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ON b*g^- CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

*In this paper, we have introduced a new class of sets called b*g^ - closed sets in topological spaces. A subset A of X is said to be b*g^ - closed if b*cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g^-open in X. Also we study some of its properties and investigate the relationship with other existing closed sets in topological spaces. As an application, we introduce a new space namely* $T_{b*g\wedge}$*-space.*

Keywords: b*g^ - closed sets, g^ - open sets, b* - closure, b* - closed sets

1. Introduction

D. Andrijevic[2] introduced b-open sets in topology and studied its properties. b*-closed sets have been introduced and investigated by Muthuvel[9]. N. Levine[8] introduced generalized closed (briefly g-closed) sets and studied their basic properties. M.K.R.S.Veerakumar[16] defined g^-closed sets in topological space and studied their properties.

Now, we introduce the concept of b*g^-closed sets and b*g^-open sets in topological space and study some of their properties. Applying these sets we obtain new space namely $T_{b*g\wedge}$ space.

2. Preliminaries

Throughout this paper (X,τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned for a subset A of (X,τ), cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition: 2.1

A subset A of a topological space (X,τ) is called

a) a semi-open[7] set if $A \subseteq cl(int(A))$

b) an α-open set[12] if $A \subseteq int(cl(int(A)))$

c) a b-open set[2] if $A \subseteq cl(int(A)) \cup int(cl(A))$

d) a regular open[14] set if $A = int(cl(A))$

The complement of a semi- open (resp. α - open, b - open, regular open) set is called semi-closed (resp. α-closed, b-closed, regular closed) set.

The intersection of all semi -closed (resp. α - closed, b-closed, regular- closed) sets of X containing A is called the semi-closure (resp. α - closure, b-closure, regular closure) of A and is denoted by scl(A) (resp. αcl(A), bcl(A), rcl(A)). The family of all semi-open (resp. α-open, b-open, regular-open) subsets of a space X is denoted by SO(X) (resp. αO(X), bO(X), rO(X)).

Definition: 2.2

1) a generalized closed set (briefly g-closed)[8] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X

2) a gs-closed set[3] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

3) a gb-closed set[1] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

4) a rb-closed set[11] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in X.

5) a gr*-closed set[6] if $rCl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

6) a g^-closed set[16] if Cl(A) whenever $A \subseteq U$ and U is semi-open in X.

7) a bg^-closed set[15] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^-open in X.

8) a g*s-closed set[13] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X.

9) a (gs)*-closed set[5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X.

10) a r*bg*-closed set[4] if $rbCl(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in X.

Definition: 2.3 A space (X,τ) is called a

a) T_b-space[3]], if every gs-closed set in it is closed.

b) T_{gs}-space[1], if every gb-closed set in it is b-closed.

c) $T_{bg\wedge}$-space[15], if every bg^-closed set in it is b-closed.

d) $T^*_{bg\wedge}$-space[15], if every bg^-closed set in it is closed.

3. b*g^ - closed sets

We introduce the following definition.

Definition: 3.1 A subset A of a topological space (X,τ) is called a b*g^ -closed set if b*cl(A) $\subseteq$ U whenever A $\subseteq$ U and and U is g^-open in X. The family of all b*g^-closed sets of X are denoted by b*g^-C(X).

Definition: 3.2 The complement of a b*g^-closed set is called b*g^-open set. The family of all b*g^-open sets of X are denoted by b*g^-O(X).

Example: 3.3 Let X={a, b, c} and τ= {X, Φ, {a}, {b,c}} then {X, Φ,{a}, {b,c}} are b*g^-closed sets and {X, Φ, {b,c}, {a}} are b*g^-open sets in X.

Proposition: 3.4 Every closed set is b*g^-closed set.

Proof: Let A be any closed set in X and U be any g^-open set such that A $\subseteq$ U. Since A is closed, cl(A)=A. Therefore, b*cl(A) $\subseteq$ cl(A) = A $\subseteq$ U. Hence A is b*g^-closed set.

The following example shows that the converse of the above proposition need not be true.

Example: 3.5 Let X={a, b, c}, τ= {X, Φ, {a}, {b}, {a,b}}

C(X)={X, Φ, {b,c}, {a,c}, {c}}

b*g^-C(X)={X, Φ, {a}, {b}, {c}, {b,c}, {a,c}}.

