SOME SPECIAL CASES OF APPELL'S AND HUMBERT'S FUNCTIONS OF MATRIX ARGUMENTS IN COMPLEX CASE

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ABSTRACT

We have proved three results for the Appell's functions of matrix arguments-one each for the function $\overline{F}_1$ and $\overline{F}_3$ and one for the function $\overline{F}_4$ in complex case.

INTRODUCTION

Appell's functions of matrix arguments have earlier been studied by Mathai [4, 5, 6] and also by Saxena, Sethi and Gupta [7]. In the present paper we have utilized Mathai's definitions for all the functions studied. Upadhyaya and Dhani have given integral representation associated with Appell's Humbert's functions of matrix arguments in the case of real symmetric positive definite. We have given in this paper further generalization of these results in the case of Hermitian positive definite matrix of complex number. All the matrices appearing in this in the paper are (pxp) real Hermitian positive definite matrices and the meanings of all the other symbols used are the same as in the works of Mathai [3, 4].

1. Preliminary Definitions

Definition 1.1: The Appell's function $\overline{F}_1 = \overline{F}_1$ (a, b, b′; c; $-\overline{X}-\overline{Y}$) of matrix argument is defined as that function for which the M-transform (Matrix-transform) is the following:

$$M(\bar{F}_1) = \int \bar{X} > 0 \int \bar{Y} > 0 \left|\bar{X}\right| \rho_1^{-p} \left|Y\right| \rho_2^{-p} F_1(a, b, b'; c; -\bar{X} - \bar{Y})\, d\bar{X}\, d\bar{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)\tilde{\Gamma}_p(b-\rho_1)\tilde{\Gamma}_p(b-\rho_2)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(b')\tilde{\Gamma}_p(c-\rho_1-\rho_2)} \quad \ldots. \quad (1.1)$$

for Re $(a - \rho_1 - \rho_2, b - \rho_1, b' - \rho_2, c - \rho_1 - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.2: $\bar{F}_2 = \bar{F}_2(a, b, b'; c; -\bar{X} - \bar{Y})$

$$M(\bar{F}_2) = \int \bar{X} > 0 \int \bar{Y} > 0 \left|\bar{X}\right| \rho_1^{-p} \left|Y\right| \rho_2^{-p} F_2(a, b, b'; c, c; -\bar{X} - \bar{Y})\, d\bar{X}\, d\bar{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(c')\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)\tilde{\Gamma}_p(b-\rho_1)\tilde{\Gamma}_p(b'-\rho_2)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(b')\tilde{\Gamma}_p(c-\rho_1)\tilde{\Gamma}_p(c-\rho_2)} \quad \ldots. \quad (1.2)$$

for Re $(a - \rho_1 - \rho_2, b - \rho_1, b' - \rho_2, c - \rho_1, c' - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.3: $\bar{F}_3 = \bar{F}_3(a, a', b, b'; c; -\bar{X} - \bar{Y})$

$$M(\bar{F}_3) = \int \bar{X} > 0 \int \bar{Y} > 0 \left|\bar{X}\right| \rho_1^{-p} \left|Y\right| \rho_2^{-p} F_3(a, a'\, b, b'; c; -\bar{X} - \bar{Y})\, d\bar{X}\, d\bar{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1(a'-\rho_2)\tilde{\Gamma}_p(b-\rho_1)\tilde{\Gamma}_p(b'-\rho_2)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(a')\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(b')\tilde{\Gamma}_p(c-\rho_1-\rho_2)} \quad \ldots. \quad (1.3)$$

for Re $(a - \rho_1, a' - \rho_2, b - \rho_1, b' - \rho_2, c - \rho_1 - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.4: $\bar{F}_4 = \bar{F}_4\ (a, b; c, c'; -\bar{X} - \bar{Y})$

$$M(\bar{F}_4) = \int \bar{X} > 0 \int \bar{Y} > 0 \left|\bar{X}\right| \rho_1^{-p} \left|Y\right| \rho_2^{-p} F_4(a, b; c, c'; -\bar{X} - \bar{Y})\, d\bar{X}\, d\bar{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(c')\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)\tilde{\Gamma}_p(b-\rho_1-\rho_2)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(c-\rho_1)\tilde{\Gamma}_p(c'-\rho_2)} \quad \ldots. \quad (1.4)$$

