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CONTROLLING OF CHAOS IN THE LOGISTIC MAP

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ABSTRACT

The map $f(x) = ax(1-x)$, is known as the logistic map where a is the control parameter. This map has been extensively studied by many researchers and rich contents of bifurcations and chaos have been explored. Logistic map is one of the well known maps and has become a standard map for studying bifurcations and chaos of discrete dynamical systems. In general, x and the control parameter a of logistic map are restricted in $0 \leq x \leq 1$ and $0 < a \leq 4$ so that each x in the interval [0,1] is mapped onto the same interval [0,1]. It is known that there are stable fixed points $x^* = 0$ and $x^* = 1 - {}^{1}/_{a}$ in the interval of our interest. After that, we have period-doubling bifurcation at a = 3, 3.4494897, 3.54409... . These numerical results are well known and can be reproduced through computer programs. In logistic map the accumulation point is given by a = 3.566945672..... . In this paper discussion is related with controlling of chaos for different values of "a" by proportional pulse method in one dimensional cascade and by OGY method for two dimensional cascade. Chaos occurrence has been advanced or delayed by immigration-emigration technique.

Keywords: Proportional Pulse method; OGY method; Immigration-Emigration technique.

1.1 Introduction:

The discretized version of a continuous dynamical system known as logistic equation representing the growth of some population may be called as logistic map. The map is given by $x_{n+1} = a\, x_n(1 - x_n)$, where '$a$' is the control parameter and x_n is the proportionate growth in the population. It has been a wonderful example of one dimensional model which shows many complicated behavior despite its simple structure. Period doubling route to chaos is one of the wonderful characteristics it shows with the increase of the parameter 'a'. The difference equation attains a chaotic character when the control parameter is just less than 3.6. This means that the long term behavior cannot be predicted with the help of the periodic points. Hence here it comes the need of controlling of chaos. The period doubling route to chaos for the logistic map is shown in the following figure:

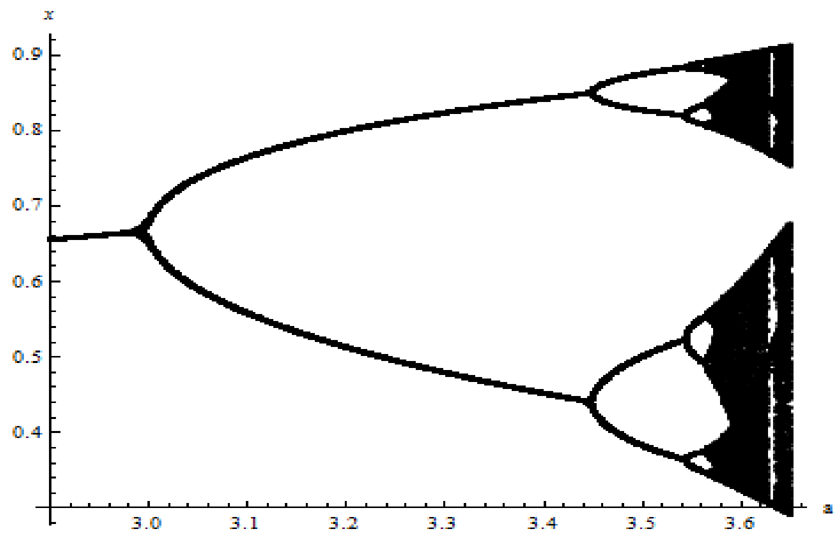

Fig 1.11:

The first control technique was OGY method [1**9**] proposed by Ott et.al in 1990. A lot of techniques have come after that. We have taken the periodic proportional pulse method [**3,14**] to control chaos in logistic map. The proportional pulse method was discussed by Matias and Guemez [**14**]. After that N.P.Chau [**3**] discussed in a similar manner but gave some restrictions on the initial conditions by which chaos can be controlled. However, he has also shown that for large periodic value sometimes chaos is suppressed and sometimes not.

