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MAGNETOHYDRODYNAMIC UNSTEADY FLOW OF A DUSTY CONDUCTING FLUID IN A RECTANGULAR CHANNEL WITH TIME DEPENDENT PRESSURE GRADIENT

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ABSTRACT

In the present paper, the unsteady flow of a dusty conducting viscous incompressible fluid in a long rectangular channel under the influence of uniform magnetic field and a time dependent pressure gradient has been studied. The particular cases when the pressure gradient is (i) an absolute constant,(ii) periodic function of time, (iii) an exponentially decreasing function of time and (iv) $Cte^{-\lambda t}$*, have been discussed in detail.*

KEYWORDS: Magnetohydrodynamic flow, Dusty fluid, Pressure gradient rectangular channel.

INTRODUCTION

Interest in problem of mechanics of systems with more than one phase has developed rapidly in recent years. The study of fluids having uniform distribution of solid spherical particles is of interest in a wide range of areas of technical importance. These areas include fluidization (flow through packed beds), flow in rocket tubes, where small carbon or metallic fuel particles are present, environmental pollution, the process by which rain drops are formed by the coalescence of small droplets, which might be considered as solid particles for the purpose of examining their movement prior to coalescence, combustion, and more recently, blood flow in capillaries.

Saffman (1962) has expressed a model equation describing the influence of dust particles on the motion viscous fluids. Later on the large number of dusty viscous flow problems have been investigated by Marble (1963); Michael and Miller (1966); Michael and Norey (1968); Verma and Mathur (1973); Gupta (1979); Srivastava (2002); Sanyal and Dasgupta (2002); Gireesha and Bagewadi (2003,2007);Gireesha, Bagewadi and Venkatesh (2007); Gireesha,Venkatash and Bagewadi (2009) and Elangovan and Ratchagar (2009) etc. through channels of various cross-sections under the influence of time dependent pressure gradient. The basic theory of multiphase fluid flow has been given by Soo (1967).

Singh, Lal and Sharma (1990); Bhatnagar and Bhardwaj (1998); Mal and Sengupta (2003); Mishra and Bhola (2005); Varshney and Singh (2006); Kumar, Jha and Shrivastava (2006,2006); Singh, Singh and Jha (2009); Agrawal, Agrawal and Varshney (2012) , Agrawal and Singh (2012) etc. have discussed the unsteady flow of dusty fluid through porous medium in the different type of channels with time dependent pressure gradient. Tripathi, Sharma and Singh (2013) studied magnetohydrodynamic unsteady flow of a dusty conducting fluid through porous medium in a rectangular channel with time dependent pressure gradient. Rathod and Parveen (2015) have discussed the time dependent pressure gradient effect on unsteady MHD couette flow and heat transfer of a couple stress fluid.

The present paper is concerned with the flow problem of a conducting viscous incompressible fluid with embedded non-conducting small identical spherical particles in a long rectangular channel under the influence of uniform magnetic field applied perpendicularly to the flow of fluid and a time varying pressure gradient, taking the fluid and dust particles to be initially at rest. The expressions for velocities of conducting fluid and non-conducting particles are obtained by using Finite Fourier Cosine and Laplace transforms. The particular cases when (i)the pressure gradient is an absolute constant, (ii)the the pressure gradient is a periodic function of time, (iii) the pressure gradient is an exponentially decreasing function of time, and (iv) the pressure gradient is $Cte^{-\lambda t}$, have also been discussed in detail

EQUATION OF THE PROBLEM

Using the rectangular cartesian coordinate system, the walls of the channel are taken to be the planes $x = \pm a$ and $y = \pm b$. The fluid and dust particles velocities u(x,y,t) and v(x,y,t) respectively, are in z-direction i.e. along the axis of rectangular channel. A uniform magnetic field is applied perpendicular to the planes $y = \pm b$ of rectangular channel. Taking the number density of non-conducting dust particles to be constant throughout the motion, the appropriate momentum equations of motion, after introducing the electromagnetic force obtained by Soo (1968), are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{K_0 N_0}{\rho}(v - u) - \frac{\sigma B_0^2}{\rho}u \qquad \text{... (1)}$$

