



## VISCO-ELASTIC MHD FLOW WITH HEAT AND MASS TRANSFER PAST A POROUS PLATE IN PRESENCE OF CHEMICAL REACTION AND HEAT GENERATION

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### ABSTRACT

*An analysis of unsteady, two-dimensional free convective MHD visco-elastic flow with heat and mass transfer past a semi-infinite moving vertical porous plate with variable suction in presence of homogeneous first-order chemical reaction and temperature dependent heat generation is presented. The equations governing the flow field are solved by perturbation technique. Expressions for velocity, temperature, mass concentration and skin friction coefficient are obtained. The velocity field and the skin friction coefficient are illustrated graphically to observe the visco-elastic effects in combination with other flow parameters involved in the solution. It is observed that the flow field is significantly affected by the visco-elastic parameter.*

**Key words:** Free convection, visco-elastic, perturbation technique, Prandtl number, skin friction, porous plate, Chemical reaction, MHD.

**2010 Mathematics Subject Classification:** 76A10.

### 1 Introduction

The free convective flow with heat and mass transfer for an electrically conducting fluid past a porous plate in presence of a magnetic field has been studied in a large scale because of its application in plasma studies, nuclear reactors, geothermal energy extractions and the boundary layer control in the field of aerodynamics. The heat and mass transfer problems in combination with chemical reaction are of great importance in many processes and have attracted the attention

of a large number of scholars. A reaction is said to be of the order  $n$ , if the reaction rate is proportional to the  $n$ -power of concentration. In particular, a reaction is said to be first-order, if the rate of reaction is directly proportional to concentration itself. In well mixed system, the reaction is heterogeneous if it takes place at an interface and homogeneous, if it takes place in solution. Helmy [1] has studied the effects of magnetic field on hydrodynamic flow past plate without heat transfer. The MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in presence of a pressure gradient for large and small Prandtl numbers, using the method of matched asymptotic expansion has been studied by Gribben [2]. The effects of Hall currents on hydromagnetic free convective boundary layer flow in a porous medium past a plate, using harmonic analysis has been studied by Takhar and Ram [3]. The problem of an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction has been solved by Kim [4]. Takhar and Ram [5] also have studied the free convective MHD flow of water with heat transfer at  $4^{\circ}\text{C}$  through a porous medium. Takhar and Beg [6] have studied the non-Darcy mixed convective flow of an incompressible viscous fluid past a porous flat plate in a saturated porous medium. An approximate solution for the two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate has been obtained by Soundalgekar [7]. Raptis and Kafousias [8] have analyzed the effect of a magnetic field upon the steady free convective flow through a porous medium bounded by an infinite vertical plate at constant temperature with constant suction velocity. Raptis [9] has also studied the case of time-dependent two-dimensional natural convective flow of an incompressible, electrically-conducting viscous fluid with heat transfer in a highly porous medium bounded by an infinite vertical porous plate. Soundalgekar [10] has studied the effect of free convection on steady MHD flow past a vertical porous plate and its unsteady part has been studied by Gulab and Mishra [11]. Soundalgekar and Takhar [12] have studied the effect of the temperature of oscillating plate on combined convective flow past a semi-infinite vertical plate. Georgantopoulos *et al.* [13] have investigated the effect of free convective MHD oscillatory flow with mass transfer past an infinite vertical porous plate. Singh and his co-workers [14] have analyzed the unsteady MHD free convective flow through a porous medium confined between two infinite vertical parallel oscillating porous plates with different amplitudes. Vajravelu and Hadyinicolaou [15] have studied the convective flow of an electrically conducting fluid with heat transfer at stretching surface with uniform stream velocity. Chamkha [16] has studied the three-dimensional free convective MHD flow on a vertical stretching surface

with heat generation/absorption. Chambre and Young [17] have investigated a first order chemical reaction in the neighbourhood of a horizontal plate. Apelblat [18] has obtained an analytical solution for mass transfer with a chemical reaction of first order. Das *et al.* [19] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Das *et al.* [20] have also obtained the mass transfer effects on moving isothermal vertical plate in the presence of a chemical reaction. Bala and Barma [21] have studied the unsteady MHD flow with heat and mass transfer past a semi-infinite moving porous plate with variable suction in presence of homogeneous chemical reaction. Choudhury and Das [22] have studied the MHD boundary layer flow of a non-Newtonian fluid past a flat plate.