In section 1.2 Chau's method has been applied to control chaos in logistic map at the parameter beyond accumulation point. In section 1.3 OGY method has been applied to control chaos. In section 1.4 OGY technique has been applied together with Chau's technique and chaos could be controlled successfully. In section 1.5 immigration-emigration technique has been applied, however in this case chaos could not be controlled although it could be advanced or delayed.

1.2 Controlling of chaos in logistic map using Chau's technique[13]:

Theory of Chau's technique is as follows. Let the difference equation be $x_{n+1} = f(x_n, a)$, where 'a' is some control parameter. Let x^* be one of the periodic points of period p. Then, $x^* = f^p(x^*, a)$. For stability we have $\left|\frac{df^p}{dx}_{x=x*}\right| < 1$. Let us assume that the map shows chaos at some parameter say $a = a_0$ and so all the periodic points are unstable. So, at $a = a_0$, $\left|\frac{df^p}{dx}_{x=x*}\right| > 1$. We define a new composite function as $G(x) = kf^p(x)$. Then a fixed point x_s will satisfy the equation,

$$G(x_s) = kf^p(x_s) = x_s \qquad \textbf{(1.2.1)}$$

We consider another new function $C^p(x) = \frac{x}{f^p(x)}\frac{df^p}{dx}$. So, for stability of G at x_s, $|c^p(x_s)| < 1$. We draw the curve $c^p(x)$ and choose the value of x such that $|c^p(x)| < 1$. That value of x can be made as periodic point of period p for the function G for some suitable value of k which can be easily calculated using the equation (1.2.1). For period $p = 1$, it is shown as follows:

$p = 1$:

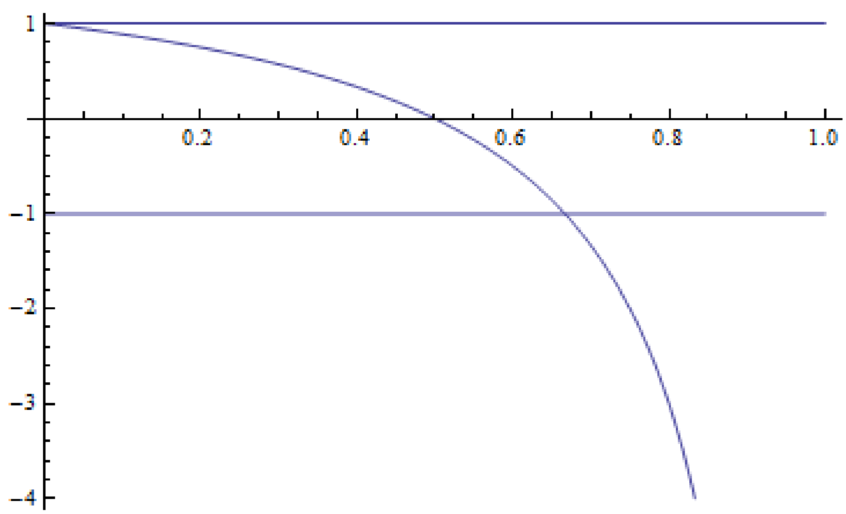

Fig 1.2.a: The curves are $C^1(x)$, y=1 and y= -1.The figure shows positive values of x for which $C^1(x)$ lies between -1 and 1.

A value of x is chosen in such a way that $|c^1(x)| < 1$ (the above figure will help in choosing the value of x) and we can get the value of k satisfying equation (1.2.1). Let x = 3.6 be one such point and the corresponding value of k is 0.555556

