. A $\rightarrow$ B represents A implies B but not conversely.



1. sbĝ - open map	2. open map	3. α – open map
4. Regular open map	5. rb – open map	6. b – open map
7. sg – open map	8. gs – open map	9. gb – open map
10. g*b – open map	11. bĝ - open map	

5. sbĝ – HOMEOMORPHISM

We introduce the following definition.

Definition 5.1: A bijection f: (X,τ) → (Y,σ) is called a sbĝ – homeomorphism if f is both sbĝ – continuous map and sbĝ- open map.
That is, Both f and $f^{-1}$are sbĝ – continuous map.

Example 5.2: Let X = Y = {a,b,c}
τ = { X, ϕ, {a}} and σ = { Y, ϕ, {b}}
Define a map f: (X,τ) → (Y,σ) by f(a) = b, f(b) = c, f(c) = a
sbĝ – C(X) = {X,ϕ,{b},{c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{b},{a,b},{b,c}}
C(Y) = {Y,ϕ,{a,c}}
O(X) = {X,ϕ,{a}}

Here, the inverse image of a closed set {a,c} in Y is {b,c} which is sbĝ – closed set in X and the image of an open set {a} in X is {b} which is sbĝ –open in Y. Hence, f is sbĝ – homeomorphism.

Proposition 5.3:
a) Every homeomorphism is sbĝ – homeomorphism
b) Every sbĝ – homeomorphism is b – homeomorphism
c) Every sbĝ – homeomorphism is gs – homeomorphism
d) Every sbĝ – homeomorphism is gb – homeomorphism
e) Every sbĝ – homeomorphism is g*b – homeomorphism

f) Every sbĝ –homeomorphism is bĝ – homeomorphism

Proof:

a) Let f: (X,τ) → (Y,σ) be a homeomorphism. Then f is continuous and open map. By Proposition 3.5(a) and Proposition 4.4(a), f is sbĝ – continuous and sbĝ – open map. Hence, f is sbĝ – homeomorphism.

b) Let f: (X,τ) → (Y,σ) be a sbĝ - homeomorphism. Then f is sbĝ- continuous and sbĝ - open map. By Proposition 3.7(a) and Proposition 4.6(a), f is b – continuous and b – open map. Hence, f is b – homeomorphism.

c) Let f: (X,τ) → (Y,σ) be a sbĝ - homeomorphism. Then f is sbĝ- continuous and sbĝ - open map. By Proposition 3.7(c) and Proposition 4.6(c), f is gs – continuous and gs – open map. Hence, f is gs – homeomorphism.

d) Let f: (X,τ) → (Y,σ) be a sbĝ - homeomorphism. Then f is sbĝ- continuous and sbĝ - open map. By Proposition 3.7(d) and Proposition 4.6(d), f is gb – continuous and gb – open map. Hence, f is gb – homeomorphism.

e) Let f: (X,τ) → (Y,σ) be a sbĝ - homeomorphism. Then f is sbĝ- continuous and sbĝ - open map. By Proposition 3.7(e) and Proposition 4.6(e), f is g*b – continuous and g*b – open map. Hence, f is g*b – homeomorphism.

f) Let f: (X,τ) → (Y,σ) be a sbĝ - homeomorphism. Then f is sbĝ- continuous and sbĝ - open map. By Proposition 3.7(f) and Proposition 4.6(f), f is bĝ – continuous and bĝ – open map. Hence, f is bĝ – homeomorphism.

The following examples show that the converse of the above proposition need not be true.

Example 5.4:

a) Let X = Y = {a,b,c}, τ = {X, ϕ, {a,c}}, σ = {Y, ϕ, {a}, {a,b}, {a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
sbĝ – C(X) = {X,ϕ,{b},{c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}
C(Y) = { Y,ϕ,{b},{c},{b,c}}
O(X) = {X,ϕ,{a,c}}

Here, the inverse image of closed sets {c},{b,c} in Y are {c},{b,c} which are sbĝ – closed in X but not closed in X. So f is sbĝ – continuous but not continuous. Also, the image of an open set {a,c} in X is {a,c} which is sbĝ – open set in Y. Thus, f is sbĝ – open map. Therefore, f is sbĝ – homeomorphism, but not a homeomorphism.

b) Let X = Y = {a,b,c}, τ = {X, ϕ, {a},{b},{a,b},{a,c}}, σ = {Y, ϕ,{a},{b,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c
sbĝ – C(X) = {X,ϕ,{b},{c},{a,c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{a},{b,c}}
C(Y) = { Y,ϕ,{a},{b,c}}
O(X) = {X,ϕ,{a},{b},{a,b},{a,c}}
b – C(X) = {X,ϕ,{b},{c},{a,c},{b,c}}
b – O(Y) = {Y,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}