for Re $(a - \rho_1 - \rho_2, b - \rho_1 - \rho_2, c - \rho_1 - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.5: The Humbert's function $\overline{\Phi}_1 = \overline{\Phi}_1$ (a, b; c; – $\overline{X} - \overline{Y}$) of matrix arguments is defined as that function which has the following M-transform:

$$M(\overline{\Phi}_1) = \int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1 - p} |\overline{Y}|^{\rho_2 - p} \, \overline{\Phi}_1(a, b; c; -\overline{X}, -\overline{Y}) \, d\overline{X} \, d\overline{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)\tilde{\Gamma}_p(b-\rho_1)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(c-\rho_1-\rho_2)} \quad \ldots. \quad (1.5)$$

for Re $(a - \rho_1 - \rho_2, b - \rho_1, c - \rho_1 - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.6: $\overline{\Psi}_1 = \overline{\Psi}_1$ (a, b; c; – $\overline{X} - \overline{Y}$)

$$M(\overline{\Psi}_1) = \int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1 - p} |\overline{Y}|^{\rho_2 - p-1} \, \overline{\Psi}_1(a, b; c, c'; -\overline{X}, -\overline{Y}) \, d\overline{X} \, d\overline{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(c')\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)\tilde{\Gamma}_p(b-\rho_1)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(c-\rho_1)\tilde{\Gamma}_p(c'-\rho_2)} \quad \ldots. \quad (1.6)$$

for Re $(a - \rho_1 - \rho_2, b - \rho_1, c - \rho_1\, c' - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.7: $\overline{\Psi}_2 = \overline{\Psi}_2$ (a; c, c′; – $\overline{X} - \overline{Y}$)

$$M(\overline{\Psi}_2) = \int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1 - p} |\overline{Y}|^{\rho_2 - p-1} \, \overline{\Psi}_2(a, c, c'; -\overline{X}, -\overline{Y}) \, d\overline{X} \, d\overline{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(c')\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1-\rho_2)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(c-\rho_1)\tilde{\Gamma}_p(c'-\rho_2)} \quad \ldots. \quad (1.7)$$

for Re $(a - \rho_1 - \rho_2, c - \rho_1, c' - \rho_2, \rho_1, \rho_2) > p-1$

Definition 1.8: $\overline{\Xi}_2 = \overline{\Xi}_2$ (a; b; c; – $\overline{X} - \overline{Y}$)

$$M(\overline{\Xi}_2) = \int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1 - p} |\overline{Y}|^{\rho_2 - p-1} \, \overline{\Xi}_2(a, b; c; -\overline{X}, -\overline{Y}) \, d\overline{X} \, d\overline{Y}$$

$$= \frac{\tilde{\Gamma}_p(c)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(a-\rho_1)\tilde{\Gamma}_p(b-\rho_1)}{\tilde{\Gamma}_p(a)\tilde{\Gamma}_p(b)\tilde{\Gamma}_p(c-\rho_1-\rho_2)} \quad \ldots. \quad (1.8)$$

for Re $(a - \rho_1, b - \rho_1, c - \rho_2, \rho_1, \rho_2) > p-1$

2. Appell's Functions of Matrix Arguments.

THEOREM 2.1:

$|\bar{P}|\beta'\ F_1(\alpha, \beta, \beta'; \gamma; -\bar{X}, -\bar{P}^{-1/2}\bar{Y} - \bar{P}^{-1/2})$

$$= \frac{1}{\tilde{\Gamma}_p(\beta')} \int \bar{T} > 0 e^{-tr(\bar{P}\bar{T})} |\bar{T}|^{\beta'-P}\ \bar{\Phi}_1(\alpha, \beta; \gamma; -\bar{X} - \bar{T}^{1/2}\bar{Y}\ \bar{T}^{1/2}) \quad \ldots(2.1)$$

for Re $(\beta') > -1$

PROOF: Taking the M-transform of the right side of eq. (2.1) with respect to the variables $\bar{X}$ and $\bar{Y}$ and the parameters ρ_1 and ρ_1 respectively, we have,

$$\int \bar{X} > 0 \int \bar{Y} > 0 \left|\bar{X}\right|\rho_1^{-p} \left|\bar{Y}\right|\rho_2^{-p}\ \bar{\Phi}_1(\alpha, \beta; \gamma; -\bar{X} - \bar{T}^{1/2}\bar{Y}\ \bar{T}^{1/2}) d\bar{X}\, d\bar{Y} \quad \ldots(2.2)$$

Which, under the transformation

$\bar{Y}_1 = \bar{T}^{1/2}\bar{Y}\ \bar{T}^{1/2}$ (with $d\bar{Y}_1 = |\bar{T}|^{p} d\bar{Y}$ and $|\bar{Y}_1| = |\bar{T}||\bar{Y}|$)

and using the definition (1.5) yields,

$$|\bar{T}|^{-\rho_2} \frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\alpha-\rho_1-\rho_2)\tilde{\Gamma}_p(\beta-\rho_1)}{\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)} \quad \ldots (2.3)$$

Substituting this expression the right side of eq. (2.1) and then integrating out T in the resulting expression by using a Gamma integral gives,

$$|\overline{P}|^{\beta'-\rho_2} \frac{\tilde{\Gamma}_p(\beta'-\rho_1)\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\alpha-\rho_1-\rho_2)\tilde{\Gamma}_p(\beta-\rho_1)}{\tilde{\Gamma}_p(\beta')\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)} \quad \ldots (2.4)$$

Now, Taking the M-transform of the left side of eq. (2.1) with respect to the variables $\overline{X}$ and $\overline{Y}$ and the parameters ρ_1 and ρ_1 respectively, we get,

$$\int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1-p} |\overline{Y}|^{\rho_2-p} |\overline{P}|^{-\beta'} \times \overline{F}_1(\alpha, \beta, \beta\times; \gamma; -\overline{X} - \overline{P}^{-1/2}\overline{Y}\,\overline{P}^{-1/2}) d\overline{X}\, d\overline{Y} \quad \ldots(2.5)$$

Which, under the transformation

$\overline{Y}_2 = \overline{P}^{-1/2}\overline{Y}\,\overline{P}^{-1/2}$ (with $d\overline{Y}_2 = |\overline{P}|^{-p} d\overline{Y}$ and $|\overline{Y}_2| = |\overline{P}|^{-1}|\overline{Y}|$)

and then using the definition (1.1) leads us to the same result as in eq. (2.4).

THEOREM 2.2: For p = 2,

$$|\overline{P}|-\alpha' \;\; \overline{F}_3(\alpha, (\alpha' + 1)/2, \beta, (2\alpha' + 1)/4; \gamma; -\overline{X}, -4\overline{P}^{-1}\overline{Y} - \overline{P}^{-1})$$

$$= \frac{1}{\tilde{\Gamma}_p(\alpha')} \int_{\overline{T}>0} e^{-tr(\overline{P}\,\overline{T})} |\overline{T}|^{\alpha'-P} \;\; \overline{\Xi}_2(\alpha, \beta; \gamma; -\overline{X} - \overline{T}\,\overline{Y}\,\overline{T}') d\overline{T} \quad \ldots(2.6)$$

where Re $(\alpha') > p - 2$.