The object of the present author is to study the unsteady MHD flow of Walters liquid (Model B') with heat and mass transfer past a semi infinite moving vertical porous plate with variable suction in presence of chemical reaction and heat generation and also to observe the visco-elastic effects on the momentum and thermal fields along with other flow parameters.

The constitutive equation for Walters liquid (Model B') is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik}, \quad \sigma'_{ik} = 2\eta_0 e^{ik} - 2K_0 e^{ik} \quad (1)$$

where  $\sigma_{ik}$  is the stress tensor,  $p$  is isotropic pressure,  $g_{ik}$  is the metric tensor of a fixed coordinate system  $x^i$ ,  $v^i$  is the velocity vector, the contravariant form of  $e^{ik}$  is given by

$$e^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^i_{,m} e^{im} - v^i_{,m} e^{mk} \quad (2)$$

It is the convected derivative of the deformation rate tensor  $e^{ik}$  defined by

$$2e_{ik} = v_{i,k} + v_{k,i} \quad (3)$$

Here  $\eta_0$  is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \quad \text{and} \quad K_0 = \int_0^\infty \tau N(\tau) d\tau \quad (4)$$

$N(\tau)$  being the relaxation spectrum as introduced by Walters [23, 24]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau) d\tau, \quad n \geq 2 \quad (5)$$

have been neglected.

## 2 Formulaion of the Problem

A two-dimensional unsteady laminar flow of a visco-elastic fluid characterized by Walters liquid (Model B') with heat and mass transfer, past a semi-infinite vertical porous moving plate in the presence of chemical reaction is considered. A uniform transverse magnetic field is applied perpendicular to the plate (figure 1). The  $\bar{x}$ -axis is taken vertically upwards along the plate and the  $\bar{y}$ -axis is taken perpendicular to it. All the fluid properties except the density in the buoyancy force term are assumed to be constant.

We make the following assumptions:

- (i) The viscous dissipation is neglected.
- (ii) The induced magnetic field is negligible as the applied magnetic field and the magnetic Reynolds number are very small.
- (iii) Viscous and Darcy's resistance terms are considered with constant permeability of the porous medium.
- (iii) The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equations.
- (iv) The porous plate moves with constant velocity in the direction of fluid flow, and the free stream velocity is followed by an exponentially increasing small perturbation law.
- (v) The plate temperature, concentration and free stream velocity are assumed to be exponentially varying with time.

