AN APPLICATION OF $\overline{H}$ -FUNCTION AND GENERALIZED POLYNOMIALS IN THE STUDY OF ANGULAR DISPLACEMENT IN A SHAFT II

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ABSTRACT

The aim of present paper is to discuss the application of certain products involving $\overline{H}$ -function ([8], [9]) and a generalized polynomials [12] in obtaining a solution of the partial differential equation

$$\frac{\partial^2 \phi}{\partial t^2} = R^2 \frac{\partial^2 \phi}{\partial x^2}$$

concerning to a problem of angular displacement in a shaft. The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis, applied mathematics and mathematical physics.

1. Introduction. As an example of the application of $\overline{H}$ -function ([8],[9]) and generalized polynomials in applied mathematics, we consider the problem of determining the twist ϕ (x,t) in a shaft of circular section with its axis along the x-axis. Now the displacement ϕ (x,t) due to initial twist must satisfy the boundary value problem, if we assume that both the ends x = 0 and x = ω of the shaft are free.

$$\frac{\partial^2 \phi}{\partial t^2} = R^2 \frac{\partial^2 \phi}{\partial x^2}, \qquad \ldots(1.1)$$

where R is a constant.

$$\frac{\partial \phi(0,t)}{\partial x} = 0$$

and $\psi(x,0) = f(x)$, …(1.2)

Let

$$f(x) = \left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots, y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right]$$

$$.\overline{H}_{p,q}^{m,n}\left[z\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right], \qquad \ldots(1.3)$$

The generalized polynomials defined by Srivastava [12] is as follows:

$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}[x_1,\ldots,x_s] = \sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$A[n_1,\alpha_1;\ldots;n_s,\alpha_s]\, x_1^{\alpha_1}\ldots x_s^{\alpha_s}, \qquad \ldots(1.4)$$

where $n_{i'} = 0,1,2,\ldots\ \forall\ i' = (1,\ldots, s)$, $m_1,\ldots, m_s$ arbitrary positive integers and the coefficient $A[n_1,\alpha_1;\ldots;n_s,\alpha_s]$ are arbitrary constants, real or complex.

The $\overline{H}$ -function defined by Inayat-Hussain ([8],[9]) as

$$\overline{H}_{p,q}^{m,n}[z] = \overline{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j;B_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;A_j)_{m+1,q}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\phi(\xi)\, z^{\xi}\, d\xi, \qquad \ldots(1.5)$$

where $i = \sqrt{-1}$,

$$\psi(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{B_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{A_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)}, \qquad \ldots(1.6)$$

where a_j ($j = 1,\ldots,p$) and b_j ($j = 1,\ldots,q$) are complex parameters, $a_j \geq 0$ ($j = 1,\ldots,p$) (not all zero simultaneously) and the exponents B_j ($j = 1,\ldots,n$) and A_j ($j = m+1,\ldots,q$) can take on non-integer values, when these exponents take integer values, the $\overline{H}$ -function reduces to the familiar H-function due to Fox [7].

The condition for the absolute convergence of the defining integral for the $\overline{H}$ -function given by Buschman and Srivastava [2] as

$$\Omega \equiv \sum_{j=1}^{m} \beta_j + \sum_{j=1}^{n} B_j \alpha_j - \sum_{j=m+1}^{q} A_j \beta_j - \sum_{j=n+1}^{p} \alpha_j > 0 , \qquad \ldots(1.7)$$

and $$|\arg(z)| < \frac{1}{2}\pi\Omega , \qquad \ldots(1.8)$$

where Ω is given by (1.7).

The behaviour of the $\bar{H}$-function for small values of $|z|$ follows easily from a result recently given by Rathie [11]. We have

$$\bar{H}_{p,q}^{m,n}[z] = 0(|z|^{\alpha}), \; \alpha = \min_{1\le j\le m} [\mathrm{Re}(b_j/\beta_j)], \; |z| \to 0 , \qquad \ldots(1.9)$$

2. The main integral: The integral to be established here is

$$\int_0^{\omega} \left(\cos\frac{\pi\mu x}{\omega}\right)\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots,\right.$$

$$\left. y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right] \bar{H}_{p,q}^{m,n}\left[z\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right] dx$$

$$= \frac{\omega\, 2^{2\mu-\sigma+2\sum_{i=1}^{s} k_i\alpha_i}}{\Gamma(2\mu)\sqrt{\pi}} \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$.A[n_1,\alpha_1;\ldots;n_s,\alpha_s]\, y_1^{\alpha_1}\ldots y_s^{\alpha_s}\, \bar{H}_{p+2,q+1}^{m+1,n+1}\left[z\,4^h \left| \begin{matrix} (1-\mu+\frac{\sigma}{2}-\sum_{i=1}^{s} k_i\alpha_i, h; 1), \\ (\frac{1}{2}-\mu+\frac{\sigma}{2}-\sum_{i=1}^{s} k_i\alpha_i, h), \end{matrix}\right.\right.$$