Here {a}, {b} are b*g^-closed sets but not closed sets in X.

Proposition: 3.6

i) Every semi-closed set is b*g^-closed

ii) Every α-closed set is b*g^-closed.

iii) Every regular-closed set is b*g^-closed.

Proof: i) Let A be any semi-closed set in X such that $A \subseteq U$ where U is g^-open. Since A is semi-closed, $b^*cl(A) = scl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence, A is b*g^-closed.

ii) Let A be any α-close set in X such that $A \subseteq U$ where U is g^-open. Since A is α-closed, $b^*cl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence A is b*g^-closed.

iii) Let A be any regular-closed set in X such that $A \subseteq U$. Where U is g^-open. Since, A is regular-closed $b^*cl(A) \subseteq rcl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence, A is b*g^-closed.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.7 Let X= {a, b, c} and τ = {X, Φ, {a}}

S-C(X)={X, Φ, {b,c}, {c}, {b}}

α-C(X)={X, Φ, {b,c},{c}, {b}}

r-C(X)={X, Φ}

b*g^-C(X)={X, Φ, {b}, {c}, {a,b}, {b,c}, {a,c}}.

Here, {a,b}, {a,c} are b*g^-closed sets but not semi-closed, α-closed, regular-closed.

Proposition: 3.8 Every b*g^-closed set is gs-closed.

Proof: Let A be any b*g^-closed set and U be any open set such that $A \subseteq U$. Since "Every open set is g^-open set" we have $scl(A) \subseteq b^*cl(A) \subseteq U$. Therefore, $scl(A) \subseteq U$ where U is open in X. Hence, A is gs-closed.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.9 Let X={a, b,c}, τ={X, Φ, {a,b}, {c}}

b*g^-C(X)={X, Φ, {a,b}, {c}}

gs-C(X)={X, Φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}}.

Here {a}, {b}, {b,c}, {a,c} are gs-closed sets but not b*g^-closed sets.

Proposition: 3.10 Every b*g^-closed set is bg^-closed.

Proof: Let A be any b*g^-closed set in X and U be any g^-open set such that $A \subseteq U$. Now $bcl(A) \subseteq b^*cl(A) \subseteq U$. Therefore, $bcl(A) \subseteq U$ where U is g^-open in X. Hence A is bg^-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.11 Let X={a, b, c}, τ={X, Φ, {a,c}}

b*g^-C(X)={X, Φ, {b}, {a,b}, {b,c}}

bg^-C(X)={ X, Φ, {a}, {b}, {c}, {a,b}, {b,c}}.

Here {a}, {c} are bg^-closed sets but not b*g^-closed sets.

Proposition: 3.12 Every b*g^-closed set is gb-closed set.

Proof: Let A be any b*g^-closed set in X and U be any open set such that $A \subseteq U$. Since "Every open set is g^-open set" we have $bcl(A) \subseteq b^*cl(A) \subseteq U$ where U is open in X. Hence A is gb-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.13 Let X={a, b, c}, τ={X, Φ, {a}, {b,c}}

gb-C(X)={X, Φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}}

b*g^-C(X)={X, Φ, {a}, {b,c}}.

Here {b}, {c}, {a,b}, {a,c} are gb-closed sets but not b*g^-closed sets.

Proposition: 3.14 Every r*bg*-closed set is b*g^-closed set.

Proof: Let A be any r*bg*-closed set in X and U be any g^-open set in X such that $A \subseteq U$. Now, $b^*cl(X) \subseteq rbcl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence, A is b*g^-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.15 Let X= {a, b, c}, τ={X, Φ, {a}, {a,b}}

b*g^-C(X)={X, Φ, {b}, {c}, {b,c}, {a,c}}

r*bg*-C(X)={X, Φ, {b,c}}.

Here {b}, {c}, {a,c} are b*g^-closed sets but not r*bg*-closed sets.

Proposition: 3.16 Every gr*-closed set is b*g^-closed set.

Proof: Let A be any gr*-closed set and U be any g^-open set such that $A \subseteq U$. Since "Every g^-open set is g-open" we have $b^*cl(A) \subseteq rcl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$ where U is g^-open in X. Hence, A is b*g^-closed set.

The converse of the above proposition need not be true.