$$\frac{\partial v}{\partial t} = \frac{K_0}{M}(u - v) - \frac{k'}{\rho}\frac{\partial p}{\partial z} \qquad \text{... (2)}$$

where u and v denote the velocities of fluid and dust particle respectively; p is the fluid pressure; M, the mass of a particle; K_0, the Stokes resistance coefficient, which for spherical particle of radius r is $6\pi\mu r$, μ being the viscosity of the fluid; N_0, the number density of particle; t, the time; ρ, ρ_p and $\bar{\rho}_p$ are density of fluid, mass density of particle and material density of the particle respectively; $\upsilon = \mu/\rho$, the kinematic viscosity of the fluid; B_0, the magnetic inductivity; σ, the electric conductivity, $k' = \rho/\rho_p$.

It is assumed that effect of the induced magnetic field and the electric field produced by motion of the electrically conducting field is negligible and no external field is applied. The dust particles are non-conducting.

Introducing the non-dimensional quantities

$$x^* = \frac{x}{a}, \; y^* = \frac{y}{a}, \; z^* = \frac{z}{a}, \; p^* = \frac{a^2}{\rho\nu^2}p, \; t^* = \frac{\nu}{a^2}t,$$

$$u^* = \frac{ua}{\nu}, \quad v^* = \frac{va}{\nu}$$

Equations (1) and (2) become (dropping stars)

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta(v - u) - H^2 u \qquad \ldots (3)$$

$$\frac{\partial v}{\partial t} = \gamma'(u - v) - k'Q\frac{\partial p}{\partial z} \qquad \ldots (4)$$

where

$$\beta = \frac{K_0'}{\gamma} = \frac{N_0 K_0 a^2}{\rho\nu}, \; K_0' = \frac{N_0 M}{\rho}, \; \gamma = \frac{M\nu}{K_0 a^2}\partial, \; \gamma' = \frac{1}{\gamma}, \; Q = \frac{\nu}{a}$$

and $H = aB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number)

Initially, the fluid and particles are rest. The flow takes place under the influence of time dependent pressure gradient with no-slip boundary conditions. From symmetric consideration, the flow in region $x \geq 0, y \geq 0$, is considered. Accordingly, the boundary conditions are:

$$\left.\begin{array}{ll} t > 0 \quad u(1, y, t) = 0 & 0 \leq y \leq h \\ \qquad\; v(1, y, t) = 0 & \\ \frac{\partial u}{\partial x} = 0, \; \frac{\partial v}{\partial x} = 0 & \text{at } x = 0 \end{array}\right\} \qquad \ldots (5)$$

And

$$\left.\begin{array}{l}\left.\begin{array}{l}u(x,h,t)=0\\ v(x,h,t)=0\end{array}\right\}\quad 0\le x\le 1\\ \dfrac{\partial u}{\partial y}=0,\quad \dfrac{\partial v}{\partial y}=0\qquad \text{at } y=0\end{array}\right\}\qquad \ldots(6)$$

where h = b/a

SOLUTION OF THE PROBLEM

For solving the problem, we choose the finite cosine transform defined as

$$\bar{u}(m,y,t)=\int_0^1 u(x,y,t)\cos q_m x\,dx \qquad \ldots(7)$$

$$\bar{\bar{u}}(x,n,t)=\int_0^h u(x,y,t)\cos q_n y\,dy \qquad \ldots(8)$$

where

$$q_m=\frac{2m+1}{2}\pi,\qquad q_n=\frac{2n+1}{2h}\pi$$

It can be shown that the inversion formula for finite cosine transforms defined by (7) and (8) are given by

$$u(x,y,t)=2\sum_{m=0}^{\infty}\bar{u}(m,y,t)\cos q_m x \qquad \ldots(9)$$

and

$$u(x,y,t)=\frac{2}{h}\sum_{n=0}^{\infty}\bar{\bar{u}}(x,n,t)\cos q_n y \qquad \ldots(10)$$