$$\left.\left. \begin{matrix} (a_j,\alpha_j;B_j),(a_j,\alpha_j)_{n+1,p},(\sigma-\sum_{i=1}^{s} k_i\alpha_i, 2h) \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;A_j)_{m+1,q} \end{matrix}\right.\right], \qquad \ldots(2.1)$$

where $k_i > 0\,(i = 1,\ldots, s)$, $h > 0$, $2\omega > \mathrm{Re}\left(\sigma - 2k\frac{b_j}{\beta_j}\right) > 0$ $(j = 1,\ldots,m)$, m is an arbitrary positive integer and the coefficient $A(n_1,\alpha_1;\ldots;n_s,\alpha_s]$ are arbitrary constants, real or complex.

3. Evaluation of (2.1): The integral in (2.1) can be established by making use of the $\overline{H}$ -function in terms of Mellin-Barnes contour integral given by (1.5) and the definition of a generalized polynomials given by (1.4), then interchanging the order of summation and integration, evaluate the inner integral with the help of a result given by Chaurasia and Gupta ([2], p.59]), and we arrive at the desired result.

4. Solution of the problem posed: The solution of the problem to be established is

$$\phi(x,t) = \frac{1}{2^{\sigma}\sqrt{\pi}} \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$.A(n_1,\alpha_1;...;n_s,\alpha_s]\, y_1^{\alpha_1} \cdots y_s^{\alpha_s} \frac{2^{2\nu+2\sum_{i=1}^{s} k_i\alpha_i+1}}{\Gamma(2\nu)} H^{m+1,n+1}_{p+2,q+1}$$

$$.\left[z\,4^{h} \left| \begin{matrix} (1-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s} k_i\alpha_i, h;1), (a_j,\alpha_j;B_j), (a_j,\alpha_j)_{n+1,p}, (\sigma-\sum_{i=1}^{s} k_i\alpha_i, 2h) \\ (\frac{1}{2}-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s} k_i\alpha_i, h), (b_j,\beta_j)_{1,m}, (b_j,\beta_j;A_j)_{m+1,q} \end{matrix} \right. \right] \left(\cos\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\nu Rt}{\omega}\right),$$

…(4.1)

which holds true under the same conditions needed for (2.1).

5. Derivation on (4.1): The solution of the problem can be written as ([6], Churchill 1941, p.125 (4)].

$$\phi(x,t) = \frac{1}{2}a_0 + \sum_{\nu=1}^{\infty} a_\nu \left(\cos\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\nu Rt}{\omega}\right), \qquad \text{…(5.1)}$$

where a_ν = (ν = 0,1,2,…) are the coefficients in the Fourier cosine series for f(x) in the interval (0,ω). If t = 0 then by virtue of (1.3), we get

$$\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S^{m_1,\ldots,m_s}_{n_1,\ldots,n_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots, y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right]$$

$$\overline{H}^{m,n}_{p,q}\left[z\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right] = \frac{1}{2}a_0 + \sum_{\nu=1}^{\infty} a_\nu\left(\cos\frac{\pi x}{\omega}\right), \qquad \text{…(5.2)}$$

Multiplying both sides of (5.2) by $\left(\cos\frac{\pi\mu x}{\omega}\right)$ and integrating with respect to x from 0 to ω, we have

$$\int_0^{\omega}\left(\cos\frac{\pi\mu x}{\omega}\right)\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right]$$

$$.\overline{H}_{p,q}^{m,n}\left[z\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right]=\frac{1}{2}a_0\int_0^{\omega}\left(\cos\frac{\pi\mu x}{\omega}\right)dx$$

$$+\sum_{\nu=1}^{\infty}a_\nu\int_0^{\omega}\left(\cos\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\mu x}{\omega}\right)dx\,, \qquad \ldots(5.3)$$

Using (2.1) along with orthogonality property of the cosine functions, we get

$$a_\nu=\sum_{\alpha_1=0}^{[n_s/m_s]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s n_s}}{\alpha_s!}A[n_1,\alpha_1;\ldots;n_s,\alpha_s]\,y_1^{\alpha_1}\cdots y_s^{\alpha_s}$$

$$.\frac{2^{2\nu-\sigma+\sum_{i=1}^{s}k_i\alpha_i+1}}{\Gamma(2\nu)\sqrt{\pi}}\overline{H}_{p+2,q+1}^{m+1,n+1}\left[z\,4^h\left|\begin{matrix}(1-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1),(a_j,\alpha_j;B_j),\\(\frac{1}{2}-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),(b_j,\beta_j)_{1,m},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j)_{n+1,p},(\sigma-\sum_{i=1}^{s}k_i\alpha_i,2h)\\(b_j,\beta_j;A_j)_{m+1,q}\end{matrix}\right]\,, \qquad \ldots(5.4)$$

Using (5.1) and (5.4), we arrive at the desired solution in (4.1).

