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INTUITIONISTIC FUZZY B-IDEAL ON B-ALGEBRAS

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ABSTRACT

In this paper, we introduce the concept of intuitionistic fuzzy B-ideals in B-algebra. Homomorphism and anti-homomorphism functions are satisfied while applying the intuitionistic fuzzy B-ideal concept. Intuitionistic Fuzzy B-ideal is also applied in Cartesian product.

Keywords: B-algebra, B-ideals, Fuzzy B-ideals, Intuitionistic fuzzy B-ideals, Homomorphism, Anti-homomorphism, Cartesian product.

1. INTRODUCTION

After the introduction of fuzzy subsets by L.A.Zadeh[7], several researchers explored on the generalization of the notion of fuzzy subset.J.R.Cho and H.S.Kim[4] discussed relations between B-algebras and other topics, especially quasi-groups. H.K.Park and H.S.Kin[5] introduced the notion of Quadratic B-algebras. Sun ShinAhn and KeumseongBang[3] have discussed the fuzzy subalgebra in B-algebra.C.Yamini and S.Kailasavalli[1] introduced the notion of Fuzzy B-ideals.Atanassov[6] introduced the concept of intuitionistic fuzzy sets, which is a significant extension of fuzzy set theory by Zadeh. JiayinPeng[2] introduced the notion of Intuitionistic fuzzy B-algebras. In this paper we introduce the Intuitionistic fuzzy B-ideals and investigate how to deal with the homomorphism, anti-homomorphism, Cartesian product of Intuitionistic fuzzy B-ideals and strongest intuitionistic fuzzy relation.

2. PRELIMINARIES

In this section we give some basic definitions and preliminaries of B-algebras and introduce intuitionistic fuzzy B-ideals.

Definition 2.1: A B-algebra is a non-empty set X with a constant 0 and a binary operation '*' satisfying the following axioms:

(i) $x * x = 0$

(ii) $x * 0 = x$

(iii) $(x * y) * z = x * (z * (0 * y)), for\ all\ x, y, z \in X$

For brevity we also call X a B-algebra. In X we can define a binary relation "≤" by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2:A non-empty subset I of a B-algebra X is called a subalgebra of X if $x * y \in I$ for any $x, y \in I$.

Definition 2.3: Let α be a fuzzy set in a B-algebra. Then α is called a fuzzy subalgebra of X if $\alpha(x * y) \geq \alpha(x) \wedge \alpha(y)\ for\ all\ x, y \in X$.

Definition 2.4: An intuitionistic fuzzy set $S = \{< x, \alpha_S(x), \beta_S(x) >/x \in X\}$ of X is said to be an intuitionistic fuzzy B-algebra if it satisfies

$$\alpha_S(x * y) \geq \alpha_S(x) \wedge \alpha_S(y)$$

$$\beta_S(x * y) \leq \beta_S(x) \vee \beta_S(y)$$

Definition 2.5: A non-empty subset I of a B-algebra X is called a B-ideal of X if it satisfies for $x, y, z \in X$

(i) $0 \in I$

(ii) $(x * y) \in I and\ (z * x) \in I$ implies $(y * z) \in I$

Definition 2.6: Let $(X, * ,0)$ be a B-algebra, a fuzzy set α in X is called a fuzzy B-ideal of X if it satisfies the following axioms

(i) $\alpha(0) \geq \alpha(x)$

(ii) $\alpha(y * z) \geq \alpha(x * y) \wedge \alpha(z * x), for\ all\ x, y, z \in X$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The fuzzy set α given by $\alpha(0) = 0.8, \alpha(1) = 0.5, \alpha(2) = 0.2$ is a fuzzy B-ideal.