Multiplying equations (3) and (4) by $\cos q_m x.\cos q_n y$ and then integrating twice within the limits 0 to 1 and 0 to h and using the boundary conditions (5) and (6), it is found

$$\frac{\partial U}{\partial t}=\frac{(-1)^{m+n}}{q_m q_n}f(t)-(q_m^2+q_n^2)U+\beta(V-U)-H^2U \qquad \ldots(11)$$

$$\frac{\partial V}{\partial t}=\gamma'(U-V)-\frac{(-1)^{m+n}k'Q}{q_m q_n}f(t) \qquad \ldots(12)$$

where

$$U = \int_0^1 \int_0^h u(x, y, t) \cos q_m x . \cos q_n y \, dx \, dy$$

$$V = \int_0^1 \int_0^h v(x, y, t) \cos q_m x . \cos q_n y \, dx \, dy$$

$$\text{and} - \frac{\partial p}{\partial z} = f(t)$$

Again, applying Laplace transform to equations (11) and (12) under the transform initial condition

$U = 0, \quad V = 0, \text{ at } \quad t = 0$

It is found

$$s\bar{U} = \frac{(-1)^{m+n}}{q_m q_n} \bar{f}(s) - (q_m^2 + q_n^2)\bar{U} + \beta(\bar{V} - \bar{U}) - H^2\bar{U} \qquad \ldots(13)$$

$$s\bar{V} = \gamma'(\bar{U} - \bar{V}) - \frac{(-1)^{m+n} k' Q}{q_m q_n} \bar{f}(s) \qquad \ldots(14)$$

where $\bar{U}, \bar{V}$ and $\bar{f}(s)$ are the Laplace transforms of the respective quantities.

Solving equations (13) and (14), it is found

$$\bar{U} = \frac{(-1)^{m+n}}{q_m q_n} \frac{(s + \gamma' - k'Q\beta)\bar{f}(s)}{(s - \alpha_1)(s - \alpha_2)} \qquad \ldots(15)$$

$$\bar{V} = \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma'(s + \gamma' - k'Q\beta)}{(s + \gamma')(s - \alpha_1)(s - \alpha_2)} - \frac{k'Q}{(s + \gamma')} \right] \bar{f}(s) \qquad \ldots(16)$$

where

$$\alpha_1 = -\frac{1}{2}\left[(\gamma' + \beta + H^2 + q_m^2 + q_n^2) + \{(\gamma' + \beta + H^2 + q_m^2 + q_n^2)^2 - 4\gamma'(q_m^2 + q_n^2 + H^2)\}^{1/2}\right]$$

and

$$\alpha_2 = -\frac{1}{2}\left[(\gamma' + \beta + H^2 + q_m^2 + q_n^2) - \{(\gamma' + \beta + H^2 + q_m^2 + q_n^2)^2 - 4\gamma'(q_m^2 + q_n^2 + H^2)\}^{1/2}\right]$$

are the roots of the equation

$$s^2 + (\gamma' + \beta + H^2 + q_m^2 + q_n^2)s + \gamma'(H^2 + q_m^2 + q_n^2) = 0$$

Now to obtain u and v, it may invert the Laplace transform by convolution theorem and then applying the inversion formulae for the finite cosine transforms, it is found

$$u = \frac{4}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\int_0^t f(t-\eta)\{A_1 e^{\alpha_1 \eta} + A_2 e^{\alpha_2 \eta}\}d\eta\right]$$

$$\times \cos q_m x . \cos q_n y \qquad \ldots (17)$$

and

$$v = \frac{4}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\int_0^t f(t-\eta)\left\{\gamma'(B_1 e^{\alpha_1 \eta} + B_2 e^{\alpha_2 \eta}) + B_3 e^{-\gamma' \eta}\right\}d\eta\right]$$

$$\times \cos q_m x . \cos q_n y \qquad \ldots (18)$$

where

$$A_1 = \frac{(\alpha_1 + \gamma' - k'Q\beta)}{(\alpha_1 - \alpha_2)}\,, \qquad A_2 = -\frac{(\alpha_2 + \gamma' - k'Q\beta)}{(\alpha_1 - \alpha_2)},$$

$$B_1 = \frac{A_1}{(\alpha_1 + \gamma')}\,, \quad B_2 = \frac{A_2}{(\alpha_2 + \gamma')}\,, \quad B_3 = -\left\{1 + \frac{\gamma'\beta}{(\alpha_1 + \gamma')(\alpha_2 + \gamma')}\right\}k'Q$$