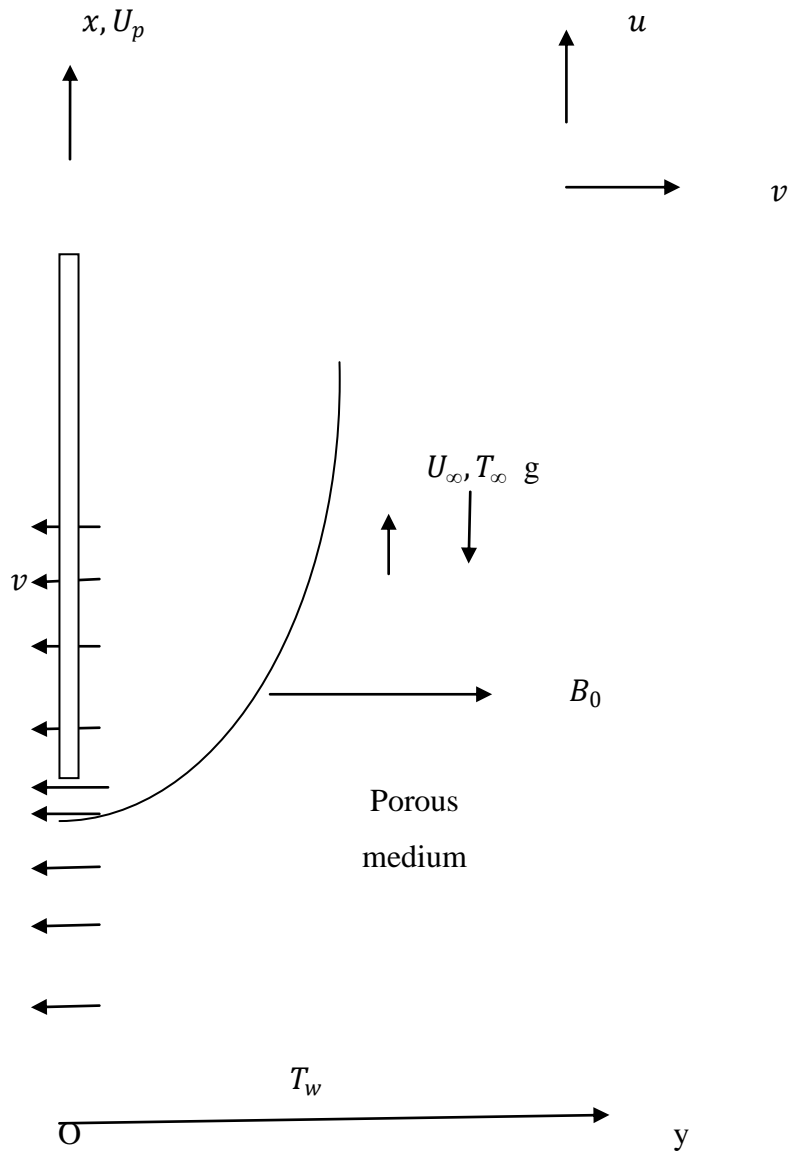


Fig-1: Physical configuration of the problem.

With these assumptions, the equations governing the mass, momentum, energy and concentration are given as follows:

Equation of continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (6)$$

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{K_0}{\rho} \left( \frac{\partial^3 \bar{u}}{\partial \bar{t} \partial \bar{y}^2} + \bar{v} \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} \right) + g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty) - \nu \frac{\bar{u}}{\bar{K}} - \frac{\sigma}{\rho} \bar{B}_0^2 \bar{u} \quad (7)$$

The fourth, sixth and seventh terms on the right hand side of equation (7) represent the buoyancy effects, the bulk matrix linear resistance, i.e. Darcy term, and the MHD term respectively.

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + Q(\bar{T} - \bar{T}_\infty) \quad (8)$$

Concentration equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - \bar{K}_1(\bar{C} - \bar{C}_\infty) \quad (9)$$

The boundary conditions for the velocity, temperature and concentration fields are,

$$\bar{u} = \bar{U}_p, \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty)e^{\bar{n}\bar{t}}, \bar{C} = \bar{C}_w + \varepsilon(\bar{C}_w - \bar{C}_\infty)e^{\bar{n}\bar{t}} \text{ at } \bar{y} = 0 \quad (10)$$

$$\bar{u} \rightarrow \bar{U}_\infty = U_0(1 + \varepsilon e^{\bar{n}\bar{t}}), \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \text{ as } \bar{y} \rightarrow \infty \quad (11)$$

where  $\nu = \frac{\eta_0}{\rho}$  is the kinematic viscosity,  $\alpha$  is the thermal diffusivity,  $\bar{K}$  is the permeability of the porous medium,  $\beta$  is the volumetric co-efficient of expansion for heat transfer,  $\bar{\beta}$  is the volumetric co-efficient of expansion for the fluid,  $\rho$  is the density,  $\sigma$  is the electrical conductivity of the fluid,  $g$  is the acceleration due to gravity,  $\bar{T}$  is the temperature,  $\bar{T}_\infty$  is the temperature of the fluid outside the boundary layer.

From the equation (6), it is observed that the suction velocity normal to the plate is a function of time only and we take it as

$$\bar{v} = -V_0(1 + \varepsilon A e^{\bar{n}\bar{t}}) \quad (12)$$

where  $A$  is the suction velocity parameter and is a real positive constant,  $\varepsilon$  and  $A\varepsilon$  are small (less than unity) and  $\bar{V}_0$  is a scale of suction velocity and has non-zero positive constant value.

