International Research Journal of Mathematics, Engineering and IT
Vol. 3, Issue 4, April 2016 IF- 3.563 ISSN: (2349-0322)

Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

INTUITIONISTIC FUZZY BG-IDEAL ON BG-ALGEBRAS

C. Alphina[1], S. Mohamed Yusuff Ansari[2]

[1] M.Phil. Research Scholar, Department of Mathematics,
Jamal Mohamed College, Tiruchirappalli – 20.
[2]Assistant Professor, Department of Mathematics,
Jamal Mohamed College, Tiruchirappalli – 20

ABSTACT

In this paper, we introduce the concept of intuitionistic fuzzy BG-ideals in BG-algebra and investigate some of their properties. Homomorphism and epimorphism functions are satisfied while applying the intuitionistic fuzzy BG-ideal concept. Intuitionistic Fuzzy BG-ideal is also applied in Cartesian product.

KEYWORDS

BG-algebra, Sub BG - algebra and BG-ideals, Fuzzy BG –ideals, Intuitionistic fuzzy B-ideals.

1. Introduction

AFTER the introduction of the concept of fuzzy sets byZadeh several researches were conducted on thegeneralizations of the notion of fuzzy sets.The idea of " Intuitionistic fuzzy set " was first publishedby Atanassov, as a generalization of the notion of the fuzzy set. In this paper, using the Atanssov's idea, we establish theideals in BG-algebras, and investigate some of their properties. Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK – algebras and BCI – algebras. It is known that the class of BCK – algebras is a proper subclass of the class of BCI –algebras. J. Neggers and H.S.Kim introduced a new notion, called B – algebra. C.B.Kim and H.S.Kim introduced the notion of the BG – algebra which is a generalization of B – algebra. In this paper, we classify the Intuitionistic fuzzy BG – ideals in BG – Algebra.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1

A nonempty set X with a constant 0 and a binary operation '*' is called a BG-Algebra if it satisfies the following axioms.

1. $x * x = 0$
2. $x * 0 = x$
3. $(x * y) * (0 * y) = x, \forall\, x, y \in X.$

Example

Let X= {0, 1, 2, 3} be the set with the following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (X, *, 0) is a BG- Algebra.

Definition 2.2

Let S be a non-empty subset of a BG-algebra X, then S is called a sub algebra of X if $x * y \in S, for\ all\ x, y \in S$.

Definition 2.3

A non-empty subset I of a BG- algebra X is called an ideal of X if

(i) $0 \in I$

(ii) $x * y \in I \; and \; y \in I$ imply that $x \in X \; fo \; allx, y \in X$.

Definition 2.4

Let X be a BG-algebra and I be a subset of X ,then I is called a BG-ideal of X if it satisfies following conditions:

1. $0 \in I$
2. $x * y \;\; \in I \; and \; y \;\; \in I \Rightarrow \;\; x \in \;\; I$
3. $x \in I \; and \; y \in X \Rightarrow x * y \in I \;, I \times X \subseteq I.$

Definition 2.5

A mapping $f: X \rightarrow Y$ of a BG-algebra is called a homomorphism if $f(x * y) = f(x) * f(y) \forall \; x, y \in X$

Remark:

If $f: X \rightarrow Y$ is a homomorphism of BG-algebra, then $f(0) = 0$.

Definition 2.6

Let X be a non-empty set. A fuzzy set A of X is given by

$$A = \{(x, \alpha_A(x)) | x \in X\}$$

Where α_A: $X \rightarrow [0,1]$ is the membership function, and complement of A denoted by $\tilde{A}$,is the fuzzy set in X with $\widetilde{\alpha_A}(x) = 1 - \alpha_A(x) \; \forall \; x \in X$.

Definition 2.7

An intuitionistic fuzzy set A in a non-empty set X is defined by

$$A = \{(x, \alpha_A(x), \beta_A(x)) | x \in X\},$$

Where the functions $\alpha_A: X \rightarrow [0,1], and \; \beta_A: X \rightarrow [0,1]$ denoted the degree of membership and the degree of non-membership, respectively, and

$$0 \leq \alpha_A(x) + \beta_A(x) \leq 1 \quad \forall\, x \in X.$$

Remark:

We use the symbol $A = (\alpha_A, \beta_A)$ for the intuitionistic fuzzy set.