PROOF: Taking the M-transform of the right side of eq. (2.6) with respect to the variables $\overline{X}$ and $\overline{Y}$ and the parameters ρ_1 and ρ_1 respectively, we have,

$$\int_{\overline{X}>0} \int_{\overline{Y}>0} |\overline{X}|^{\rho_1-p} |\overline{Y}|^{\rho_2-p} \;\; \overline{\Xi}_2(\alpha, \beta; \gamma; -\overline{X} - \overline{T}\,\overline{Y}\,\overline{T}') d\overline{X}\, d\overline{Y} \quad \ldots(2.7)$$

Applying the transformation $\overline{Y}_1 = \overline{T}\,\overline{Y}\,\overline{T}'$ with $\overline{Y}_1 = |\overline{T}|^{p+1} d\overline{Y}$ and $|\overline{Y}_1| = |\overline{T}|^2 |\overline{Y}|$ to the above expression and then using definition (1.8) produces,

$$|\bar{T}|^{-\rho_2} \frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\alpha-\rho_1)\tilde{\Gamma}_p(\beta-\rho_1)}{\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)} \quad \dots (2.8)$$

Substituting this expression the right side of eq. (2.6) and then integrating out $\bar{T}$ in the resulting expression by using a Gamma integral leads us to,

$$|\bar{P}|^{(\alpha'-\rho_2)} \frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\alpha-\rho_1)\tilde{\Gamma}_p(\beta-\rho_1)\tilde{\Gamma}_p(\alpha-\rho_2)}{\tilde{\Gamma}_p(\alpha')\tilde{\Gamma}_p(\alpha)\tilde{\Gamma}_p(\beta\beta\tilde{\Gamma}_p(\gamma-\rho_1-\rho_2)} \quad \dots (2.9)$$

Now, Taking the M-transform of the left side of eq. (2.6) with respect to the variables $\bar{X}$ and $\bar{Y}$ and the parameters ρ_1 and ρ_1 respectively, we get,

$$\int \bar{X}>0 \int \bar{Y}>0 \left|\bar{X}\right|\rho_1^{-p} \left|\bar{Y}\right|\rho_2^{-p} |\bar{P}|^{-\alpha'} \times$$

$$\bar{F}_3[\alpha, (\alpha'+1)/2, \beta, (2\alpha'+1)/4; \gamma; -\bar{X}, -4\bar{P}^{-1}\bar{Y}-\bar{P}^{-1})d\bar{X}\, d\bar{Y} \quad \dots (2.10)$$

Making use of the transformation

$\bar{Y}_2 = 4\bar{P}^{-1}\bar{Y}-\bar{P}^{-1}$ $d\bar{Y}_2 = 4^{p-1}|\bar{P}|^{-p} d\bar{Y}$ and $|\bar{Y}_2| = 4p\,|\bar{P}|^{-2}\,|\bar{Y}|$ in above expression and then using the definition (1.3) along with the observation that for p = 2,

$$4^{-p\rho_2} \frac{\tilde{\Gamma}_p[(\alpha'+1)/2-\rho_2]\tilde{\Gamma}_p[(2\alpha'+1)/4-\rho_2]}{\tilde{\Gamma}_p[(\alpha'+1)/2]\tilde{\Gamma}_p[(2\alpha'+1)/4]} = \frac{\tilde{\Gamma}_p[(\alpha'+2-\rho_2)]}{\tilde{\Gamma}_p[(\alpha')]} \quad \dots (2.11)$$

from eq. (6.13) page 84 of Mathai [4], finally leads us to same result as in eq. (2.9) above.

It is to be noted that this result is different from the corresponding result in the scalar case.

THEOREM 2.3:

$$|\bar{P}|^{-\alpha'}\ \bar{F}_4(\alpha, \beta; \gamma,\gamma'; -\bar{P}^{-1/2}\bar{X}\ \bar{P}^{1/2}, -\bar{P}^{-1/2})$$

$$= \frac{1}{\tilde{\Gamma}_p(\alpha')} \int_{\bar{T}>0} e^{-tr(\bar{P}\bar{T})} |\bar{T}|^{\alpha'-P} \bar{\Psi}_2(\beta;\gamma,\gamma';-\bar{T}^{1/2}\bar{X}\bar{T}^{1/2},-\bar{T}^{1/2}\bar{Y}\bar{T}^{1/2}) d\bar{T} \qquad \ldots(2.12)$$

where Re $(\alpha') > p - 1$.