PARTICULAR CASES

(i) When the pressure gradient is constant

Substituting f(t)=C (where C is an absolute constant) in the above equation and on simplifying, velocities of the fluid and dust particles are

$$u = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma' - k'Q\beta}{\alpha_1\alpha_2} + \frac{A_1}{\alpha_1}e^{\alpha_1 t} + \frac{A_2}{\alpha_2}e^{\alpha_2 t}\right]\cos q_m x . \cos q_n y$$

$$\ldots (19)$$

$$v = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\left(\frac{\gamma' - k'Q\beta}{\alpha_1\alpha_2} - \frac{k'Q}{\gamma'}\right) + \frac{\gamma' B_1}{\alpha_1}e^{\alpha_1 t}\right.$$

$$\left. + \frac{\gamma' B_2}{\alpha_2}e^{\alpha_2 t} - \frac{k'QB_3}{\gamma'}e^{-\gamma' t}\right]\cos q_m x . \cos q_n y \qquad \ldots (20)$$

(ii) When the Pressure Gradient is periodic Function of Time

Substituting $f(t) = C\sin \omega t$ (where C and ω are constants) in the above equations and on simplifying, velocities of the fluid and the dust particles are

$$u = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\omega A_1}{(\alpha_1^2+\omega^2)}e^{\alpha_1 t} + \frac{\omega A_2}{(\alpha_2^2+\omega^2)}e^{\alpha_2 t}\right.$$

$$\left. + \left\{\frac{\omega^2 + (\gamma' - k'Q\beta)^2}{(\alpha_1^2+\omega^2)(\alpha_2^2+\omega^2)}\right\}^{1/2} \sin(\omega t - \psi_1)\right] \cos q_m x . \cos q_n y \qquad \ldots (21)$$

and

$$v = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma'\omega B_1}{(\alpha_1^2+\omega^2)}e^{\alpha_1 t} + \frac{\gamma'\omega B_2}{(\alpha_2^2+\omega^2)}e^{\alpha_2 t} + \frac{\omega B_3}{(\gamma'^2+\omega^2)}e^{-\gamma' t}\right.$$

$$+ \gamma' \left\{\frac{\omega^2 + (\gamma' - k'Q\beta)^2}{(\alpha_1^2+\omega^2)(\alpha_2^2+\omega^2)(\gamma'^2+\omega^2)}\right\}^{1/2} \sin(\omega t - \psi_2)$$

$$\left. - \frac{k'Q}{(\gamma'^2+\omega^2)^{1/2}}\sin(\omega t - \psi_3)\right] \cos q_m x . \cos q_n y \qquad \ldots (22)$$

respectively,

where

$$\psi_1 = \tan^{-1}\left(-\frac{\omega}{\alpha_1}\right) + \tan^{-1}\left(-\frac{\omega}{\alpha_2}\right) - \tan^{-1}\frac{\omega}{(\gamma' - k'Q\beta)} ,$$

$$\psi_2 = \tan^{-1}\left(-\frac{\omega}{\alpha_1}\right) + \tan^{-1}\left(-\frac{\omega}{\alpha_2}\right) + \tan^{-1}\left(\frac{\omega}{\gamma'}\right) - \tan^{-1}\frac{\omega}{(\gamma' - k'Q\beta)} ,$$

$$\psi_3 = \tan^{-1}\left(\frac{\omega}{\gamma'}\right)$$

(iii) When the Pressure Gradient is Exponentially Decreasing Function Time

Substituting $f(t) = Ce^{-\lambda t}$ (where C and λ are constants) in the above equations and on simplifying, velocities of fluid and dust particles are

$$u = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma' - k'Q\beta - \lambda}{(\alpha_1+\lambda)(\alpha_2+\lambda)}e^{-\lambda t} + \frac{A_1}{(\alpha_1+\lambda)}e^{\alpha_1 t}\right.$$

$$\left. + \frac{A_2}{(\alpha_2+\lambda)}e^{\alpha_2 t}\right] \cos q_m x . \cos q_n y \qquad \ldots (23)$$