Definition 2.8

Let A be an intuitionistic fuzzy set in BG- algebra. Then A is called an intuitionistic fuzzy sub-algebra of X if

$$\alpha_A(x * y) \geq min\{\alpha_A(x), \alpha_A(y)\} \;;$$

$$\beta_A(x * y) \leq max\{\beta_A(x), \beta_A(y)\} \quad \forall\, x, y \in X.$$

Remark:

For every intuitionistic fuzzy sub-algebra A in X, we have the following properties:

(i) $\alpha_A(0) \geq \alpha_A(x)$

(ii) $\beta_A(0) \leq \beta_A(x) \;\; for\ all\ x \in X.$

Definition 2.9

Let A be an intuitionistic fuzzy set in X. For $t \in [0,1]$, the set

$$A_t = \{x \in X | \alpha_A(x) \geq t; 1 - \beta_A(x) \geq t\}$$

is called a level subset of A.

Definition 2.10

An intuitionistic fuzzy set $A = (\alpha_A, \beta_A)$ in X is called intuitionistic fuzzy BG- ideal, if it satisfies the following inequalities.

1. $\alpha_A(0) \geq \alpha_A(x); \beta_A(0) \leq \beta_A(x)$
2. $\alpha_A(x) \geq min\{\alpha_A(x * y), \alpha_A(y)\}\,;\; \beta_A(x) \leq max\{\beta_A(x * y), \beta_A(y)\}$
3. $\alpha_A(x * y) \geq min\{\alpha_A(x), \alpha_A(y)\}\,;\; \beta_A(x * y) \leq max\{\beta_A(x), \beta_A(y)\} \forall\, x, y \in X.$

Definition 2.11

Let A and B be the intuitionistic fuzzy sets in X. The Cartesian product $A \times B: X \times X \to [0,1]$ is defined by

$$\alpha_{A\times B}(x,y) = min\{\alpha_A(x), \alpha_B(y)\}$$

$$\beta_{A\times B}(x,y) = max\{\beta_A(x), \beta_B(y)\}$$

Theorem 3.1

If Aand B are intuitionistic fuzzy BG-ideals of a BG-algebra X×X then A×B is an intuitionistic fuzzy BG-ideal of X×X.

Proof:

For any $(x,y) \in X \times X$ we have,

$$\alpha_A(0,0) = min\{\alpha_A(0), \alpha_B(0)\}$$

$$\geq min\ \{\alpha_A(x), \alpha_B(y)\}$$

$$= \alpha_{A\times B}(x,y)$$

$$\alpha_{A\times B}(0,0) \geq \alpha_{A\times B}(x,y)$$

$$\beta_{A\times B}(0,0) = max\ \{\beta_A(0), \beta_B(0)\}$$

$$\leq max\{\beta_A(x), \beta_B(y)\}$$

$$= \beta_{A\times B}(x,y)$$

$$\beta_{A\times B}(0,0) \leq \beta_{A\times B}(x,y)$$

Let $(x_1, x_2)\ and\ (y_1, y_2) \in X \times X$ then

$$\alpha_{A\times B}(x_1, x_2) = min\{\ \alpha_A(x_1), \alpha_B(x_2)\}$$

$$\geq min\{min\{\alpha_A(x_1 * y_1), \alpha_A(y_1)\}, min\{\alpha_B(x_2 * y_2), \alpha_B(y_2)\}\}$$

$$= min\{min\{\alpha_A(x_1 * y_1), \alpha_B(x_2 * y_2)\}, min\{\alpha_A(y_1), \alpha_B(y_2)\}\}$$

$$= min\{\alpha_{A\times B}(x_1 * y_1, x_2 * y_2), \alpha_{A\times B}(y_1, y_2)\}$$

$$= min\{\alpha_{A\times B}[(x_1,x_2)*(y_1,y_2)], \alpha_{A\times B}(y_1,y_2)\}$$

$$\therefore \alpha_{A\times B}(x_1,x_2) \geq min\{\alpha_{A\times B}[(x_1,x_2)*(y_1,y_2)], \alpha_{A\times B}(y_1,y_2)\}$$

And

$$\beta_{A\times B}(x_1,x_2) = max\,\{\,\beta_A(x_1), \beta_B(x_2)\}$$

$$\leq max\big\{max\{\beta_A(x_1*y_1), \beta_A(y_1)\}, max\{\beta_B(x_2*y_2), \beta_B(y_2)\}\big\}$$

$$= max\big\{max\{\beta_A(x_1*y_1), \beta_B(x_2*y_2)\}, max\{\beta_A(y_1), \beta_B(y_2)\}\big\}$$

$$= max\{\beta_{A\times B}(x_1*y_1, x_2*y_2), \beta_{A\times B}(y_1,y_2)\}$$

$$= max\{\beta_{A\times B}[(x_1,x_2)*(y_1,y_2)], \beta_{A\times B}(y_1,y_2)\}$$

$$\therefore \beta_{A\times B}(x_1,x_2) \leq max\{\beta_{A\times B}[(x_1,x_2)*(y_1,y_2)], \beta_{A\times B}(y_1,y_2)\}$$