PROOF: Taking the M-transform of the right side of eq. (2.12) with respect to the variables $\bar{X}$ and $\bar{Y}$ and the parameters ρ_1 and ρ_1 respectively, we have,

$$\int_{\bar{X}>0} \int_{\bar{Y}>0} |\bar{X}|^{\rho_1 - p} |\bar{Y}|^{\rho_2 - p} \times$$

$$\bar{\Psi}_2(\beta;\gamma,\gamma';-\bar{T}^{1/2}\bar{X}\bar{T}^{1/2},-\bar{T}^{1/2}\bar{Y}\bar{T}^{1/2}) d\bar{X}\, d\bar{Y} \qquad \ldots(2.13)$$

Which, under the transformations

$\bar{X}_1 = \bar{T}^{1/2}\bar{X}\bar{T}^{1/2}$, $\bar{Y}_1 = \bar{T}^{1/2}\bar{Y}\bar{T}$ with $d\bar{X}_1 = |\bar{T}|^{p} d\bar{X}$, $d\bar{Y}_1 = |\bar{T}|^{p} d\bar{Y}$

and $|\bar{X}_1| = |\bar{T}||\bar{X}|$, $|\bar{Y}_1||\bar{T}||\bar{Y}|$

and then using the definition (1.7) yields,

$$|\bar{T}|^{-\rho_1-\rho_2} \frac{\tilde{\Gamma}_p(\gamma)\tilde{\Gamma}_p(\gamma')\tilde{\Gamma}_p(\rho_1)\tilde{\Gamma}_p(\rho_2)\tilde{\Gamma}_p(\beta-\rho_1-\rho_2)}{\tilde{\Gamma}_p(\beta)\tilde{\Gamma}_p(\gamma-\rho_1)\tilde{\Gamma}_p(\gamma'-\rho_2)} \qquad \ldots(2.14)$$

Substituting this expression the right side of eq. (2.12) and then integrating out $\bar{T}$ in the resulting expression by using a Gamma integral produces,

$$|\bar{P}|^{(\alpha-\rho_1-\rho_2)} \frac{\bar{\Gamma}_p(\gamma')\bar{\Gamma}_p(\gamma)\bar{\Gamma}_p(\rho_1)\bar{\Gamma}_p(\rho_2)\bar{\Gamma}_p(\alpha-\rho_1-\rho_2)\bar{\Gamma}_p(\beta-\rho_1-\rho_2)}{\bar{\Gamma}_p(\alpha)\bar{\Gamma}_p(\beta)\bar{\Gamma}_p(\gamma-\rho_1)\bar{\Gamma}_p(\gamma'-\rho_2)} \qquad \ldots(2.15)$$

Now, Taking the M-transform of the left side of eq. (2.12) with respect to the variables $\bar{X}$ and $\bar{Y}$ and the parameters ρ_1 and ρ_1 respectively, we get,

$$\int_{\bar{X}>0} \int_{\bar{Y}>0} |\bar{X}|^{\rho_1 - p} |\bar{Y}|^{\rho_2 - p} |\bar{P}|^{-\alpha'} \times$$

$$|\bar{P}|^{-\alpha}\bar{F}_4[\alpha, \beta; \gamma,\gamma'; -\bar{P}^{-1/2}\bar{X}\bar{P}^{-1/2}, \bar{P}^{-1/2}\bar{Y}\bar{P}^{-1/2})\, d\bar{X}\, d\bar{Y} \qquad \ldots(2.16)$$

Which, under the transformation

$\bar{X}_2 = 4\bar{P}^{-1/2}\bar{X}\bar{P}^{-1/2}$, $\bar{Y}_2 = \bar{P}^{-1/2}\bar{Y}\bar{P}^{-1/2}$ with $d\bar{X}_2 = |\bar{P}|^{-p} d\bar{X}$,

$d\bar{Y}_2 = |\bar{P}|^{-p} d\bar{Y}$ and $|\bar{X}_2| = |\bar{P}|^{-1}|\bar{X}|$, $|\bar{Y}_2| = |\bar{P}|^{-1}|\bar{Y}|$

and then using the definition (1.4) leads us to the same result as in eq. (2.15)

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