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AXIAL VIBRATION OF A CANTILEVER BEAM WITH A TIP MASS

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ABSTRACT

In this study free axial vibration of a cantilever beam with a tip mass is analyzed. The boundary conditions are written for the fixed end and the end with the tip mass after the differential equation of motion is solved by separation of variables method. The frequency values for the first three vibration modes of the beam are obtained for various values of concentrated mass and presented in the tables. The frequency values for the concentrated mass values of zero are also compared with the ones of cantilever beam without tip mass and nearly the exact values are obtained with negligible error percentages.

Keywords: Axial vibration, Cantilever Beam, Tip Mass, Frequency

1. Introduction

In practice, the representation of a beam by a discrete model is an idealized model; however, in fact, beams have continuously distributed mass and elasticity. Mostly, especially for the axially vibration, beams are modeled as continuous systems having infinite number of degreed of freedom [1-10].

In this study, the free vibration analysis of a uniform axially vibrating cantilever beam with a tip mass is made. The differential equation of motion of the axially vibrating beam is solved by separation of variables method [11] and the displacement function is obtained. The boundary conditions are written for the fixed end and the tip mass. The natural frequencies for the first three modes are obtained for the various values of the tip mass. The results obtained for the tip mass value of zero are compared with the frequency values of the cantilever beam without a tip mass. The axially vibrating beam considered in the study is assumed to be homogeneous and isotropic.

2. Solution of Equation of Motion for an Axıally Vıbratıng Beam

An axially vibrating beam, given in Figure.1, with the distributed mass m, the length L, the modulus of elasticity E, the cross-section area A and the axial rigidity AE has a dimensional differential equation of motion for free vibration as [12]

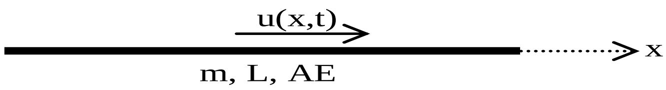

Figure 1: An Axially Vibrating Beam

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{m}{AE}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

(1)

where u(x,t) is the displacement function of the beam in terms of both displacement x and time t. Application of the separation of variables method to Eq. (1) as in the form of Eq. (2) is commonly used in vibration analysis of beams.

$$u(x,t) = X(x).T(t) = X(x).[A.\sin(\omega t) + B.\cos(\omega t)]$$

(2)

In Eq. (2), X(x) is the eigenfunction named as shape function, T(t) is time function, ω is the eigenvalue of the solution named as natural frequency and A, B are constants.

The derivatives used in Eq. (1) can, therefore, be written as

$$\frac{\partial^2 u(x,t)}{\partial x^2} = u''(x,t) = X''(x).[A.\sin(\omega t) + B.\cos(\omega t)] = X''(x).T(t)$$

(3)

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \ddot{u}(x,t) = X(x).(-\omega^2)[A.\sin(\omega t) + B.\cos(\omega t)] = -\omega^2.X(x).T(t)$$

(4)

where ($^{//}$) and ($\ddot{}$) denote the second order derivative due to x and t, respectively. Substitution of Eq. (3) and Eq. (4) in Eq. (1) gives the governing equation of motion in the form as

$$X''(x).T(t) + \frac{m\omega^2}{\text{AE}}X(x).T(t) = 0 \quad X''(x) + \frac{m\omega^2}{\text{AE}}X(x) = 0 \quad 0 \le x \le L$$

(5)

for $\alpha^2 = \frac{m\omega^2}{\text{AE}}$ $\qquad X''(x) + \alpha^2 X(x) = 0$

(6)

The characteristic equation and the solution of Eq. (6) is given as follows as D being d/dz:

$$D^2 + \alpha^2 = 0 \quad \rightarrow \quad D_{1,2} = \pm i\alpha \qquad (7)$$

$$X(x) = C_1.\sin(\alpha x) + C_2.\cos(\alpha x) \qquad (8)$$

Eq. (8) gives the shape function of the axially vibrating beam due to the displacement variable, x. Therefore, from Eq. (2), the displacement function of the axially vibrating beam has the form of Eq. (9).

$$u(x,t) = X(x).T(t) = [C_1.sin(\alpha x) + C_2.cos(\alpha x)].T(t)$$

(9)

3. Boundary Conditions

Figure 2: Axially Vibrating a Cantilever Beam with a Tip Mass

Two boundary conditions have to be written for the cantilever beam with a tip mass in Figure.2 since two integration constants (C_1, C_2) are obtained in the solution of second order differential equation of motion. The boundary conditions written for the left and the right ends of axially vibrating beam are given, respectively, as [13]

for x=0 $\quad u(x = 0, t) = 0$

(10)

for x=L $\quad N(x = L, t) = AEu'(x = L, t) = -M.\ddot{u}(x = L, t)$

(11)

where M is the tip mass value and N(x,t) is the axial force. If Eq. (9) and its derivative are substituted into Eq. (10) and Eq. (11) one gets the following relation between the coefficient matrix and the integration constants.

$$[k].\{C\} = 0; \quad [\propto.\cos(\propto L) - \propto_M^2.\sin(\propto L)].\{C_1\} = \{0\}$$

(12)

$$|k| = |\propto.\cos(\propto L) - \propto_M^2.\sin(\propto L)| = 0$$

(13)

where $\propto_M^2 = \frac{M.\omega^2}{AE}$. For non-trivial solution equating the determinant of the coefficient matrix of Eq. (12) to zero, as in Eq. (13), will give the eigenfrequencies of the axially vibrating cantilever beam with a tip mass. These frequencies are computed by a program written by the author considering the secant method [14].

