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ON FUZZY COMPLETELY PRIME HYPER BI-Γ-IDEALS IN Γ-HYPERNEAR –RINGS

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ABSTRACT

In this paper, we introduce completely prime hyper bi-Γ-ideals in Γ-hypernear-rings and completely prime radical and obtain their properties. Also we introduce fuzzy completely prime hyper bi-Γ-ideals in Γ-hypernear –rings and investigated some of their properties.

1.Introduction

The concept of Γ-ring was introduced by Nobusawa and generalized by Barnes [1]. The concept of Γ-near-ring, a generalization of both near–ring and Γ-ring was introduced by Satyanarayana [6], subsequently, the ideal theory of Γ- near-rings was developed by authors like Satyanarayana [6] and G.L.Booth [3]. N.Meenakumari and T.Tamilzh Chelvam [4] introduced Fuzzy bi-ideals in gamma-near-rings. N.Meenakumari and T.Tamilzh Chelvam [5] introduced C-Prime bi-ideals in gamma-near-rings. Bijan Davvaz, Jianming Zhan and Kyung Ho kim [2] introduced Γ-hypernear-ring which is the generalization of hypernear-rings. In this paper, we introduce completely prime hyper bi-Γ-ideals in Γ-hypernear-rings and completely prime radical and obtain their properties. Also we introduce fuzzy completely prime hyper bi-Γ-ideals in Γ-hypernear –rings and investigated some of their properties.

2.Preliminaries

Definiation 2.1

A Γ-hyper-near-ring is a triple (M,+, Γ)

I. Γ- is a non empty set of binary operation such that (M, +,α) is a hyper near-ring for each α Є Γ
II. x α (yβ z) = (xαy) β z for all x, y, z Є M and α, β Є Γ

Definition 2.2

A subset A of M is called left (respectively right) Γ-hyperideal of M if it satisfies

I. A is a normal sub hypergroup of (M, +)
II. u α x Є A (res. (u+ x) α v- u α v Є A) for all x Є A, α Є Γ, and u, v Є M.

Definition 2.3

A subset A of M is called a Γ- hyperideal of M if it is both left Γ-hyperideal and right Γ-hyperideal.

Definition 2.4

A hyper subgroup H of (M, +) is called a hyper bi-Γ- ideal if $H\Gamma M \Gamma H \subseteq H$.

Definition 2.5

A fuzzy set μ of M is called a fuzzy hyper bi-Γ-ideal of M if

I. $\text{Inf}_{z \in x-y}\ \mu(z) \geq \min\{\mu(x), \mu(y)\}$
II. $\mu(x\ \alpha\ y\ \beta\ z) \geq \min\{\{\mu(x), \mu(z)\}$ for all x, y, z Є M, α, β Є Γ

3. Completely prime Γ - hyperideal

Definintion.3.1

An Γ - hyperideal(hyper bi-Γ- ideal) I of a Γ - hypernear-ring is said to be completely prime Γ - hyperideal(hyper bi-Γ- ideal) if a, b Є M and γ Є Γ and a γ b Є I $\Rightarrow$ a Є I or b Є I.

Definition 3.2

An Γ - hyperideal I of a Γ - hypernear-ring is said to be completely semi prime Γ- hyperideal if a Є M, a γ a ЄI $\Rightarrow$ a ЄI

Definition 3.3

Let A be proper Γ- hyperideal of M. The intersection of all completely prime Γ- hyperideals of M containing A is called compeletely prime hyper radical of A and is denoted by C- rad (A).

Therorem 3.4

C- rad (A) is Γ-hyperideal of M.

Proof:

(i) Let x Є rad(A), n Є M

x Є rad(A) = ∩P

x Є P for all P

Since P is a normal sub hypergroup, this implies that

n+ x-n Є P for all P, n Є M

$\Rightarrow$ n +x-n Є ∩P, n Є M

$\Rightarrow$ n+x-nЄ rad(A), n Є M.