Here, the image of open sets {a},{a,b} are {b},{a,b} which are b-open sets in Y but not sbĝ – open set in Y. So f is b – open map but not sbĝ – open map. Also, the inverse image of a closed sets {a},{b,c} in Y are {b},{a,c} which are b – closed in X. Thus, f is b – continuous map. Therefore, f is b – homeomorphism but not sbĝ – homeomorphism.

c) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}}, σ = {Y, ϕ,{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
sbĝ – C(X) = {X,ϕ,{b},{c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{a,c}}
C(Y) = { Y,ϕ,{b}}
O(X) = {X,ϕ,{a}}
gs – C(X) = {X,ϕ,{b},{c},{a,b},{a,c},{b,c}}
gs – O(Y) = {Y,ϕ,{a},{c},{a,c}}

Here, the image of an open set {a} is {a} which is gs-open set in Y but not sbĝ – open set in Y. So f is gs – open map but not sbĝ – open map. Also the inverse image of a closed set {b} in Y is {b} which is gs – closed in X. Thus, f is gs – continuous map. Therefore, f is gs – homeomorphism but not sbĝ – homeomorphism.

d) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {a,b}}, σ = {Y, ϕ, {a}, {b}, {a,b}, {a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
sbĝ – C(X) = {X,ϕ,{b},{c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{a},{b},{a,b},{a,c}}
C(Y) = { Y,ϕ,{b},{c},{a,c},{b,c}}
O(X) = {X,ϕ,{a},{a,b}}
gb – C(X) = { X,ϕ,{b},{c},{a,c},{b,c}}
gb – O(Y) = {Y,ϕ,{a},{b},{a,b},{a,c}}

Here, the inverse image of a closed set {a,c} in Y is {a,c} which is gb – closed set in X but not sbĝ – closed set in X. So f is gb – continuous map but not sbĝ – continuous map. Also, the image of the open sets {a}, {a,b} in X are {a},{a,b} which are gb – open set in Y. Thus, f is gb – open map. Therefore, f is gb – homeomorphism but not sbĝ – homeomorphism.

e) Let X = Y = {a,b,c}, τ = {X, ϕ, {a}, {b}, {a,b}, {a,c}}, σ = {Y, ϕ, {a},{a,b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
sbĝ – C(X) = {X,ϕ,{b},{c},{a,c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}
C(Y) = { Y,ϕ,{c},{b,c}}
O(X) = {X,ϕ,{a},{b},{a,b},{a,c}}
g*b – C(X) = { X,ϕ,{b},{c},{a,c},{b,c}}
g*b – O(Y) = {Y,ϕ,{a},{b},{a,b},{a,c}}

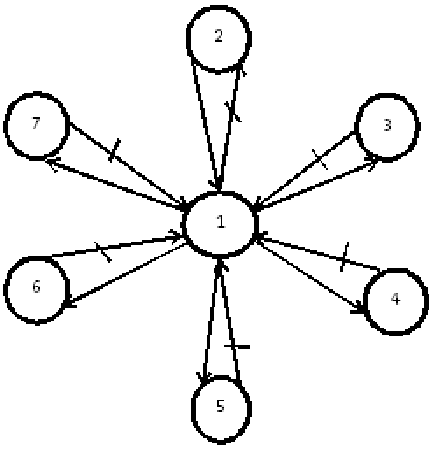
Here, the image of an open set {b} is {b} which is g*b – open set in Y but not sbĝ – open set in Y. So f is g*b – open map but not sbĝ – open map. Also, the inverse image of a closed sets {c},{b,c} in Y are {c},{b,c} which are g*b – closed sets in X. Thus f is g*b – continuous map. Therefore, f is g*b – homeomorphism but not sbĝ – homeomorphism.

f) Let X = Y = {a,b,c}, τ = {X, ϕ, {c}, {a,b}}, σ = {Y, ϕ, {a}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c
sbĝ – C(X) = {X,ϕ,{c},{a,b}}
sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}

C(Y) = { Y,ϕ,{b,c}}
O(X) = {X,ϕ,{c},{a,b}}
bĝ – C(X) = { X,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}
bĝ – O(Y) = {Y,ϕ,{a},{b},{c},{a,b},{a,c}}

Here, the inverse image of a closed set {b,c} in Y is {b,c} which is bĝ – closed set in X but not sbĝ – closed set in X. So, f is bĝ – continuous map but not sbĝ – continuous map. Also, the image of an open set {a} in X is {a} which is bĝ –open set in Y but not sbĝ – open set in Y. Thus, f is bĝ – open map but not sbĝ – open map. Therefore, f is bĝ – homeomorphism but not sbĝ – homeomorphism.

Remark 5.7: The following diagram shows the relationships of sbĝ – homeomorphism with other known existing homeomorphisms. A $\rightarrow$ B represents A implies B but not conversely.



1. sbĝ – homeomorphism
2. homeomorphism
3. b - homeomorphism
4. gs – homeomorphism
5. gb – homeomorphism
6. g*b – homeomorphism
7. bĝ – homeomorphism.