6. Special Cases:

(1) Taking m = 1, n = 3 = p = q and replacing z by – z and using

$$g(\gamma,\eta,\tau,P;z)=\frac{a_{d-1}\,\Gamma(P+1)\,\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^P\,2^{2+P}\sqrt{\pi}\,\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

$$.\overline{H}_{3,3}^{1,3}\left[-z\left|\begin{matrix}(1-\gamma,1;1),(1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1;1+P)\\(0,1),\left(-\frac{\tau}{2},1;1\right),(-\eta,1;1+P)\end{matrix}\right.\right],$$

where $a_d = \dfrac{2^{1-d}\,\pi^{-d/2}}{\Gamma(d/2)}$ [9, p.4121, eq. (5)]

The above function is connected with a certain class of Feynman integrals.

We obtain from (2.1)

(a) $$\int_0^{\omega}\left(\cos\frac{\pi\mu x}{\omega}\right)\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right] g\left[\gamma,\eta,\tau,P\,;z\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right]dx$$

$$=\frac{a_{d-1}\Gamma(P+1)\,\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^P\,2^{2+P}\,\pi\,\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}\;\frac{\omega\, 2^{2\mu-\sigma+2\sum_{i=1}^{s}k_i\alpha_i}}{\Gamma(2\mu)}$$

$$.\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}\,A\,[n_1,\alpha_1;\ldots;n_s,\alpha_s]\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$.\overline{H}_{5,4}^{2,4}\left[-z\,4^h\left|\begin{matrix}(1-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h\,;1),\,(1-\gamma,1;1),\,(1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1;1+P),\,(\sigma-\sum_{i=1}^{s}k_i\alpha_i,2h)\\(\frac{1}{2}-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h)\,,(0,1),\;\;(-\frac{\tau}{2},1;1),\,(-\eta,1;1+P)\end{matrix}\right.\right] \quad \ldots(6.1)$$

valid under the same conditions as those required for (2.1).

And from (4.1), we obtain

(b) $$\phi(x,t)=\frac{1}{2^{\sigma}\sqrt{\pi}}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$.A\,[n_1,\alpha_1;\ldots;n_s,\alpha_s]\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}\;\frac{a_{d-1}\Gamma(P+1)\,\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^P\,2^{2+P}\,\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

$$.\frac{2^{2\nu+2\sum_{i=1}^{s}k_i\alpha_i+1}}{\Gamma(2\nu)}\overline{H}_{5,4}^{2,4}\left[-z\,4^h\left|\begin{matrix}(1-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h\,;1),\,(1-\gamma,1;1),\,(1-\gamma+\frac{\tau}{2},1;1),\\(\frac{1}{2}-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),\,(0,1),\;\;(-\frac{\tau}{2};1),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-\eta,1;1+P),\ (\sigma-\sum_{i=1}^{s}k_i\alpha_i,2h)\\ (-\eta,1;1+P)\end{matrix}\right] \cos\left(\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\nu\ Rt}{\omega}\right), \qquad \ldots(6.2)$$

valid under the same conditions as those required for (4.1).

(2) Taking m = 1, n = 3 = p, q = 2 and replacing z by – $(1+\varepsilon)^{-2}$ and using

$$\beta F(d;\varepsilon) = -\frac{1}{4\pi^{d/2}(1+\varepsilon)^2}\overline{H}_{3,2}^{1,3}\left[-(1+\varepsilon)^{-2}\left|\begin{matrix}(0,1;1),\ (0,1;1),\ (-1/2,1;d)\\ (0,1),\ (-1,1;1+d)\end{matrix}\right.\right], \text{ [9,p.4121,eq.(5)]}$$

The above function is the exact partition function of the Gaussian model in statistical mechanics.