Now

$$\alpha_{A\times B}[(x_1,x_2)*(y_1,y_2)] = \alpha_{A\times B}(x_1*y_1, x_2*y_2)$$

$$= min\{\alpha_A(x_1*y_1), \alpha_B(x_2*y_2)\}$$

$$\geq min\big\{min\{\alpha_A(x_1), \alpha_A(y_1)\}, min\{\alpha_B(x_2), \alpha_B(y_2)\}\big\}$$

$$= min\big\{min\{\alpha_A(x_1), \alpha_B(x_2)\}, min\{\alpha_A(y_1), \alpha_B(y_2)\}\big\}$$

$$= min\{\alpha_{A\times B}(x_1,x_2), \alpha_{A\times B}(y_1,y_2)\}$$

$$\alpha_{A\times B}[(x_1,x_2)*(y_1,y_2)] \geq min\{\alpha_{A\times B}(x_1,x_2), \alpha_{A\times B}(y_1,y_2)\}$$

And

$$\beta_{A\times B}[(x_1,x_2)*(y_1,y_2)] = \beta_{A\times B}(x_1*y_1, x_2*y_2)$$

$$= max\{\beta_A(x_1*y_1), \beta_B(x_2*y_2)\}$$

$$\leq max\big\{max\{\beta_A(x_1), \beta_A(y_1)\}, max\{\beta_B(x_2), \beta_B(y_2)\}\big\}$$

$$= max\big\{max\{\beta_A(x_1), \beta_B(x_2)\}, max\{\beta_A(y_1), \beta_B(y_2)\}\big\}$$

$$= max\{\beta_{A\times B}(x_1,x_2),\beta_{A\times B}(y_1,y_2)\}$$

$$\beta_{A\times B}[(x_1,x_2)*(y_1,y_2)] \le max\{\beta_{A\times B}(x_1,x_2),\beta_{A\times B}(y_1,y_2)\}$$

Therefore A×B is an intuitionistic fuzzy BG-ideal of X×X.

Theorem 3.2

Let A and B be intuitionistic fuzzy sets in a BG- algebra such that A×B is an intuitionistic fuzzy BG- ideal of X×X . Then

(i) Either $\alpha_A(0) \ge \alpha_A(x)\ or\ \alpha_B(0) \ge \alpha_B(x)\ \forall\ x \in X.$

(ii) Either $\beta_A(0) \le \beta_A(x)\ or\ \beta_B(0) \le \beta_B(x)\ \forall\ x\ \in X.$

(iii) If $\alpha_A(0) \ge \alpha_A(x)\ for\ all\ x\ \in X, then\ \alpha_B(0) \ge \alpha_A(x)\ or\ \alpha_B(0) \ge \alpha_B(x)$

(iv) If $\beta_A(0) \le \beta_A(x)\ for\ all\ x\ \in X, then\ \beta_B(0) \le \beta_A(x)\ or\ \beta_B(0) \le \beta_B(x)$

(v) If $\alpha_B(0) \ge \alpha_B(x)\ for\ all\ x\ \in X, then\ \alpha_A(0) \ge \alpha_A(x)\ or\ \alpha_A(0) \ge \alpha_B(x)$

(vi) If $\beta_B(0) \le \beta_B(x)\ for\ all\ x\ \in X, then\ \beta_A(0) \le \beta_A(x)\ or\ \beta_A(0) \le \beta_B(x)$

Proof:

(i) Assume $\alpha_A(x) > \alpha_A(0)\ and\ \alpha_B(y) > \alpha_B(0)\ for\ some\ x, y\ \in X$

Then $\alpha_{A\times B}(x,y) = min\{\alpha_A(x),\alpha_B(y)\}$

$$> min\{\alpha_A(0),\alpha_B(0)\}$$

$$= \alpha_{A\times B}(0,0)$$

$$\Rightarrow \alpha_{A\times B}(x,y) > \alpha_{A\times B}(0,0)\ \forall x,y\ \in X$$

Which is a contradiction to A× B is an intuitionistic fuzzy BG- ideal of X×X.