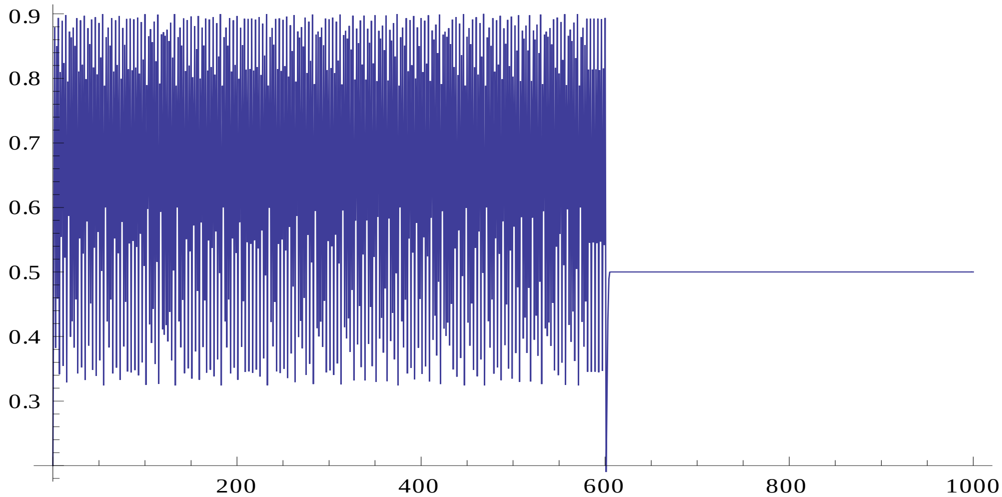

Fig 1.2.b: Abscissa represents no. of iterations; ordinate represents value of x at every iteration.

In the figure 1.2.b, up to 600 iterations it has been observed that chaos occurs, and after that the control is switched on. As the point x = 3.6 is stable for the function G, it has been observed from the above experiment that the system reaches the desired fixed point. The same procedure can be repeated to obtain desired number of periodic points. However in practice it is a bit difficult to obtain higher periodic points in the long run for the system.

For $p = 2$:

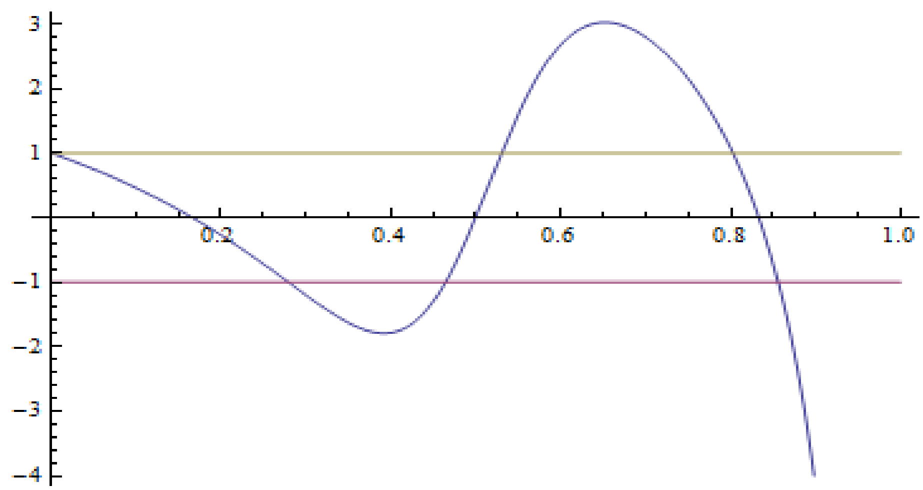

Fig 1.2.c: The curves are $C^2(x)$, y=1 and y= -1. The figure shows positive values of x for which $C^2(x)$ lies between -1 and 1.

After taking a suitable value of x such that $|C^2(x)| < 1$, suitable value of k is obtained such that x becomes periodic point of period two and then applying periodic pulse in the system periodic nature is obtained. The following figure shows the chaotic nature of the map up to iteration number 4000 and after that chaos control is switched on to obtain periodic nature of period two.