Example: 3.17 Let X={a, b, c}, τ={X, Φ, {b}}

b*g^-C(X)={X, Φ, {a}, {c}, {a,c}}

gr*-closed= { X, Φ, {a,c}}.

Here {a}, {c} are b*g^-closed sets but not gr*-closed sets.

Proposition: 3.18 Every g*s-closed set is b*g^-closed set.

Proof: Let A be any g*s-closed set in X and U be any g^-open set such that $A \subseteq U$. Since, "Every g^-open is gs-open" we have $b^*cl(A) = scl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence A is b*g^-closed set.

The converse of the above proposition need not be true.

Example: 3.19 Let X={a, b, c, d}, τ={X, Φ, {b}, {a,b}, {b, c, d}}

b*g^-C(X)= {X, Φ, {a}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a, b, c}, {a, b, d}, {a, c, d}}

g*s-C(X)= {X, Φ, {a}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a, c, d}}.

Here, {a, b, c}, {a, b, d} are b*g^-closed sets but not g*s-closed sets.

Proposition: 3.20 Every (gs)*-closed set is b*g^-closed set.

Proof: Let A be any (gs)*-closed set and U be any g^-open set such that $A \subseteq U$. Since, "Every g^-open is gs-open" we have $b^*cl(A) \subseteq cl(A) \subseteq U$. Therefore, $b^*cl(A) \subseteq U$. Hence A is b*g^-closed.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.21 Let X= {a,b,c,d}, τ= {X, Φ, {a}, {a,c}, {a, b, d}

b*g^-C(X)={X, Φ, {b}, {c}, {d}, {b,c}, {c,d}, {b,d}, {a, b, c}, {a, c, d}, {b, c, d}}

(gs)*-C(X)={X, Φ, {c}, {b, c, d}, {b,d}}.

Here {b}, {d}, {b,c}, {c,d}, {a, b, c}, {a, c, d} are b*g^-closed sets but not (gs)*-closed sets.

Proposition: 3.22 Every rb-closed set is b*g^-closed set.

Proof: Let A be any rb-closed set and U be any g^-open set such that A $\subseteq$ U. Now, b*cl(A) $\subseteq$ rcl(A) $\subseteq$ U. Therefore, b*cl(A) $\subseteq$ U. Hence A is b*g^-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example: 3.23 Let X= {a, b, c}, τ= {X, Φ, {a}, {a,b}, {a,c}}

rb-C(X)={X, Φ, {c}, {b,c}}

b*g^-C(X)= {X, Φ, {b}, {c}, {b,c}}.

Here {b} are b*g^-closed sets but not rb-closed sets.

Proposition: 3.24 Every r*g*-closed set is b*g^-closed set.

Proof: Let A be any r*g*-closed and U be any g^-open set such that A $\subseteq$ U. Since, "Every g^-open set is g-open set" we have b*cl(A) $\subseteq$ rcl(A) $\subseteq$ U. Therefore, b*cl(A) $\subseteq$ U. Hence, A is b*g^-closed set.

The converse of the proposition need not be true as shown by the following example..