Therefore either $\alpha_A(0) \ge \alpha_A(x)\ or\ \alpha_B(0) \ge \alpha_B(x)\ \forall\ x \in X.$

(ii) Assume $\beta_A(0) > \beta_A(x)\ and\ \beta_B(0) > \beta_B(y)\ for\ some\ x, y\ \in X.$

Then $\beta_{A\times B}(x,y) = max\{\beta_A(x)\beta_B(y)\}$

$$< max\{\beta_A(0),\beta_B(0)\}$$

$$= \beta_{A\times B}(0,0)$$

$$\Rightarrow\ \beta_{A\times B}(x,y) < \beta_{A\times B}(0,0)\ \forall\ x,y\ \in X.$$

Which is a contradiction to A×B is an intuitionistic fuzzy BG- ideal of X×X.

Therefore either $\beta_A(0) \le \beta_A(x)\ or\ \beta_B(0) \le \beta_B(x)\ \forall\ x \in X.$

(iii) Assume $\alpha_B(0) < \alpha_A(x)\ and\ \alpha_B(0) < \alpha_B(y)\ \forall\ x, y\ \in X.$

Then $\alpha_{A\times B}(0,0) = min\{\alpha_A(0), \alpha_B(0)\}$

$= \alpha_B(0)$

And $\alpha_{A\times B}(x, y) = min\{\alpha_A(x), \alpha_B(y)\} > \alpha_B(0)$

$= \alpha_{A\times B}(0,0)$

$\Rightarrow \alpha_{A\times B}(x, y) > \alpha_{A\times B}(0,0)\ \forall\ x, y\ \in X$

Which is a contradiction to A×B is an intuitionistic fuzzy BG- ideal of X×X.

Hence if $\alpha_A(0) \geq \alpha_A(x)\ for\ all\ x \in X\ then\ \alpha_B(0) \geq \alpha_A(x)\ or\ \alpha_B(0) \geq \alpha_B(x).$

(iv) Assume $\beta_B(0) > \beta_A(x)\ and\ \beta_B(0) > \beta_B(y)\ \forall\ x, y\ \in X$.

Then $\beta_{A\times B}(0,0) = max\{\beta_A(0), \beta_B(0)\}$

$= \beta_B(0)$

And $\beta_{A\times B}(x, y) = max\{\beta_A(x), \beta_B(y)\} < \beta_B(0)$

$= \beta_{A\times B}(0,0)$

$\Rightarrow \beta_{A\times B}(x, y) < \beta_{A\times B}(0,0)\ \forall\ x, y\ \in X$

Which is a contradiction to A×B is an intuitionistic fuzzy BG- ideal of X×X.

Hence if $\beta_A(0) \leq \beta_A(x)\ for\ all\ x \in X\ then\ \beta_B(0) < \beta_A(x)\ or\ \beta_B(0) \leq \beta_B(x).$

(v) Assume $\alpha_A(0) < \alpha_A(x)\ and\ \alpha_A(0) < \alpha_B(y)\ \forall\ x, y\ \in X.$

Then $\alpha_{A\times B}(0,0) = min\ \{\alpha_A(0), \alpha_B(0)\}$

$= \alpha_A(0)$

And $\alpha_{A\times B}(x, y) = min\{\alpha_A(x), \alpha_B(y)\} > \alpha_A(0)$

$=\alpha_{A\times B}(0,0)$

$$\Rightarrow \alpha_{A\times B}(x,y) > \alpha_{A\times B}(0,0)$$

Which is a contradiction to A×B is an intuitionistic fuzzy BG- ideal of X×X.

Hence if $\alpha_B(0) \geq \alpha_B(x)\ \forall\ x \in X$ then $\alpha_A(0) \geq \alpha_A(x)\ or\ \alpha_A(0) \geq \alpha_B(x)$

(vi) Assume $\beta_A(0) > \beta_A(x)\ and\ \beta_A(0) > \beta_B(y)\ \forall\ x,y\ \in X.$

Then $\beta_{A\times B}(0,0) = max\ \{\beta_A(0), \beta_B(0)\}$

$= \beta_A(0)$

And $\beta_{A\times B}(x,y) = max\{\beta_A(x), \beta_B(y)\} < \beta_A(0)$

$= \beta_{A\times B}(0,0)$

$$\Rightarrow \beta_{A\times B}(x,y) < \beta_{A\times B}(0,0)$$

Which is a contradiction to A×B is an intuitionistic fuzzy BG- ideal of X×X.

Hence if $\beta_B(0) \leq \beta_B(x)\ \forall\ x \in X$ then $\beta_A(0) \leq \beta_A(x)\ or\ \beta_A(0) \leq \beta_B(x)$

Theorem 3.3

If A and B are the intuitionistic fuzzy sets in the BG- algebra X such that A×B is an intuitionistic fuzzy BG-ideal of X×X, then A or B is an intuitionistic fuzzy BG- ideal of X.