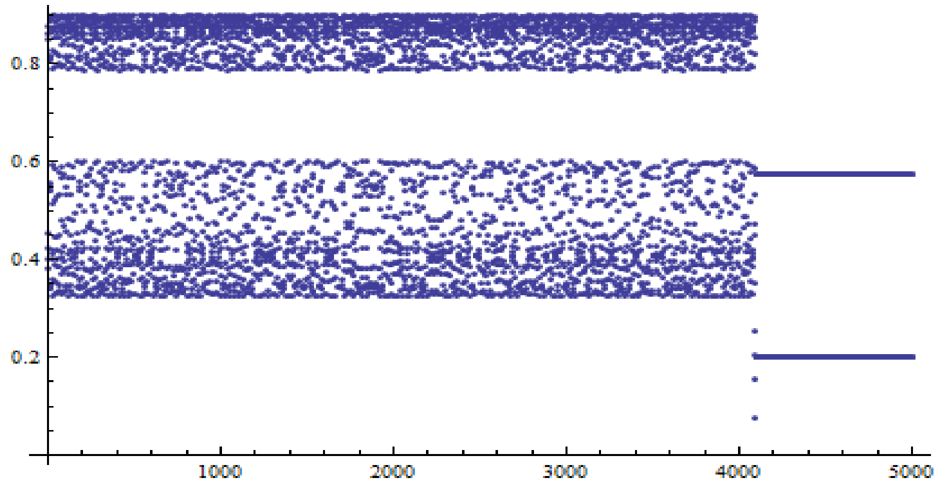

Fig 1.2.d: Abscissa represents the number of iterations, while the ordinate represents the value of x at every iteration.

Similarly at the parameter a = 3.6, $C^4(x)$ is plotted.

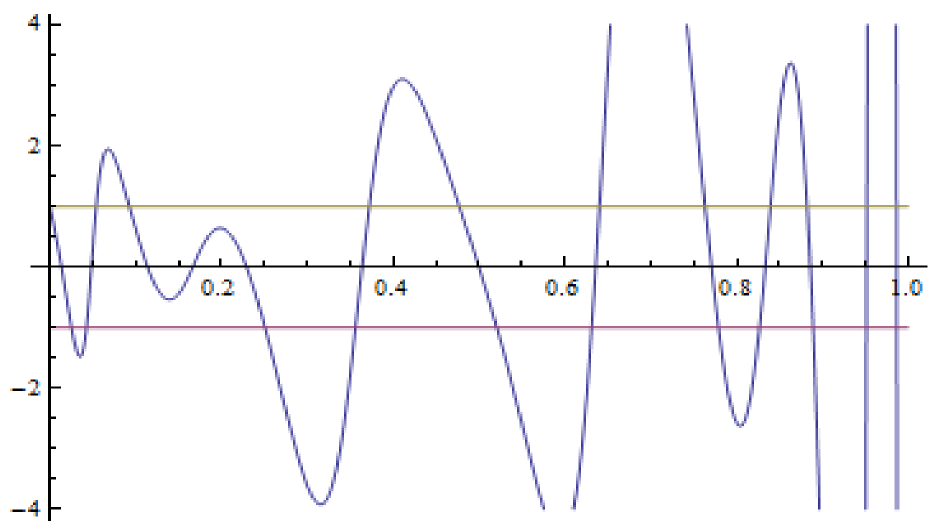

Fig 1.2.e: The curves are $C^4(x)$, y=1 and y= -1.The figure shows part of positive values of x for which $C^4(x)$ lies between -1 and 1.