(ii) Assume that x Є rad(A) , α Є Γ, u Є M

x Є rad (A) = ∩P

$\Rightarrow$ x Є P for all P

Now x Є P, α Є Γ u Є M

Since P is a left Γ-hyperideal,

$\Rightarrow$ u α x Є P for all P

$\Rightarrow$ u α x Є ∩P

$\Rightarrow$u α x Є rad (A)

(iii) Let x Є rad(A) α Є Γ, u, v Є M

Now x Є rad(A)= ∩P

$\Rightarrow$x Є P for all P

Since P is a right Γ-hyperideal,

(u+ x) α v – u α v Є P, for all P

$\Rightarrow$ (u+ x) α v – u α v Є ∩P

(u+ x) α v – u α v Є rad(A)

Hence C-rad (A) is a Γ- hyperideal.

Proposition 3.5

C-rad (A) is completely prime Γ- hyperideal.

Proof:

Let a γ b Є rad(A) = ∩P

$\Rightarrow$a γ b Є ∩P

$\Rightarrow$a γ b ЄP for all P

But P is completely prime Γ- hyperideal

$\Rightarrow$ either a Є P or b ЄP for all P

$\Rightarrow$ either a Є ∩P or b Є ∩P

$\therefore$ either a Є C- rad(A) or b Є C- rad(A)

Hence C-rad (A) is completely prime Γ- hyperideal

Theorem 3.5:

Let A be a Γ- hyperideal of M then C- rad(A) is a completely semi prime Γ-hyper ideal

Proof:

Let S= C- rad(A) = ∩P

Let a γ a ЄC- rad(A)

⇒a γ a Є∩P,

⇒a γ a Є P, for all P

Since P is completely prime Γ-hyperideal

A ЄP, for all P

⇒a Є∩P

⇒a Є C-rad(A)

Hence C-rad (A) is a completely semi prime Γ- hyperideal.

4.Fuzzy Completely prime hyper bi-Γ -ideal

Definition 4.1

A fuzzy hyper bi- Γ- ideal μ of M is called completely prime if for all x, y Є M, γ Є Γ, μ(x γ y) ≤ max {μ(x), μ(y)}.

Definition 4.2

Let μ be a fuzzy hyper bi- Γ- ideal of M. The fuzzy completely prime hyper radical of μ, denoted by √μ, is defined by √μ = ∩ { θ / θ is a fuzzy completely prime hyper bi- Γ- ideal of M containing μ}

Lemma 4.3

Let μ be a fuzzy subset of M. Then μ is a fuzzy hyper bi- Γ- ideal of M if and only if for all t Є [0, 1] each level subset μ_t, is a hyper bi-Γ- ideal of M.

Proposition 4.4

Let μ be a fuzzy subset of M. Then μ is a completely prime fuzzy hyper bi- Γ-ideal of M if and only if for all t Є [0, 1] each level subset μ_t, is a completely prime hyper bi-Γ- ideal of M.

Proof:

Suppose that μ be a completely prime fuzzy hyper bi-Γ- ideal of M. Then μ is a fuzzy hyper bi-Γ- ideal of M. By Lemma 4.3, μ_t is a hyper bi-Γ- ideal of M. Let x, y Є M and γ Є Γ such that x γ y Є μ_t. Then μ (x γ y) $\geq$ t. Since μ_t is a completely prime fuzzy hyper bi-Γ- ideal of M, we have μ(x γ y) $\leq$ max {μ(x), μ(y)} Thus max {μ(x), μ(y)}$\geq$ t which implies that μ(x) $\geq$ t or μ(y)$\geq$ t. Thus x Є μ_t or y Є μ_t.