Outside the boundary layer, equation (7) gives

$$-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{d\bar{U}_\infty}{d\bar{t}} + \frac{\nu}{\bar{K}} \bar{U}_\infty + \frac{\sigma}{\rho} \bar{B}_0^2 \bar{U}_\infty \quad (13)$$

We introduce the non-dimensional quantities,

$$u = \frac{\bar{u}}{U_0}, v = \frac{\bar{v}}{V_0}, y = \frac{V_0 \bar{y}}{\nu}, U_\infty = \frac{\bar{U}_\infty}{U_0}, U_p = \frac{\bar{U}_p}{U_0}, t = \frac{\bar{t} V_0^2}{\nu}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, C = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, n = \frac{\bar{n} \nu}{V_0^2},$$

$$K = \frac{KV_0^2}{v^2}, \delta = \frac{Qv}{V_0^2}, P_r = \frac{v\rho C_p}{\kappa} = \frac{v}{\alpha}, G_r = \frac{vg\beta(T_w - T_\infty)}{U_0V_0^2}, G_m = \frac{vg\bar{\beta}(\bar{c}_w - \bar{c}_\infty)}{U_0V_0^2}, M = \frac{\sigma\bar{B}_0^2v}{\rho V_0^2},$$

$$S_c = \frac{v}{D}, K_2 = \frac{vK_1}{V_0^2} \quad (14)$$

where  $P_r$  is the Prandtl number,  $G_r$  is the Grashof number for heat transfer,  $G_m$  is the Grashof number for mass transfer,  $K$  is the permeability of the porous medium,  $K_2$  is the chemical reaction parameter,  $\delta$  is the heat source parameter,  $S_c$  is the Schmidt number,  $M$  is the magnetic field parameter.

The non-dimensional form of the governing equations (6) to (9) are as follows:

$$\frac{\partial v}{\partial y} = 0 \quad (15)$$

$$\frac{\partial u}{\partial t} - (1 + \varepsilon Ae^{nt}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} - K_1 \left\{ \frac{\partial^3 u}{\partial t \partial y^2} - (1 + \varepsilon Ae^{nt}) \frac{\partial^3 u}{\partial y^3} \right\} + G_r \theta + G_m C + N(U_\infty - u) \quad (16)$$

where,  $N = M + \frac{1}{K}, K_1 = \frac{K_0 V_0^2}{v^2 \rho}.$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon Ae^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} + \delta \theta \quad (17)$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon Ae^{nt}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - K_2 C \quad (18)$$

with modified boundary conditions :

$$u = U_p, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt} \quad \text{at } y = 0$$

$$u \rightarrow U_\infty = 1 + \varepsilon e^{nt}, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (19)$$

### 3 Method of solution

In order to solve the governing equations, the physical variables viz. velocity, temperature and concentration are expressed in the powers of  $\varepsilon$  and are presented as follows:

$$u(y, t) = u_0(y) + \varepsilon e^{nt} u_1(y) + o(\varepsilon^2)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + o(\varepsilon^2)$$

$$C(y, t) = C_0(y) + \varepsilon e^{nt} C_1(y) + o(\varepsilon^2) \quad (20)$$

Substituting (20) in the equations (16) to (18) and equating the harmonic and non-harmonic terms and neglecting the coefficients of powers of  $o(\varepsilon^2)$  we get,

$$K_1 u_0''' + u_0'' + u_0' - Nu_0 = -N - G_r \theta_0 - G_m C_0 \quad (21)$$

$$K_1 u_1''' + (1 - nK_1) u_1'' + u_1' - (N + n) u_1 = -(N + n) - G_r \theta_1 - G_m C_1 - Au_0' - K_1 Au_0''$$

(22)

$$\theta_0'' + P_r \theta_0' + P_r \delta \theta_0 = 0 \quad (23)$$

$$\theta_1'' + P_r \theta_1' + P_r L \theta_1 = -A P_r \theta_0' \quad (24)$$

$$C_0'' + S_c C_0' - K_2 S_c C_0 = 0 \quad (25)$$

$$C_1'' + S_c C_1' - S_c L_1 C_1 = -A S_c C_0' \quad (26)$$

where  $L = \delta - n$ ,  $L_1 = K_2 - n$ .