The diagram showing control of chaos is as follows:

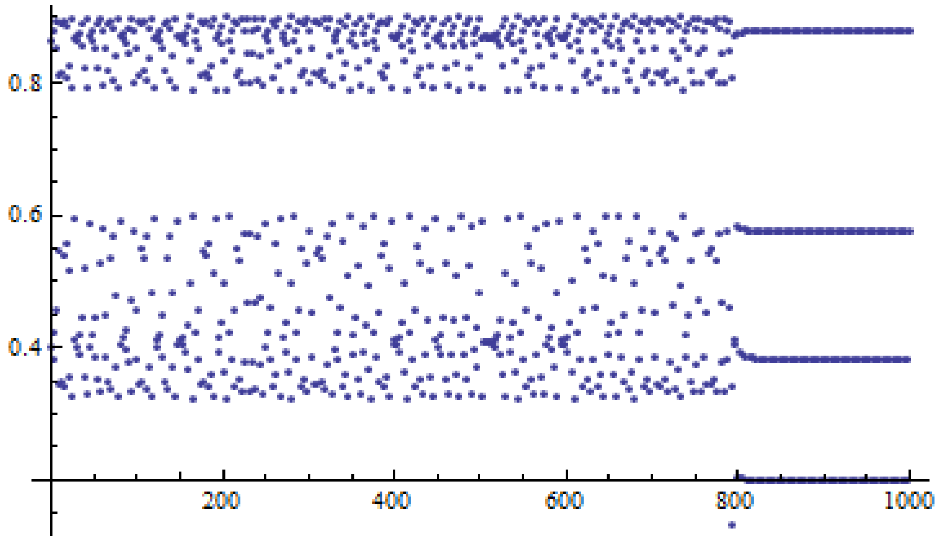

Fig 1.2.f: Abscissa represents the number of iterations, while the ordinate represents the value of x at every iteration.

For $p = 8$:

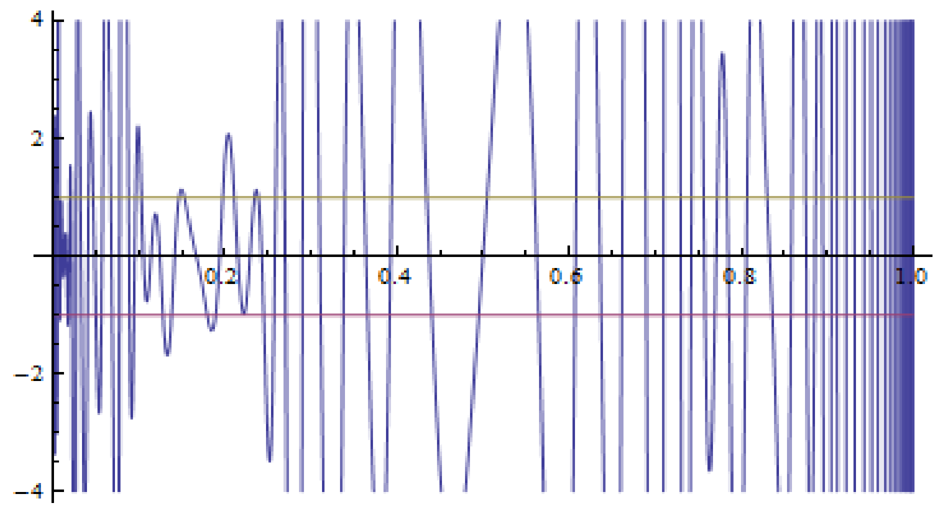

Fig 1.2.e: The curve $C^4(x)$, y=1 and y= -1.The figure shows part of positive values of x for which $C^4(x)$ lies between -1 and 1.

The diagram showing control of chaos is as follows:

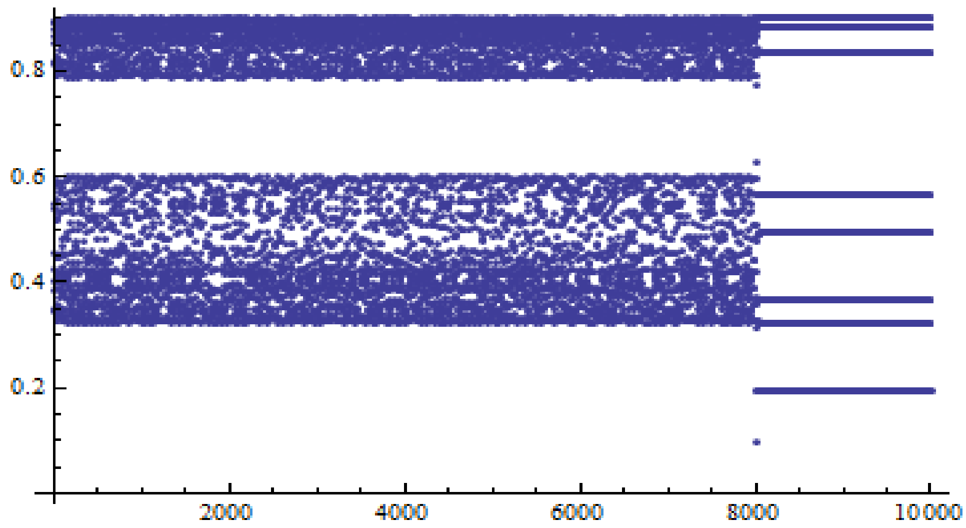

Fig 1.2.f: Abscissa represents the number of iterations, while the ordinate represents the value of x at every iteration.

1.3 Controlling of chaos in logistic map using OGY technique [13]:

The OGY method [19] applied is as follows:

Let the two dimensional map be written as:

$x_{n+1} = g(x_n, a)$. Let $\mathrm{x_u}(a)$ be an unstable fixed point of equation (1.1.2). For values of 'a' near a_0 (say) in a small neighborhood of $\mathrm{x_u}(a_0)$, the map can be approximated by a linear map given by $x_{n+1} - \mathrm{x_u}(a_0) = \mathrm{J}\big(\mathrm{x_n} - \mathrm{x_u}(a_0)\big) + C(a - a_0)$ ………...……(1.3.1)

Where J is the Jacobian and C is $\frac{\partial g}{\partial a}$, at the point $\mathrm{x_u}(a_0)$. Assuming that in a small neighborhood around the fixed point,

$a - a_0 = -K(x_n - x_u(a_0))$, where K is a constant vector of dimension 2 to be determined. Then the equation (1.3.1) becomes

$x_{n+1} - x_u(a_0) = (\mathrm{J} - \mathrm{CK})\big(\mathrm{x_n} - \mathrm{x_u}(a_0)\big)$ ………………(1.3.2)

Using equation (1.3.2) for the difference equation $(x_{n+1}, y_{n+1}) = (a\, x_n(1 - x_n), 0)$ which is the two dimensional version of logistic map, at the parameter value a = 3.86, a time series plot is shown below, where after 300 iterations, chaos control is switched on.