Proof:

First to prove that B is an intuitionistic fuzzy BG- ideal.

Given A×B is an intuitionistic fuzzy BG-ideal of X×X.

Then by theorem 3.2, we have

(i) Either $\alpha_A(0) \geq \alpha_A(x)\ or\ \alpha_B(0) \geq \alpha_B(x)\ \forall\ x \in X$ and

(ii) Either $\beta_A(0) \leq \beta_A(x)\ or\ \beta_B(0) \leq \beta_B(x)\ \ \forall\ x \in X.$

Let $\alpha_B(0) \geq \alpha_B(x)\ and\ \beta_B(0) \leq \beta_B(x)\ \forall\ x\ \in X.$

By theorem 3.2, (v) $\alpha_A(0) \geq \alpha_A(x)\ or\ \alpha_A(0) \geq \alpha_B(x)$

Now$\alpha_B(x)\ = min\{\alpha_A(0), \alpha_B(x)\}$

$=\alpha_{A\times B}(0,x)$

$\geq min\{\alpha_{A\times B}((0,x)*(0,y)),\alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0*0,x*y),\alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0,x*y),\alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0*0,x*y),\alpha_{A\times B}(0,y)\}$

$= min\{\alpha_B(x*y),\alpha_B(y)\}$

That is $\alpha_B(x) \geq min\{\alpha_B(x*y),\alpha_B(y)\}$

Now $\alpha_B(x*y) = min\{\alpha_A(0),\alpha_B(x*y)\}$

$= \alpha_{A\times B}(0,x*y)$

$= \alpha_{A\times B}(0*0,x*y)$

$= \alpha_{A\times B}((0,x)*(0,y))$

$\alpha_B(x*y) \geq min\{\alpha_{A\times B}(0,x),\alpha_{A\times B}(0,y)\}$

$=min\{\alpha_B(x),\alpha_B(y)\}$

That is $\alpha_B(x*y) \geq min\{\alpha_B(x),\alpha_B(y)\}$

By theorem 3.2 (vi), $\beta_A(0) \leq \beta_A(x)$ *or* $\beta_A(0) \leq \beta_B(x)\ \forall\ x \in X$

Now $\beta_B(x) = max\{\beta_A(0),\beta_B(x)\}$

$=\beta_{A\times B}(0,x)$

$\leq max\{\beta_{A\times B}((0,x)*(0,y)),\beta_{A\times B}(0,y)\}$

$= max\{\beta_{A\times B}(0*0,x*y),\beta_{A\times B}(0,y)\}$

$= max\{\beta_{A\times B}(0,x*y),\beta_{A\times B}(0,y)\}$

$= max\{\beta_{A\times B}(0*0,x*y),\beta_{A\times B}(0,y)\}$

$= max\{\beta_B(x*y),\beta_B(y)\}$

That is $\beta_B(x) \leq max\{\beta_B(x*y), \beta_B(y)\}$

Now $\beta_B(x*y) = max\{\beta_A(0), \beta_B(x*y)\}$

$= \beta_{A\times B}(0, x*y)$

$= \beta_{A\times B}(0*0, x*y)$

$= \beta_{A\times B}\big((0,x)*(0,y)\big)$

$\beta_B(x*y) \leq max\{\beta_{A\times B}(0,x), \beta_{A\times B}(0,y)\}$

$= max\{\beta_B(x), \beta_B(y)\}$

That is $\beta_B(x*y) \leq max\{\beta_B(x), \beta_B(y)\}$

This proves that B is an intuitionistic fuzzy BG- ideal of X.

Secondly to prove that A is an intuitionistic fuzzy BG- ideal of X.

Give A×B is an intuitionistic fuzzy BG- ideal of X×X.

Then by theorem 3.2, we have

(i) Either $\alpha_A(0) \geq \alpha_A(x)$ or $\alpha_B(0) \geq \alpha_B(x)$ $\forall\, x \in X$ and

(ii) Either $\beta_A(0) \leq \beta_A(x)$ or $\beta_B(0) \leq \beta_B(x)$ $\forall\, x \in X$

Let $\alpha_A(0) \geq \alpha_A(x)$ and $\beta_A(0) \leq \beta_A(x)$.