The corresponding boundary conditions are:

$$\begin{aligned} u_0 = U_p, u_1 = 0, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \quad \text{at } y = 0 \\ u_0 \rightarrow 1, u_1 \rightarrow 1, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, C_0 \rightarrow 0, C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (27)$$

To solve the equations (22) to (26), we use the multi-parameter perturbation scheme and the velocity components are expanded in the power of visco-elastic parameter  $K_1$  as  $K_1 \ll 1$  for small shear rate. Thus the expressions for velocity components are considered as follows:

$$\begin{aligned} u_0 &= u_{00} + K_1 u_{01} + o(K_1^2) \\ u_1 &= u_{10} + K_1 u_{11} + o(K_1^2) \end{aligned} \quad (28)$$

Using (28) in (21) and (22) and equating coefficients of  $K_1$ , neglecting terms involving  $o(K_1^2)$ , we get,

$$u_{00}'' + u_{00}' - N u_{00} = -N - G_r \theta_0 - G_m C_0 \quad (29)$$

$$u_{01}'' + u_{01}' - N u_{01} = -u_{00}''' \quad (30)$$

$$u_{10}'' + u_{10}' - (N + n) u_{10} = -(N + n) - G_r \theta_1 - G_m C_1 - A u_{00}' \quad (31)$$

$$u_{11}'' + u_{11}' - (N + n) u_{11} = -A u_{01}' - u_{10}''' + n u_{10}'' - A u_{01}''' \quad (32)$$

with relevant boundary conditions

$$\begin{aligned} u_{00} = U_p, u_{01} = 0, \quad \text{at } y = 0 \\ u_{00} \rightarrow 1, u_{01} \rightarrow 0, \quad \text{as } y \rightarrow \infty \\ u_{10} = 0, u_{11} = 0, \quad \text{at } y = 0 \\ u_{10} \rightarrow 1, u_{11} \rightarrow 0, \quad \text{as } y \rightarrow \infty \end{aligned} \quad (33)$$

The solutions of the equations (23) to (26) and (29) to (32) subject to boundary conditions (27) and (33) are attained as follows:



$$\begin{aligned}
\theta_0 &= e^{-G_2 y} \\
\theta_1 &= D_6 e^{-G_4 y} + d_2 e^{-G_2 y} \\
C_0 &= e^{-m_2 y} \\
C_1 &= D_8 e^{-m_4 y} + d_4 e^{-m_2 y} \\
u_{00} &= D_{10} e^{-F_2 y} + d_6 e^{-G_2 y} + d_8 e^{-m_2 y} + 1 \\
u_{01} &= D_{12} e^{-F_2 y} + d_{10} e^{-G_2 y} + d_{12} e^{-m_2 y} + d_{14} e^{-F_2 y} \\
u_{10} &= D_{14} e^{-F_4 y} + d_{16} e^{-G_2 y} + d_{18} e^{-G_4 y} + d_{20} e^{-F_2 y} + d_{22} e^{-m_2 y} + d_{24} e^{-m_4 y} + 1 \\
u_{11} &= D_{16} e^{-F_4 y} + d_{26} e^{-F_2 y} + d_{28} e^{-F_4 y} + d_{30} e^{-G_2 y} + d_{32} e^{-G_4 y} + d_{34} e^{-m_2 y} + d_{36} e^{-m_4 y}
\end{aligned} \tag{34}$$

#### 4 Results and Discussion

The expressions for velocity, temperature and concentration for the flow field are given by

$$u = u_{00} + K_1 u_{01} + \varepsilon e^{nt} (u_{10} + K_1 u_{11}) \tag{35}$$

$$\theta = \theta_0 + \varepsilon e^{nt} \theta_1 \tag{36}$$

$$C = C_0 + \varepsilon e^{nt} C_1 \tag{37}$$

The non-dimensional skin friction at the plate  $y=0$  is given by

$$\begin{aligned}
\sigma_w &= (u_{00}' + K_1 u_{01}' + K_1(1 + \varepsilon A e^{nt})(u_{00}'' + K_1 u_{01}'') + \varepsilon e^{nt} [u_{10}' + K_1 u_{11}' - K_1 \{n(u_{10}' + \\
&\quad K_1 u_{11}' - 1 + \varepsilon A e^{nt} u_{10}'' + K_1 u_{11}''\}]_{y=0}
\end{aligned} \tag{38}$$

The non-dimensional rate of heat transfer in the form of Nusselt number  $N_u$  is given by,

$$N_u = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = (\theta_0' + \varepsilon e^{nt} \theta_1')_{y=0} \tag{39}$$

The non-dimensional rate of mass transfer in the form of Sherwood number  $S_h$  is given by,

$$S_h = \left( \frac{\partial C}{\partial y} \right)_{y=0} = (C_0' + \varepsilon e^{nt} C_1')_{y=0} \tag{40}$$

where dash denotes differentiation with respect to  $y$ .