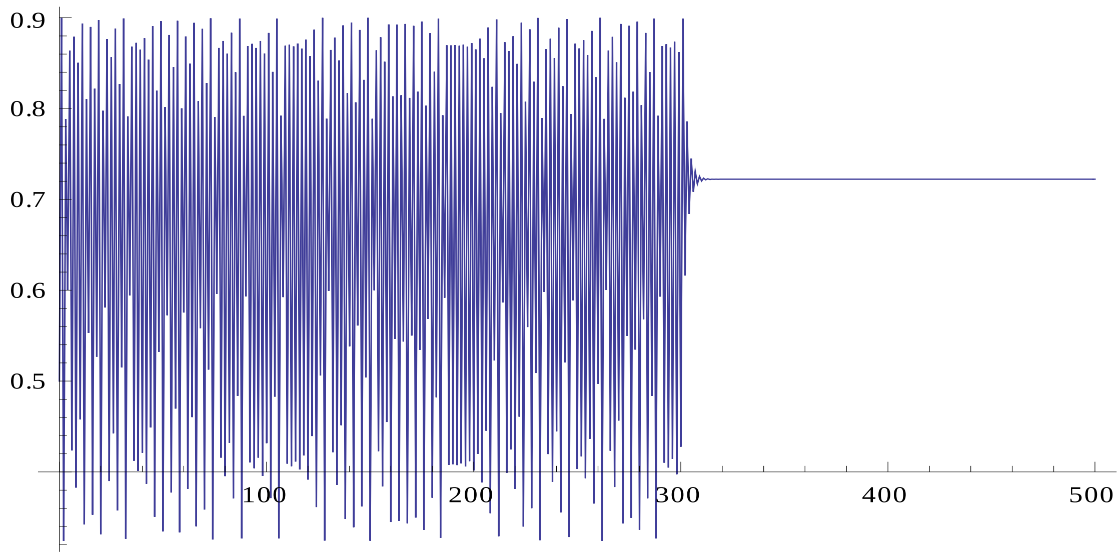

Fig 1.3a: Abscissa represents the number of iteration and the ordinate represents the value of x_n.

In case of OGY method, the same fixed point of logistic map is made stable which was unstable at the parameter value a = 3.86.

The limitation of Chau's method is that with the periodic pulse the fixed point of the original map cannot be made stable. However in case of logistic map there are infinite values of x to which the system can be headed with a suitable periodic force applied.

In case of OGY method a different point cannot be made a fixed point. Hence a natural question arises. Is it possible to stabilize those values of x for which $|C^1(x)| > 1$ in Chau's method? However the answer is yes and it is shown in the next section.

1.4 OGY in Chau's method:

The difference equation is $x_{n+1} = f(x_n, a)$, where $f(x, a) = a\,x(1 - x)$. Suppose the value of the parameter is set to form chaotic attractor or otherwise the fixed point is unstable. Then we consider the difference equation in Chau's way $x_{n+1} = F(x_n, a)$, where $F(x, a) = k\,f(x, a)$, where k is chosen in such a way that any point becomes a fixed point. By Chau's way it has been observed that infinite points are there such that $|C^1(x)| < 1$. However infinite points lie in the curve $C^1(x)$ such that $|C^1(x)| > 1$. One of those points is taken and for some suitable value of k the point is made fixed point yet unstable and then OGY is applied to stabilize that. An example is x = 0.8 is a fixed point for F which is unstable but can be made stable applying OGY on it. It is shown as bellow:

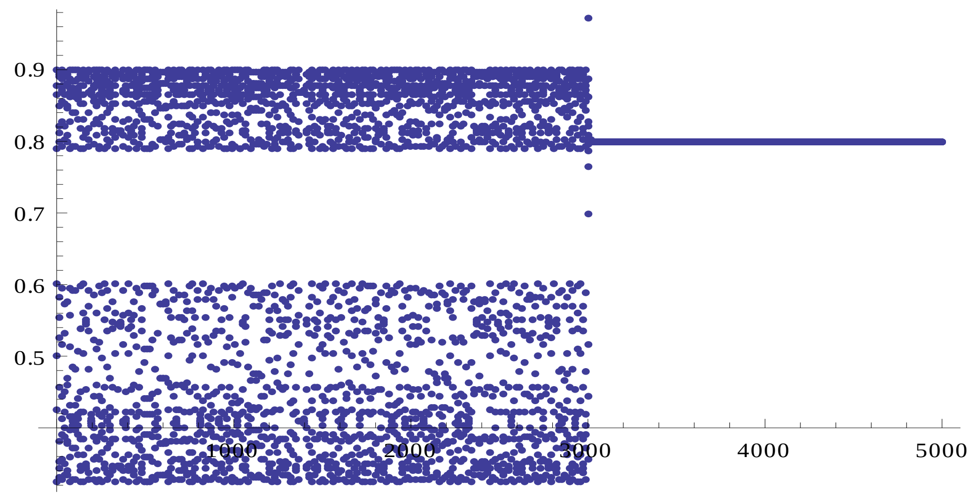

Fig:1.4 Abscissa represents the number of iteration and the ordinate represents the value of x_n.