4. Numerical Analysis

The first three natural frequencies of the axially vibrating cantilever beam with a tip mass are calculated for M values of 0, 10^{-10}, 10^{-9}, 10^{-8}, 10^{-6}, 10^{-4}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1} and 10^{2}, the beam length of L=1 m. and the modulus of elasticity of E=2100000 kg/cm^2. IPB-100, IPB-300 and IPB-600 profiles are used for numerical analysis with the mechanical properties given in Table 1 where h is height, G is weight per length, A is cross-section area and AE is axial rigidity of the corresponding profile. The distributed mass of the beam m is calculated from G/g as g being the acceleration of gravity with the value of 981 cm/sn^2.

Table 1: The Mechanical Properties of the Profiles Used in This Study

Profile	**h (cm)**	**G (kg/cm)**	**A (cm^2)**	**AE (kg)**
IPB100	10	0.081	10.3	21630000
IPB300	30	0.422	53.8	112980000
IPB600	60	1.22	156	327600000

The frequency values computed due to different values of the tip mass are presented in Table 2, Table 3 and Table 4 for, respectively, IPB-100, IPB-300 and IPB-600. The frequency values obtained for M=0 are compared with the exact frequency values obtained from the frequency equation of cantilever beam without a tip mass in the first row.

Table 2: Frequencies Computed for Different Values of the Tip Mass for IPB-100

		ω_1	ω_2	ω_3
Exact Frequencies for Cantilever Beam	$\omega_i = \frac{(2n_i - 1)}{2}\frac{\pi}{L}\sqrt{\frac{AE}{m}}$	8039.7053	24119.1160	40198.5266
M	**0**	8039.7053	24119.1160	40198.5266
	10^{-10}	8039.7052	24119.1157	40198.5261
	10^{-9}	8039.7044	24119.1131	40198.5218
	10^{-8}	8039.6956	24119.0868	40198.4779
	10^{-6}	8039	24117	40194
	10^{-4}	7944	23831	39719
	10^{-2}	4099	17306	32815
	10^{-1}	1451	16213	32226
	10^{0}	465	16093	32166
	10^{1}	148	16081	32160
	10^{2}	47	16080	32159

Table 3: Frequencies Computed for Different Values of the Tip Mass for IPB-300

		ω_1	ω_2	ω_3
Exact Frequencies for Cantilever Beam	$\omega_i = \frac{(2n_i - 1)}{2}\frac{\pi}{L}\sqrt{\frac{AE}{m}}$	8050.0567	24150.1702	40250.2836
M	**0**	8050.0567	24150.1702	40250.2836
	10^{-10}	8050.0567	24150.1701	40250.2835
	10^{-9}	8050.0565	24150.1696	40250.2827
	10^{-8}	8050.0549	24150.1646	40250.2743
	10^{-6}	8050	24150	40250
	10^{-4}	8032	24095	40157
	10^{-2}	6567	20333	35076
	10^{-1}	3139	16770	32547

	10^0	1056	16170	32236
	10^1	336	16108	32204
	10^2	107	16770	32201

Table 4: Frequencies Computed for Different Values of the Tip Mass for IPB-600

		ω_1	ω_2	ω_3
Exact Frequencies for Cantilever Beam	$\boldsymbol{\omega_i = \frac{(2n_i - 1)}{2}\frac{\pi}{L}\sqrt{\frac{AE}{m}}}$	8062.0672	24186.2015	40310.3358
M	**0**	8062.0672	24186.2015	40310.3358
	10^{-10}	8062.0672	24186.2015	40310.3358
	10^{-9}	8062.0671	24186.2013	40310.3355
	10^{-8}	8062.0665	24186.1996	40310.3329
	10^{-6}	8062	24186	40310
	10^{-4}	8056	24167	40278
	10^{-2}	7465	22451	37580
	10^{-1}	4769	17884	33223
	10^0	1774	16325	32350
	10^1	572	16145	32259
	10^2	181	16127	32250

5. Conclusions

In this study free axial vibration of a cantilever beam with a tip mass is made. The natural frequency values are obtained for different values of the tip mass and presented in tables. It can be seen from Tables 2, 3 and 4 that as the tip mass values increase from zero through a value of 10^{-6} for all profiles considered in this study the frequency values gently decrease, however, after the mentioned values the frequency values begin to decrease dramatically and rapidly. Increasing the height of the beam section causes an increase in frequency values.

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