The constants are obtained but not given here due to brevity.

The purpose of this study is to bring out the effects of visco-elastic parameter on unsteady MHD flow with heat and mass transfer past a semi infinite moving vertical porous plate with variable suction in presence of chemical reaction and heat generation as the effects of other flow parameters have been discussed by Bala and Barma [21]. The visco-elastic effect is exhibited through the non zero values of the non-dimensional parameter  $K_1$ . The corresponding results for Newtonian fluid are obtained by setting  $K_1=0$  and it is worth mentioning that these results show conformity with that of Bala and Barma [21]. The expressions for the velocity, the temperature and the concentration fields may be obtained from (36), (37) and (38) respectively. The profiles of  $u$  against  $y$  are depicted in the figures 2 to 5 and the skin friction coefficient at the plate  $y=0$  against the flow parameter  $K_2$  (chemical reaction parameter),  $S_c$  (Schmidt number) and  $A$  (suction velocity parameter) are shown in the figures 6 to 8 to observe the effects of visco-elasticity with other flow parameters involved in the solution. The numerical calculations are to be carried out for  $U_p=.5$ ,  $t=1$ ,  $M=1$ ,  $P_r=3$ ,  $K=.5$ ,  $\delta=.1$ ,  $\epsilon=.01$ ,  $n=1.5$  in all the cases.

Figures 2 to 5 reveal that the growth of visco-elasticity diminish the velocity in comparison with the Newtonian fluid ( $K_1=0$ ) at every point of the fluid flow region. Also, the velocity profile  $u$  diminishes with the rising effect of the chemical reaction parameter  $K_2$ . Again, the velocity profile  $u$  has an accelerating trend with the growth of the Grashof number for heat transfer  $G_r$  and Grashof number for mass transfer  $G_m$  for  $G_r = G_m$  (figure 4).

It is observed from figure 5 that the velocity profile  $u$  decelerates with the decrease of Grashof number for heat transfer  $G_r$  and Grashof number for mass transfer  $G_m$  for  $G_r \neq G_m$  in comparison with the case of  $G_r = G_m$  with a higher variation for  $G_r > G_m$  than for  $G_r < G_m$ .

Figures 6 and 7 illustrate that the growth of the chemical reaction parameter  $K_2$  and Schmidt number  $S_c$  diminish the viscous drag or the skin friction coefficient in both Newtonian and non Newtonian cases. Also, the skin friction experiences a decelerating trend during the rising effect of visco elastic parameter in comparison with Newtonian fluid flow phenomenon.

Again, the increases of the suction parameter  $A$ , with that of visco elastic parameter diminish the viscous drag in comparison with the Newtonian case (figure 8).

The Nusselt number which characterizes the rate of heat transfer and the Sherwood number which characterizes the rate of mass transfer of the fluid flow are not significantly affected by the variation of visco-elastic parameter.

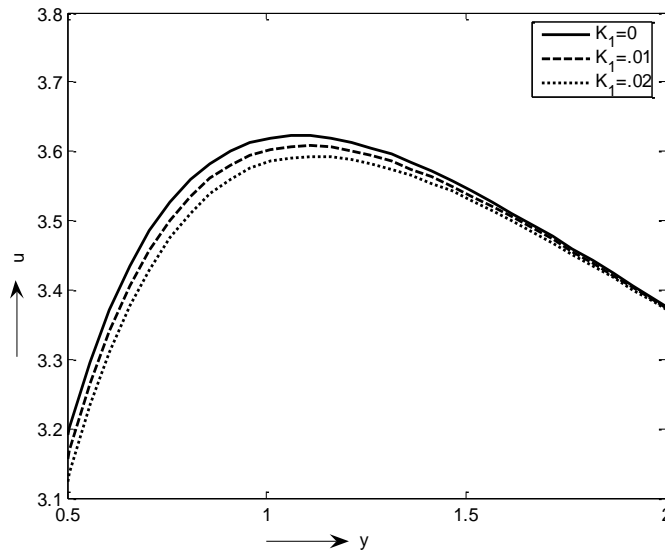


Fig-2: Velocity profile  $u$  against  $y$  for  $G_r=10$ ,  $G_m=10$ ,  $A=.5$ ,  $K_2=.1$ .