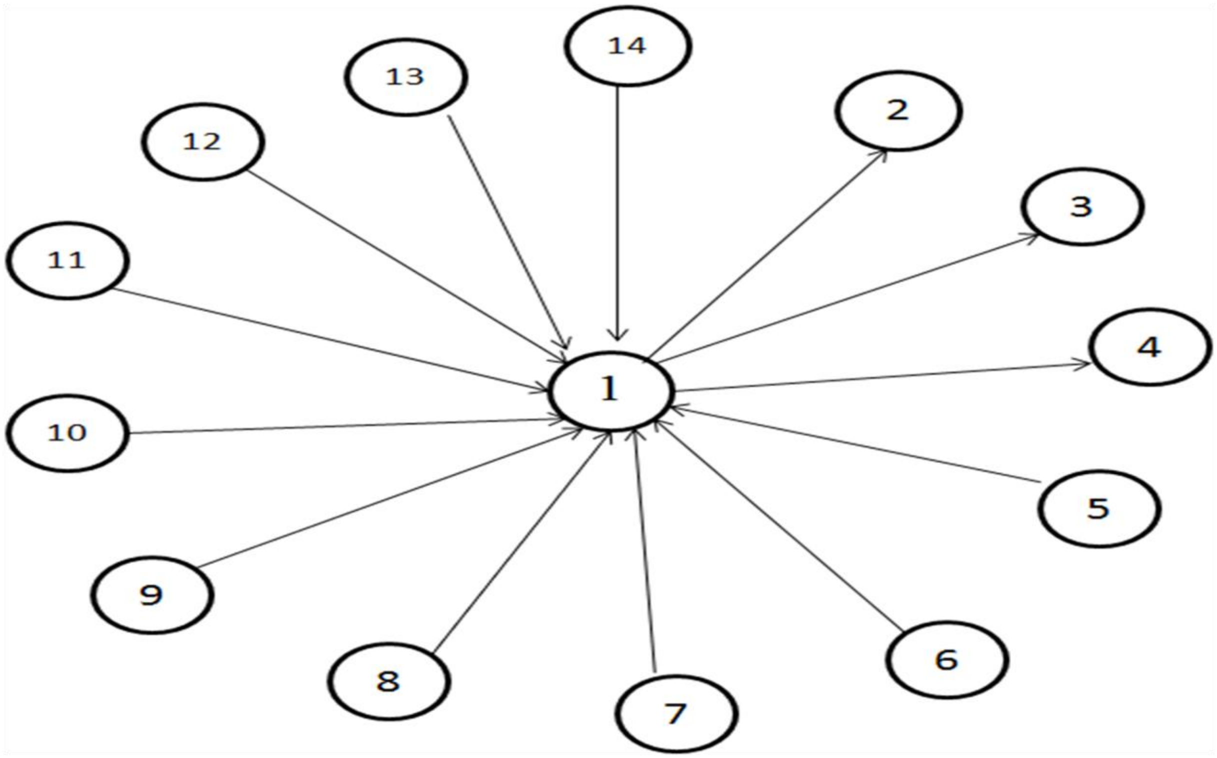
Example: 3.25 Let X={a, b, c}, τ= {X, Φ, {b}}

b*g^-C(X)={X, Φ, {a}, {c}, {a,c}}

r*g*-C(X)={X, Φ, {a,c}}.

Here {a}, {c} are b*g^-closed sets but not r*g*-closed sets.

Remark: 3.26 The following diagram shows the relationship of b*g^-closed sets with other known existing sets A→B represents A implies B but not conversely.



1. b*g^-closed set	2. closed	3.semi-closed	4. α-closed
5. regulr closed	6. gs-closed	7. bg^-closed	8.gb-closed
9. r*bg*-closed	10. gr*-closed	11. g*s-closed	12.(gs)*-closed
13. rb-closed	14. r*g*-closed		

4. CHARACTERIZATION

Lemma: 4.1 The finite union of b*g^-closed set need not be b*g^-closed set.

Example: 4.2 Let X= {a, b, c}, τ= {X, Φ, {a}, {b}, {a,b}}

b*g^-closed= {X, Φ, {a}, {b}, {c}, {a,c}, {b,c}}.

Here {a} ∪ {b}={a,b} is not b*g^-closed set.

Lemma: 4.3 The finite intersection of any two b*g^-closed set need not be b*g^-closed set.

Example: 4.4 Let X= {a, b, c} and τ={X, Φ, {a}}

b*g^-closed= {X, Φ, {b}, {c}, {b,c}, {a, b}, {a,c}}

Here {a,b} ∩ {a,c}= {a} is not b*g^-closed set.

Proposition: 4.5 Let A be a b*g^-closed set of X. Then b*cl(A)-A does not contain a non-empty g^-closed set.

Proof: Suppose A is a b*g^-closed set. Let F be a g^-closed set contained in b*cl(A)-A. Now, F^c is a g^-open set of X such that A ⊆ F^c. Since, A is b*g^-closed set we have b*cl(A) ⊆ F^c. Hence F ⊆ $(b*cl(A))^c$. Also F ⊆ b*cl(A)-A. Therefore, F ⊆ b*cl(A) ∩ $(b*cl(A))^c$ = Φ. Hence, F must be empty.

Proposition: 4.6 If A is g^-open and b*g^-closed set of X, then A is b*-closed.

Proof: Since A is g^-open and b*g^-closed. We have b*cl(A) ⊆ A. Hence, A is b*-closed.

Proposition: 4.7 The intersection of a b*g^-closed set and a b*-closed set of X is always b*g^-closed set.