Conversely assume that μ_t is a completely prime hyper bi-Γ- ideal of M for any t Є [0, 1]. Then μ_t is a hyper bi- Γ-ideal of M. Again by lemma 4.3, μ is a fuzzy hyper bi-Γ- ideal of M. Let x, y Є M, γ ЄΓ such that μ(x γ y) = t. Since μ_t is a completely prime bi- ideal of M and x γ y Є μ_t, we have x Є μ_t or y Є μ_t which implies that μ(x) $\geq$t or μ(y) $\geq$ t.Thus μ(x γ y) $\leq$ max {μ(x), μ(y)}. Hence μ is a completely prime fuzzy hyper bi-Γ- ideal of M.

Lemma 4.5

Let H be a non- empty subset of M. Then H is a hyper bi-Γ- ideal of M if and only if the characteristic function μ_H of H is a fuzzy hyper bi-Γ- ideal of M.

Proposition 4.6

Let H be a non- empty subset of M. Then H is a completely prime hyper bi-Γ- ideal of M if and only if μ_H is a completely prime fuzzy hyper bi-Γ- ideal of M.

Proof:

Suppose that H is a completely prime hyper bi-Γ-ideal of M and μ_H is the characteristic function of H . Then by Lemma 4.5, μ_H is a fuzzy hyper bi-Γ- ideal of M. Let x, y Є M and γ Є Γ. If x γ y Є H, then μ_H (x γ y) =1. Since H is a completely prime hyper bi-Γ- ideal of M and x γ y Є H, we have x Є H or y Є H. Thus μ_H (x) = 1 or μ_H (y) =1 which implies that μ_H (x γ y) $\leq$ max{μ_H (x), μ_H (y)}. If x γ y $\notin$ H, then μ_H (x γ y) = 0. Thus μ_H (x γ y) $\leq$ max {μ_H (x), μ_H (y)}. Conversely assume that μ_H is a completely prime fuzzy hyper bi-Γ- ideal of M. Then μ_H is a fuzzy hyper bi-Γ- ideal of M. By Lemma 4.5, H is a hyper bi-Γ- ideal of M. Let x, y Є M be such that x γ y Є H.

Then μ_H (x γ y) = 1. Since μ_H (x γ y) $\leq$max{μ_H (x), μ_H (y)}, we have max{μ_H (x), μ_H (y)} = 1. Thus μ_H (x) = 1 or μ_H (y) = 1. Hence x Є H or y Є H.

Proposition 4.7

If H is a hyper bi-Γ- ideal of M then for any t Є (0,1), there exists a fuzzy hyper bi-Γ- ideal μ of M such that μ_t = H .

Proposition 4.8

Let H be a completely prime hyper bi-Γ- ideal of M. For any t Є (0,1), there exists a completely prime fuzzy hyper bi-Γ- ideal of M such that μ_t = H .

Proof:

Let t Є (0,1). Then by Proposition 4.6, there exists a fuzzy hyper bi-Γ- ideal μ of M defined by

$$\mu(x) = \begin{cases} t & \text{if } x \in B \\ 0 & \text{otherwise} \end{cases}$$

such that μ_t = H. If possible suppose μ is not a completely prime fuzzy hyper bi-Γ- ideal of M. Then there exists x, y Є M and some γ Є Γ such that μ(x γ y) ≥ max{μ(x), μ(y)}. Using the definition of μ , we get μ(x) = 0, μ(y) = 0 and μ(x γ y) = t. Thus we get x γ y Є H and x ∉ H and y ∉ H. This is a contradiction since H is a completely prime hyper bi-Γ- ideal of M. Hence μ is a completely prime fuzzy hyper bi-Γ- ideal of M.

Now we omit the proofs which are straightforward.

Proposition 4.9

Let μ be a fuzzy subset of M with Im(μ) = {1,α} where α Є [0,1). Then μ is a completely prime fuzzy hyper bi-Γ- ideal of M if and only if M_μ ={x Є M / μ(x) = μ(0)} is a completely prime hyper bi-Γ- ideal of M.

Proposition 4.10

Let f : M →N be a homomorphism. If μ is a completely prime hyper bi-Γ- ideal of M, then $f^{-1}(\mu)$ is a completely prime hyper bi-Γ- ideal of M.

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