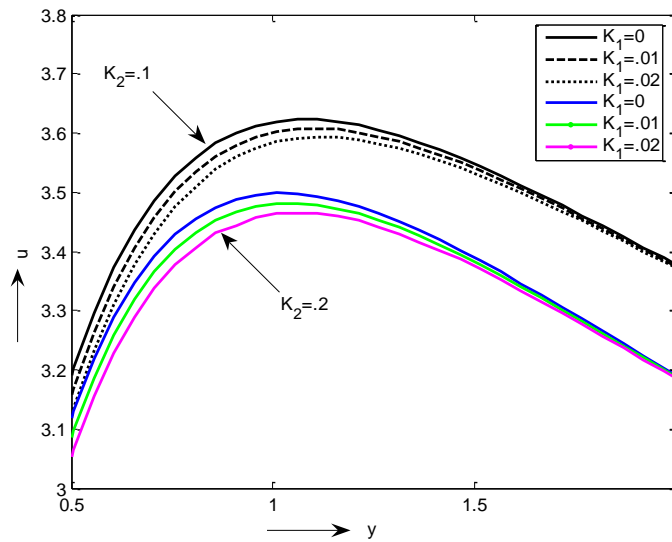


Figure 3: Velocity profile  $u$  against  $y$  for  $G_r=10$ ,  $G_m=10$ ,  $A=.5$ ,  $K_2=.1, .2$ .

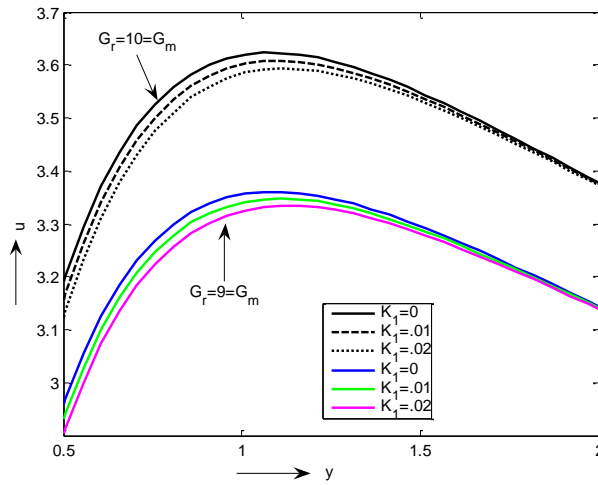


Figure 4: Velocity profile  $u$  against  $y$  for  $G_r=10, 9, G_m=10, 9, A=.5, K_2=.1$

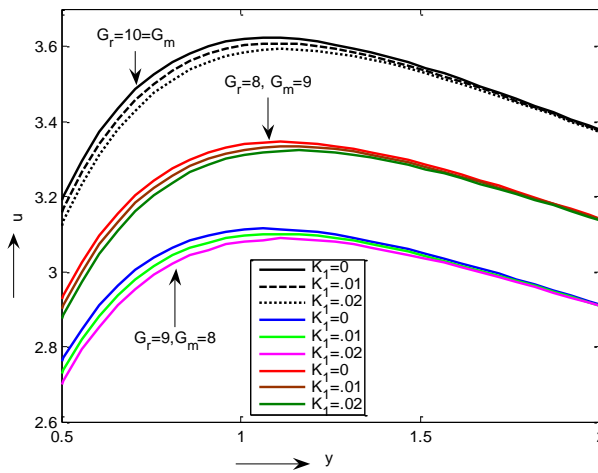


Figure 5: Velocity profile  $u$  against  $y$  for  $G_r=8, 9, 10, G_m=8, 9, 10, A=.5, K_2=.1$ .