We obtain from (2.1)

(a)
$$\int_0^{\omega}\left(\cos\frac{\pi\mu x}{\omega}\right)\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\omega}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\omega}\right)^{2k_s}\right]\beta F(d;\varepsilon)\,dx$$

$$= -\frac{\omega\, e^{2\mu-\sigma+2\sum_{i=1}^{s}k_i\alpha_i-2}}{\Gamma(2\mu)\pi^{(d+1)/2}(1+\varepsilon)^2}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$.\ A\,[n_1,\alpha_1;\ldots;n_s,\alpha_s)\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$.\overline{H}_{5,3}^{2,4}\left[-(1+\varepsilon)^{-2}\,4^h\left|\begin{matrix}(1-\mu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1),\ (0,1;1),\ (0,1;1),\ (-1/2,1;d),(\sigma-\sum_{i=1}^{s}k_i\alpha_i,2h)\\ (\frac{1}{2}-\mu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),\ (0,1),\ (-1,1;1+d)\end{matrix}\right.\right] \qquad \ldots(6.3)$$

valid under the same conditions as those required for (2.1).

And from (4.1), we have

(b)
$$\phi(x,t) = -\frac{1}{2^{\sigma+1}\pi^{(d+1)/2}(1+\varepsilon)^2}\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$.\,A\,[n_1,\alpha_1;\ldots;n_s,\alpha_s]\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}\frac{2^{2\nu+2\sum_{i=1}^{s}k_i\alpha_i}}{\Gamma(2\nu)}$$

$$.\overline{H}^{2,4}_{5,3}\left[-(1+\varepsilon)^{-2}\,4^{h}\left|\begin{matrix}(1-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1),\ (0,1;1),\ (0,1;1),\ (-1/2,1;d),\ (\sigma-\sum_{i=1}^{s}k_i\alpha_i,2h)\\ (\frac{1}{2}-\nu+\frac{\sigma}{2}-\sum_{i=1}^{s}k_i\alpha_i,h),\ (0,1),\ (-1,1;1+d)\end{matrix}\right.\right]\cos\left(\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\nu Rt}{\omega}\right), \quad \ldots(6.4)$$

valid under the same conditions as those required for (4.1).

(3) (a) On taking $n_{i'}\to 0\ (i'=1,\ldots,s)$, eq. (6.1) reduces in following form

$$\int_0^{\omega}\left(\cos\frac{\pi\mu x}{\omega}\right)\left(\sin\frac{\pi x}{2\omega}\right)^{2\mu-\sigma-1}\left(\cos\frac{\pi x}{2\omega}\right)^{\sigma-1} g\left[\gamma,\eta,\tau,P;z)\left(\tan\frac{\pi x}{2\omega}\right)^{2h}\right]$$

$$=\frac{a_{d-1}\Gamma(P+1)\,\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^{P}\,2^{2+P}\,\pi\,\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}\,\frac{\omega\,2^{2\mu-\sigma}}{\Gamma(2\mu)}\,s$$

$$.\overline{H}^{2,4}_{5,4}\left[-z\,4^{h}\left|\begin{matrix}(1-\nu+\frac{\sigma}{2},h;1),\ (1-\gamma,1;1),\ (1-\gamma+\frac{\tau}{2},1;1),\ (1-\eta,1;1+P),\ (\sigma,2h)\\ (\frac{1}{2}-\nu+\frac{\sigma}{2},h),\ (0,1),\ (-\frac{\tau}{2},1;1),\ (-\eta,1;1+P)\end{matrix}\right.\right] \quad \ldots(6.5)$$

valid under the same conditions as those required for (6.1).

(b) On taking $n_{i'}\to 0\ (i'=1,\ldots,s)$, eq. (6.2) reduces in following form

$$\phi(x,t)=\frac{1}{2^{\sigma}\pi}\,\frac{a_{d-1}\Gamma(P+1)\,\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^{P}\,2^{2+P}\,\Gamma(\gamma)\Gamma(\gamma+\frac{\tau}{2})}\,\frac{2^{2\nu+1}}{\Gamma(2\nu)}$$

$$.\overline{H}^{2,4}_{5,4}\left[-z\,4^{h}\left|\begin{matrix}(1-\nu+\frac{\sigma}{2},h;1),\ (1-\gamma,1;1),\ (1-\gamma+\frac{\tau}{2},1;1),\ (1-\eta,1;1+P),\ (\sigma,2h)\\ (\frac{1}{2}-\nu+\frac{\sigma}{2},h),\ (0,1),\ (-\frac{\tau}{2},1;1),\ (-\eta,1;1+P)\end{matrix}\right.\right].\cos\left(\frac{\pi\nu x}{\omega}\right)\left(\cos\frac{\pi\nu Rt}{\omega}\right), \quad \ldots(6.6)$$

valid under the same conditions as those required for (4.1).

4. Letting s = 2 in (2.1) and (3.1), we get a known result recently obtained by Chaurasia and Lata ([5], eq.(2.1), p.188).

5. Taking $B_j = 1 = A_j$, $s = 2$ and $K \to 0$ in (2.1) and (3.1), we find known result recently obtained by Chaurasia and Godika ([3], eq. (5), p.79).

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