By theorem 3.2 (iii),

If$\alpha_A(0) \geq \alpha_A(x)$ $for\ all\ x \in X, then$ $\alpha_B(0) \geq \alpha_A(x)$ or $\alpha_B(0) \geq \alpha_B(x)$

Now $\alpha_A(x) = min\{\alpha_B(0), \alpha_A(x)\}$

$=\alpha_{A\times B}(0,x)$

$\geq min\{\alpha_{A\times B}\big((0,x)*(0,y)\big), \alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0*0, x*y), \alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0, x*y), \alpha_{A\times B}(0,y)\}$

$= min\{\alpha_{A\times B}(0*0, x*y), \alpha_{A\times B}(0, y)\}$

$= min\{\alpha_A(x*y), \alpha_A(y)\}$

That is $\alpha_A(x) \geq min\{\alpha_A(x*y), \alpha_A(y)\}$

Now $\alpha_A(x*y) = min\{\alpha_B(0), \alpha_A(x*y)\}$

$= \alpha_{A\times B}(0, x*y)$

$= \alpha_{A\times B}(0*0, x*y)$

$= \alpha_{A\times B}\big((0, x)*(0, y)\big)$

$\alpha_A(x*y) \geq min\{\alpha_{A\times B}(0, x), \alpha_{A\times B}(0, y)\}$

$=min\{\alpha_A(x), \alpha_A(y)\}$

That is $\alpha_A(x*y) \geq min\{\alpha_A(x), \alpha_A(y)\}$

By theorem 3.2 (iv), $\beta_B(0) \leq \beta_A(x)\ or\ \beta_B(0) \leq \beta_B(x)\ \forall\ x \in X$

Now $\beta_A(x)\ = max\{\beta_B(0), \beta_A(x)\}$

$=\beta_{A\times B}(0, x)$

$\leq max\big\{\beta_{A\times B}\big((0, x)*(0, y)\big), \beta_{A\times B}(0, y)\big\}$

$= max\{\beta_{A\times B}(0*0, x*y), \beta_{A\times B}(0, y)\}$

$= max\{\beta_{A\times B}(0, x*y), \beta_{A\times B}(0, y)\}$

$= max\{\beta_{A\times B}(0*0, x*y), \beta_{A\times B}(0, y)\}$

$= max\{\beta_A(x*y), \beta_A(y)\}$

That is $\beta_A(x) \leq max\{\beta_A(x*y), \beta_A(y)\}$

Now $\beta_A(x*y) = max\{\beta_B(0), \beta_A(x*y)\}$

$= \beta_{A\times B}(0, x*y)$

$= \beta_{A\times B}(0*0, x*y)$

$$= \beta_{A\times B}((0,x) * (0,y))$$

$\beta_A(x * y) \le max\{\beta_{A\times B}(0,x), \beta_{A\times B}(0,y)\}$

$$= max\{\beta_A(x), \beta_A(y)\}$$

That is $\beta_A(x * y) \le max\{\beta_A(x), \beta_A(y)\}$.

This proves that A is an intuitionistic fuzzy BG- ideal of X.

Theorem 3.4

If A is an intuitionistic fuzzy BG- ideal of X, then A_t is a BG- ideal of X.

Proof:

Let A be an intuitionistic fuzzy BG- ideal of X. Then $\forall\ x \in X$

4. $\alpha_A(0) \ge \alpha_A(x); \beta_A(0) \le \beta_A(x)$
5. $\alpha_A(x) \ge min\{\alpha_A(x * y), \alpha_A(y)\}$; $\beta_A(x) \le max\{\beta_A(x * y), \beta_A(y)\}$
6. $\alpha_A(x * y) \ge min\{\alpha_A(x), \alpha_A(y)\}$; $\beta_A(x * y) \le max\{\beta_A(x), \beta_A(y)\}$

To prove that A_t is a BG- ideal of X.
We know that $A_t = \{x | \alpha_A(x) \ge t; 1 - \beta_A(x) \ge t\}$
Let $x, y \in A_t$ and A is an intuitionistic fuzzy BG- ideal of X.

Since $\alpha_A(0) \ge \alpha_A(x); \beta_A(0) \le \beta_A(x)$ and $x \in A_t$ we have

$$\alpha_A(0) \ge \alpha_A(x) \ge t \quad and \quad 1 - \beta_A(0) \ge 1 - \beta_A(x) \ge t$$

Which implies$0 \in A_t \forall\ t \in [0,1]$
Let $x * y \in A_t\ and\ y \in A_t$.
Therefore $\alpha_A(x * y) \ge t$; $1 - \beta_A(x * y) \ge t$

$$\alpha_A(y) \ge t\ ;\ 1 - \beta_A(y) \ge t$$

Now $\alpha_A(x) \ge min\{\alpha_A(x * y), \alpha_A(y)\}$
$\ge min\{t, t\} \ge t$

$\alpha_A(x) \ge t$

$\beta_A(x) \le max\{\beta_A(x * y), \beta_A(y)\}$

$\leq max\{1-t, 1-t\}$

$\beta_A(x) \leq 1-t$

$\therefore t \leq 1-\beta_A(x)$

That is $x \in A_t$.