Proof: Let A be a b*g^-closed set and B be a b*-closed set. Since, A is b*g^-closed, b*cl(A) ⊆ U whenever U is g^-open. Let B be such that A∩B ⊆ U where U is g^-open. Now, b*cl(A∩B) ⊆ b*cl(A) ∩ b*cl(B) ⊆ U ∩ B ⊆ U. Hence A ∩ B is b*g^-closed set. Therefore, intersection of any b*g^-closed set and a b*-closed set of X is always b*g^-closed set.

5. APPLICATIONS

As an application of b*g^-closed sets, we introduce a new space namely $T_{b*g^\wedge}$ - space.

Definition: 5.1

A space (X,τ) is called a $T_{b*g\wedge}$- space, if every b*g^-closed set in X is closed.

Proposition: 5.2

Every T_b- space is $T_{b*g\wedge}$- space.

Proof:

Let (X,τ) be T_b- space. Let A be b*g^-closed set in X. By proposition: 3.8, "Every b*g^-closed set is gs-closed", A is gs-closed. Since (X,τ) is T_b- space, A is closed. Hence (X,τ) is $T_{b*g\wedge}$- space.

The converse of the above proposition need not be true as shown in the following example.

Example: Let X={a, b, c}, τ={X, Φ, {a,b}, {c}}

b*g^-C(X)={X, Φ, {c}, {a,b}}

gs-C(X)= {X, Φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}}

C(X)= {X, Φ, {c}, {a,b}}

Hence (X, τ) is $T_{b*g\wedge}$- space but not T_b- space.

Proposition: 5.3

Every $T^*_{bg\wedge}$ - space is $T_{b*g\wedge}$- space

Proof:

Let (X,τ) be $T^*_{bg\wedge}$ - space. Let A is b*g^-closed. Since "Every b*g^-closed set is bg^-closed". Since, X is $T^*_{bg\wedge}$, A is closed. Therefore, (X,τ) is $T_{b*g\wedge}$ - space.

The converse of the above proposition need not be true as shown by the following example.

Example: Let X={a, b, c}, τ={X, Φ, {a}, {b,c}}

b*g^-C(X)= {X, Φ, {a}, {b,c}}

bg^-C(X)= {X, Φ, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}}

$C(X)= \{X, \Phi, \{a\}, \{b,c\}\}$

Hence (X, τ) is $T_{b*g\wedge}$- space but not $T^{*}_{bg\wedge}$- space.

Proposition: 5.4

Every $T_{b*g\wedge}$- space is T_{gs}- space.

Proof:

Let (X,τ) be $T_{b*g\wedge}$- space. Let A be b*g^-closed set in (X,τ). By proposition: 3.12, "Every b*g^-closed set is gb-closed", A is gb-closed. Since "Every closed set is b-closed", A is b-closed set in X. Therefore, (X,τ) is T_{gs}-space.

The converse of the above proposition need not be true as shown in the following example.

Example: Let $X= \{a, b, c\}$, $\tau= \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$

$b*g\wedge\text{-}C(X)= \{X, \Phi, \{a\}, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$

$gb\text{-}C(X)= \{X, \Phi, \{a\}, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$

$C(X)= \{ X, \Phi, \{c\}, \{b,c\}, \{a,c\}\}$

Hence, (X,τ) is T_{gs}- space but not $T_{b*g\wedge}$- space.

Proposition: 5.5

Every $T_{b*g\wedge}$- space is $T_{bg\wedge}$- space.

Proof:

Let (X,τ) be $T_{b*g\wedge}$ - space. Let A be b*g^ closed set in (X,τ). By Proposition: "Every closed set is bg^-closed set, A is bg^-closed. Since, "Every closed set is b-closed set". A is b-closed set in X. Therefore, (X,τ) is $T_{bg\wedge}$ - closed.

The converse of the above proposition need not be true as shown in the following example.

Example: Let $X= \{a, b, c\}$, $\tau= \{X, \Phi, \{a,c\}\}$

$b*g\wedge\text{-}C(X)= \{X, \Phi, \{b\}, \{a,b\}, \{b,c\}\}$

b-C(X)= { X, Φ, {a}, {b}, {c}, {b,c}, {a,b}}

C(X)= { X, Φ, {b}}

Hence, (X,τ) is $T_{bg\wedge}$- space but not $T_{b*g\wedge}$- space.

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