1.5 Immigration Emigration method [20]:

We consider the discrete time single population model $x_{n+1} = f(x_n) + c.$, where x_{n+1} represents the growth of population in the presence of constant influx say c. Then it has been observed that the effect of influx can control chaotic situation. Same technique has been applied in case of logistic map, however the chaotic scenario could not be avoided although due to consideration of immigration and emigration chaos can be delayed or made early with respect to the parameter. Some examples are as follows:

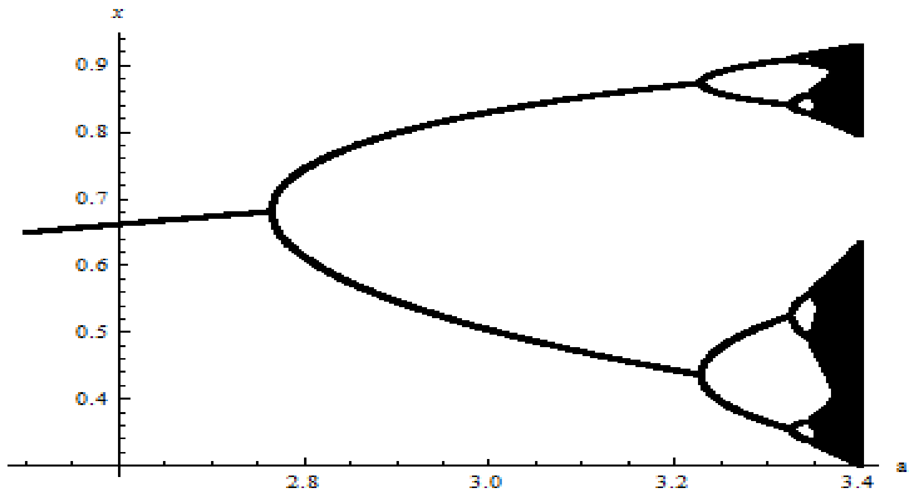

FIG:1.5 Immigration technique($f(x) = ax(1 - x) + 0.08$ has been taken).

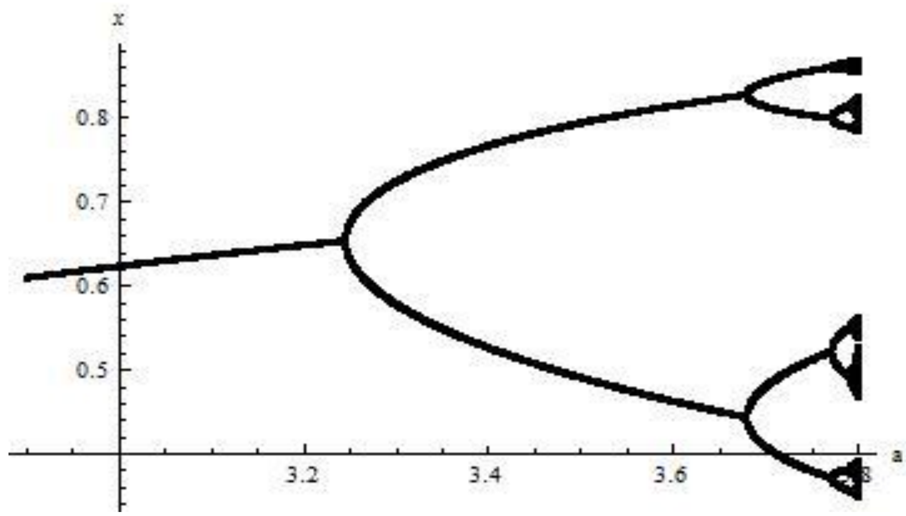

FIG: 1.6 Emigration technique ($f(x) = ax(1 - x) - 0.08$ has been taken)

Conclusion: From the above discussion it is observed that the used methods are applicable and effective to control chaos for low periodic points. It is difficult to sustain periodic points of period greater than eight as the basin of attraction of these points become small and hence when the chaos control parameter is switched on the initial value may not fall in the basin of attraction of the stable periodic points.

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