Let $x \in A_t$, $y \in X$.

Choose y in X such that $\alpha_A(y) \geq t$ and $1-\beta_A(y) \geq t$.

Since $x \in A_t \Rightarrow \alpha_A(x) \geq t,\ 1-\beta_A(x) \geq t$

We know that

$$\alpha_A(x*y) \geq min\{\alpha_A(x), \alpha_A(y)\}$$
$$\geq min\{t, t\} \geq t$$

That is $\alpha_A(x*y) \geq t$

$$\beta_A(x*y) \leq max\{\beta_A(x), \beta_A(y)\}$$

$$\leq max\{1-t, 1-t\}$$

$$\leq 1-t$$

That is$1-\beta_A(x*y) \geq t$

$\Rightarrow x*y \in A_t$

Hence A_t is an fuzzy BG- ideal of X.

Theorem 3.5:

If X be a BG- algebra $\forall\ t \in [0,1]$ $andA_t$ is a BG- ideal of X, then A is an intuitionistic fuzzy BG- ideal of X.

Proof:

Since $A_t$is a BG- ideal of X,

(i) $0 \in A_t$

(ii) $x*y \in A_t and\ y \in A_t \Rightarrow x \in A_t$

(iii) $x \in A_t, y \in X \Rightarrow x*y \in A_t$

To prove that A is an intuitionistic fuzzy BG- ideal of X.

(i) $x, y \in A_t$ $then$ $\alpha_A(x) \geq t$; $1-\beta_A(x) \geq t$

$$\alpha_A(y) \geq t\ ;\ 1 - \beta_A(y) \geq t$$

Let $\alpha_A(x) = t_1\ and\ \alpha_A(y) = t_2$.

Without loss of generality let $t_1 \leq t_2 \Rightarrow 1 - t_1 \geq 1 - t_2$.

$$\alpha_A(x) + \beta_A(x) \leq 1$$

$$t_1 + \beta_A(x) \leq 1$$

$t_1 \leq 1 - \beta_A(x)$ and also $t_2 \leq 1 - \beta_A(y)$

Then $x \in A_{t_1}$

Now $x \in A_{t_1}\ and\ y \in X\ implies\ x * y \in A_{t_1}$

That is $\alpha_A(x * y) \geq t_1\ ;\ 1 - \beta_A(x * y) \geq t_1$

$$\Rightarrow \alpha_A(x * y) \geq t_1$$

$$= min\{t_1, t_2\}$$

$$= min\{\alpha_A(x), \alpha_A(y)\}$$

$$\alpha_A(x * y) \geq min\{\alpha_A(x), \alpha_A(y)\}$$

And also $\beta_A(x * y) \leq 1 - t_1$

$$= max\{1 - t_1, 1 - t_2\}$$

$$\leq max\{1 - (1 - \beta_A(x)), 1 - (1 - \beta_A(y))\}$$

$$= max\{\beta_A(x), \beta_A(y)\}$$

That is $\beta_A(x * y) \leq max\{\beta_A(x), \beta_A(y)\}$

(ii) Let $\alpha_A(0) = \alpha_A(x * x)$

$\geq min\{\alpha_A(x), \alpha_A(x)\}$ (By proof (i))

$\geq \alpha_A(x)$

And $\beta_A(0) = \beta_A(x * x)$

$\leq max\{\beta_A(x), \beta_A(x)\}$(By proof (i))

$\leq \beta_A(x)$

$\therefore\ \alpha_A(0) \geq \alpha_A(x)\ ;\ \beta_A(0) \leq \beta_A(x)\ \ \forall\, x \in X.$

(iii) Let$\alpha_A(x) = \alpha_A\big((x * y) * (0 * y)\big)$ (by proof(i))

$\geq min\{\alpha_A(x * y), \alpha_A(0 * y)\}$

$\geq min\big\{\alpha_A(x * y), min\{\alpha_A(0), \alpha_A(y)\}\big\}$

$\geq min\{\alpha_A(x * y), \alpha_A(y)\}$ (By (ii))

$\therefore \alpha_A(x) \geq min\{\alpha_A(x * y), \alpha_A(y)\}$

Let$\beta_A(x) = \beta_A\big((x * y) * (0 * y)\big)$ (By proof (i))

$\leq max\{\beta_A(x * y), \beta_A(0 * y)\}$

$\leq max\big\{\beta_A(x * y), max\{\beta_A(0), \beta_A(y)\}\big\}$

$\leq max\{\beta_A(x * y), \beta_A(y)\}$ (By (ii))

$\therefore \beta_A(x) \leq max\{\beta_A(x * y), \beta_A(y)\}$

Hence A is an intuitionistic fuzzy BG- ideal of X.