and

$$v = \frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\left\{\frac{\gamma'(\gamma' - k'Q\beta - \lambda)}{(\gamma' - \lambda)(\alpha_1+\lambda)(\alpha_2+\lambda)} - \frac{k'Q}{(\gamma' - \lambda)}\right\}e^{-\lambda t}\right.$$

$$+\frac{\gamma' B_1}{(\alpha_1+\lambda)}e^{\alpha_1 t}+\frac{\gamma' B_2}{(\alpha_2+\lambda)}e^{\alpha_2 t}-\frac{k' Q B_3}{(\gamma'-\lambda)}e^{-\gamma' t}\Bigg]\cos q_m x.\cos q_n y \qquad \ldots(24)$$

respectively.

(iv)When $\mathbf{f(t) = Cte^{-\lambda t}}$

The velocities of the fluid and the dust particles are

$$u=\frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\Bigg[\frac{A_1}{(\alpha_1+\lambda)^2}e^{\alpha_1 t}+\frac{A_2}{(\alpha_2+\lambda)^2}e^{\alpha_2 t}$$

$$+\Bigg\{\frac{\gamma'-k'Q\beta-\lambda}{(\alpha_1+\lambda)(\alpha_2+\lambda)}t$$

$$+\frac{(2\lambda+\alpha_1+\alpha_2)(\gamma'-k'Q\beta)+\left(\alpha_1\alpha_2-\lambda^2\right)}{(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2}\Bigg\}\ e^{-\lambda t}\Bigg]$$

$$\times\cos q_m x.\cos q_n y \qquad \ldots(25)$$

and

$$v=\frac{4C}{h}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\Bigg[\frac{\gamma' B_1}{(\alpha_1+\lambda)^2}e^{\alpha_1 t}+\frac{\gamma' B_2}{(\alpha_2+\lambda)^2}e^{\alpha_2 t}+\frac{B_3}{(\gamma'-\lambda)^2}e^{-\gamma' t}$$

$$+\Bigg\{\frac{(\gamma'-k'Q\beta-\lambda)\gamma' t}{(\gamma'-\lambda)(\alpha_1+\lambda)(\alpha_2+\lambda)}+\frac{(\gamma'-k'Q\beta)D}{\alpha_1\alpha_2\lambda}$$

$$-\frac{k'Q\{(\gamma'-\lambda)t-1\}}{(\gamma'-\lambda)}\Bigg\}e^{-\lambda t}\Bigg]\cos q_m x.\cos q_n y \qquad \ldots(26)$$

respectively,

where

$$D=[\{(\alpha_1-\alpha_2)(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2(\gamma'-\lambda)^2(\gamma'-k'Q\beta)$$

$$+\alpha_2\gamma'\lambda^2(\gamma'-k'Q\beta+\alpha_1)(\alpha_2+\gamma')(\alpha_2+\lambda)^2(\gamma'-\lambda)^2-\alpha_1\gamma'\lambda^2$$

$$\times(\gamma'-k'Q\beta+\alpha_2)(\alpha_1+\gamma')(\alpha_1+\lambda)^2(\gamma'-\lambda)^2+\alpha_1\alpha_2\lambda^2 k'Q\beta$$

$$\times(\alpha_1-\alpha_2)(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2+\alpha_1\alpha_2\gamma'(k'Q\beta-\gamma'+\lambda)(\alpha_1-\alpha_2)$$

$$\times(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)(\alpha_2+\lambda)(\gamma'-\lambda)\}/\{(\gamma'-k'Q\beta)$$

$$\times(\alpha_1-\alpha_2)(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2(\gamma'-\lambda)^2\}].$$

DISCUSSION

Evidently the velocities of fluid and the dust particles will be slower due to the applied uniform magnetic field.

In the particular case (iv), if we put $\lambda = 0$, the velocities of the fluid and the dust particles can be obtained for a linearly time dependent pressure gradient.

If H=0, all the velocities expressions for fluid and dust particles can be obtained in the absence of magnetic field under the influence of various pressure gradient and also if $K' = 0$, the results are in agreement with those of Gupta and Gupta (1976).

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