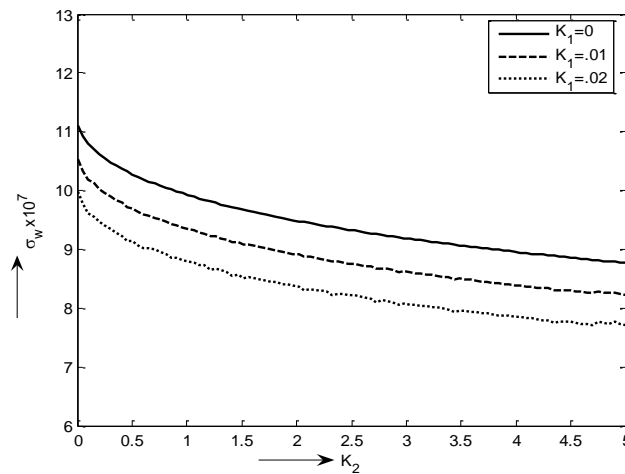


Figure 6: Skin friction coefficient  $\sigma_w$  at the plate  $y=0$  against  $K_2$  for  $G_r=10$ ,  $G_m=10$ ,  $A=.5$ ,  $S_c=.1$ .

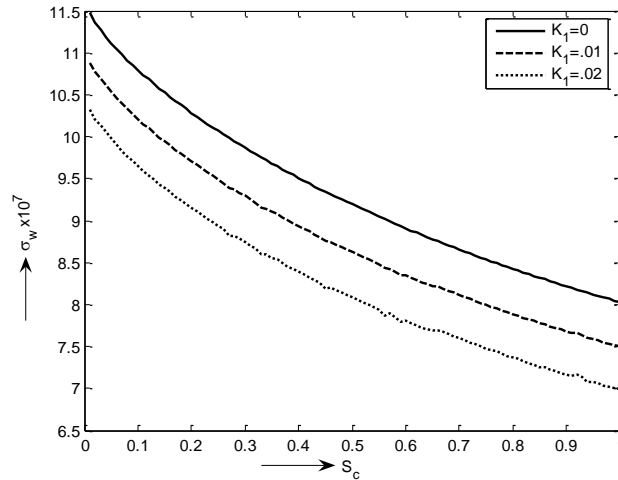


Figure 7: Skin friction coefficient  $\sigma_w$  at the plate  $y=0$  against  $S_c$  for  $G_r=10$ ,  $G_m=10$ ,  $A=.5$ ,  $K_2=.1$ .

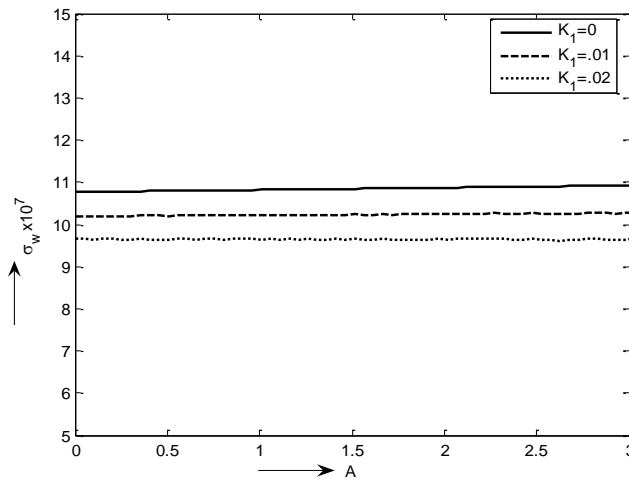


Figure 8: Skin friction coefficient  $\sigma_w$  at the plate  $y=0$  against  $A$  for  $G_r=10$ ,  $G_m=10$ ,  $S_c=.1$ ,  $K_2=.1$ .

## Conclusion

The effects of visco-elastic parameter on the unsteady MHD flow with heat and mass transfer past a semi infinite moving vertical porous plate with variable suction in presence of chemical reaction and heat generation have been studied in this problem. Some significant points of the present study are listed as below:

- The velocity profile accelerates and then diminishes in both Newtonian and non-Newtonian cases.
- The growth of visco-elastic parameter declines the velocity at every point of the fluid flow region.
- The magnitude of the velocity profile enhances with the simultaneous rise of the Grashof numbers for heat and mass transfer.
- The magnitudes of the velocity profile diminish with the decrease of Grashof numbers for heat and mass transfer for unequal values in comparison with those for equal values.
- The skin friction coefficient at the plate against chemical reaction parameter, Schmidt number and suction parameter decreases along with the increasing values of visco-elastic parameter in compared to Newtonian fluid.
- The Nusselt number and the Sherwood number are not significantly affected by the visco-elastic parameter.

## 5 References

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