Definition 3.1

Let $f: x \rightarrow Y$ be a mapping of BG-algebra and A be an intuitionistic fuzzy set of Y then

A^f is the pre-image of A under f if

$$\alpha_{A^f}(x) = \alpha_A\big(f(x)\big)\ and\ \beta_{A^f}(x) = \beta_A\big(f(x)\big)\ \forall\, x \in X.$$

Theorem 3.6:

Let f:X→Y be a homomorphism of BG-algebra if A is an intuitionistic fuzzy BG-ideal of Y then A^f is intuitionistic fuzzy BG-ideal of X.

Proof:

For any $x, y \in X$, we have $\alpha_{A^f}(x) = \alpha_A\big(f(x)\big)$

$$\leq \alpha_A(0)$$

$$=\alpha_A\big(f(0)\big)$$

$$=\alpha_{A^f}(0)$$

Similarly $\beta_{A^f}(x) = \beta_A(f(x)$

$$\geq \beta_A(0)$$

$$= \beta_A(f(0))$$

$$=\beta_{A^f}(0)$$

Let $x,y \in X$ then,

$$min\{\alpha_{A^f}(x * y), \alpha_{A^f}(y)\} = min\{\alpha_A\big(f(x * y)\big), \alpha_A(f(y))\}$$

$$=min\{\alpha_A\big(f(x) * f(y)\big), \alpha_A\big(f(y)\big)\}$$

$$\leq \alpha_A\big(f(x)\big)$$

$$=\alpha_{A^f}(x)$$

That is $\alpha_{A^f}(x) \geq min\{\alpha_{A^f}(x * y), \alpha_{A^f}(y)\}$

And $max\ \{\beta_{A^f}(x * y), \beta_{A^f}(y)\}\ = max\{\beta_A\big(f(x * y)\big), \beta_A(f(y))\}$

$$= max\{\beta_A\big(f(x) * f(y)\big), \beta_A\big(f(y)\big)\}$$

$$\geq \beta_A\big(f(x)\big) = \beta_{A^f}(x)$$

That is $\beta_{A^f}(x) \leq max\{\beta_{A^f}(x * y), \beta_{A^f}(y)\}$

Now $min\{\alpha_{A^f}(x), \alpha_{A^f}(y)\} = min\{\alpha_A\big(f(x)\big), \alpha_A\big(f(y)\big)\}$

$$\leq \alpha_A\big(f(x) * f(y)\big)$$

$$= \alpha_A\big(f(x * y)\big)$$

$$=\alpha_{A^f}(x * y)$$

That is $\alpha_{A^f}(x * y) \geq min\{\alpha_{A^f}(x), \alpha_{A^f}(y)\}$

And $max\{\beta_{A^f}(x), \beta_{A^f}(y)\} = max\{\beta_A\big(f(x)\big), \beta_A\big(f(y)\big)\}$

$$\geq \beta_A\big(f(x) * f(y)\big)$$

$$= \beta_A\big(f(x * y)\big)$$

$$=\beta_{A^f}(x * y)$$

That is $\beta_{A^f}(x * y) \leq max\{\beta_{A^f}(x), \beta_{A^f}(y)\}$

Hence A^f is an intuitionistic fuzzy BG-ideal of X.

CONCLUSION

In this paper we have discussed about Intuitionistic Fuzzy BG-Ideals, Homomorphism, Cartesian product of Intuitionistic Fuzzy BG-Ideals.

References

[1] Ahn S.S. and Lee D, (2004), Fuzzy subalgebras of BG algebras, Commun. Korean Math.Soc, vol 19 No2, 243 – 251.

[2]Akram .M and Dar K.H.,(2005) On Fuzzy d-algebras, Journal of mathematics, Vol.37 pp 61 – 76 .

[3] Jun Y.B, Roh E.H. and Kim H.S., (1998), On BH – algebra ,sci. mathematica1 pp 347 -354.

[4] Neggers and Kim. H.S, (2002), On – B- Algebras, Math,Vensik, vol 54 pp 21-29.

[5] Neggers and Kim H.S, (1999), On – d- Algebras, Math,Slovaca , vol 49 pp 19-26.