Proposition 5.8: For any bijection f: (X,τ) → (Y,σ) the following statements are equivalent

a) Its inverse map f^{-1}: Y → X is sbĝ – continuous
b) f is a sbĝ – open map
c) f is a sbĝ – closed map.

Proof: (a) $\Rightarrow$ (b) :

Let G be any open set in X.

Since f^{-1} is sbĝ – continuous, f(G) is sbĝ – open in Y. So, f is a sbĝ – open map.

(b) $\Rightarrow$ (c) :

Let F be any closed set in X. Then F^c is open in X. Since f is sbĝ – open, $f(F^c)$ is sbĝ – open in Y. So, f(F) is sbĝ – closed in Y. Therefore, f is a sbĝ – closed map.

(c) $\Rightarrow$ (a) :

Let F be any closed set in X. Since f is a sbĝ – closed map, f(F) is closed in Y. So, $(f^{-1})^{-1}(F)$ is closed in Y. Therefore, f^{-1} is sbĝ – continuous.

6. REFERENCES

[1] Ahmad Al. Omari and Mohd.SalmiMD. Noorani, On Generalized b-closed sets, *Bull. Malaysian Mathematical Sciences Society*,(2) 32(1) (2009),19-30.

[2] D. Andrijevic, On b-open sets, *Mat. Vesnik.*,48(1996), no. 1-2, 59-64.

[3] S.P.Arya and T.M.Nour, Characterizations of S-Normal spaces, *Indian J.Pure Appl. Math.*, Vol 21(1990).

[4] K.Bala Deepa Arasi & S.Navaneetha Krishnan, On sbĝ-closed sets in Topological spaces, *IJMA* – 6(10), 2015, 115-121.

[5] P.Bhattacharya and B.K.Lahiri, Semi-generalized closed sets in Topology, *Indian J. Math.*, 29(1987), 375-382.

[6]M.Caldas and E.Ekici, Slightly γ continuous functions Bol. Soc, Parana Mat (3)22(2004) No.2,63-74.

[7] R.Devi R., Maki H., and Balachandran K., Semi-generalized homeomorphisms and generalized semi-homeomorphism in topological spaces, *Indian J. Pure. Appl. Math., 26(3) (1995), 271-284.*

[8] R.Devi, H.Maki and K.Balachandran, Semi generalized Closed maps and generalized semi-closed maps, MEM. Fac.Sci.Kochi Univ. Sec A Math 14(1993) 41-54.

[9] N Levine, Semi-open sets and semi-continuity in topological spaces *Amer.Math. Monthly*, 70(1963), 36-41.

[10] N Levine, Generalized closed sets in topology *Rend.Circ.Mat.Palermo*, 19(1970) 89-96.

[11] H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 15(1994),57-63.

[12] Nagaveni and A.Narmadha, On regular b-closed sets in Topological spaces, *Heber. Internationalconference on Application of Mathematics & Statistics*, *HICAMS*-2012, 5-7, Jan 2012, 81-87.

[13] A.Narmadha and Nagaveni, On regular b-open sets in Topological spaces, *Int. Journal of Math. Analysis*, 7(19)(2013) 937-948.

[14] New Class of rg*b – continuous functions in toplogicals spaces, International Journal of Fuzzy Mathematical Archive.

[15] ONjastad, On some classes of nearly open sets, *Pacific J Math.*, 15(1965),

[16] Stone.M,Application of the theory of Boolean rings to general topology, *Trans. Amer. Maths. Soc.*, 41(1937) 374-481.

[17] R.Subasree and M.Maria Singam , On b$\hat{g}$ - Closed Sets in Topological Spaces, *IJMA*, 4(7)(2013),168-173.

[18] P.Sundaram, H.Maki and K.Balachandran, Semi generalized continuous maps and Semi $T_{1/2}$ spaces *Bull. Fukuoka Univ Ed Part III 40(1991) 33-40*

[19] R.Subasree and M.Maria Singam, On b$\hat{g}$ – continuous maps and b$\hat{g}$ – open maps in Topological spaces, *IJERT*, Vol 2(10), October 2013.

[21] M.K.R.S.Veerakumar, (2003), $\hat{g}$ – closed sets in Topological Space, *Bull. Allahabad. Math. Soc.*, Vol.18, 99-112.

[22]N.V. Velicko, H-closed topological spaces, *Amer. Math.Soc. Transl.*, 78(1968), 103-118.

[23] D.Vidhya and Parimelazhagan, g*b-closed sets in Topological spaces, *Int. J. Contemp. Math. Sciences*, 7(27), 2012,1305-1312.

[24]D.Vidhya & R.Parimelazhagan, g*b homeomorphism and contra g*b – continuous maps in Topological spaces, *International Journal of computer Applications*, Volume 58 – No.14, November 2012.