Definition 2.7: An intuitionistic fuzzy set S of a B-algebra X is said to be an intuitionistic fuzzy B-ideal if it satisfies the following conditions

(i) $\alpha_S(0) \geq \alpha_S(x)$

(ii) $\beta_S(0) \leq \beta_S(x)$

(iii) $\alpha_S(y * z) \geq \alpha_S(x * y) \wedge \alpha_S(z * x)$

(iv) $\beta_S(y * z) \leq \beta_S(x * y) \vee \beta_S(z * x), for\ all\ x, y, z \in X$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The intuitionistic fuzzy subset S given by $\alpha_S(0) = 0.9, \alpha_S(1) = 0.6, \alpha_S(2) = 0.3$ and $\beta_S(0) = 0.1, \beta_S(1) = 0.4, \beta_S(2) = 0.7$ is an intuitionistic fuzzy B-ideal.

3. HOMOMORPHISM AND ANTI HOMOMORPHISM OF B-ALGEBRA

Definition 3.1: Let $(X, *, 0)$ and $(Y, \Delta, 0')$ be B-algebras. A mapping $f: X \to Y$ is called a homomorphism if $f(x * y) = f(x)\Delta f(y), for\ all\ x, y \ \in X$

Definition 3.2: Let $(X, *, 0)$ and $(Y, \Delta, 0')$ be B-algebras. A mapping $f: X \to Y$is called a antihomomorphismif $f(x * y) = f(y)\Delta f(x), for\ all\ x, y \ \in X.$

Definition 3.3: For any homomorphism $f: X \rightarrow Y$ the set $\{x \in x / f(x) = 0'\}$ is called the kernel of *f*, denoted by $Ker(f)$ and the set $\{f(x)/x \in X\}$ is called the image of 'f ' denoted by $Im(f)$.

Definition 3.4: Let $f: X \rightarrow Y$ be a mapping of B-algebras and α be a fuzzy set of Y. The map α^f is the pre-image of α under *f*, if $\alpha^f(x) = \alpha(f(x))\ for\ all\ x\ in\ X$.

Definition 3.5: Let $f: X \rightarrow Y$ be a mapping of B-algebras and S be an intuitionistic fuzzy set of Y. The map S^f is the pre-image of S under *f*, if

(i) $\alpha_S^f(x) = \alpha_S(f(x))$

(ii) $\beta_S^f(x) = \beta_S(f(x))\ for\ all\ x\ in\ X$

Definition 3.6: Let $f: X \rightarrow X$ be an endomorphism and α be a fuzzy set in X. We define a new fuzzy set in X by α^f in X as $\alpha_S^f(x) = \alpha_S(f(x))\ for\ all\ x\ in\ X$.

Definition 3.7: Let $f: X \rightarrow X$ be an endomorphism and S be an intuitionistic fuzzy set in X. We define a new fuzzy set in X by S^f in X as

$$\alpha_S^f(x) = \alpha_S(f(x))$$

$$\beta_S^f(x) = \beta_S(f(x))\ for\ all\ x\ in\ X$$

Theorem 3.8: Let *f*be an endomorphism of a B-algebra X. If S is an intuitionistic fuzzy B-ideal of X, then so s^f.

Proof:

(i) $\alpha_S^f(0) = \alpha_S(f(0))$

$\geq \alpha_S(f(x))$

$= \alpha_S^f(x)$

$\Rightarrow \alpha_S^f(0) \geq \alpha_S^f(x)$

(ii) $\beta_S^f(0) = \beta_S(f(0))$

$\leq \beta_S(f(x))$

$= \beta_S^f(x)$

$$\Rightarrow \beta_S^f(0) \leq \beta_S^f(x)$$

(iii) $\alpha_S^f(y * z) = \alpha_S(f(y * z))$

$= \alpha_S(f(y) * f(z))$

$\geq \alpha_S(f(x) * f(y)) \wedge \alpha_S(f(z) * f(x))$

$= \alpha_S(f(x * y)) \wedge \alpha_S(f(z * x))$

$= \alpha_S^f(x * y) \wedge \alpha_S^f(z * x)$

$$\Rightarrow \alpha_S^f(y * z) \geq \alpha_S^f(x * y) \wedge \alpha_S^f(z * x)$$

(iv) $\beta_S^f(y * z) = \beta_S(f(y * z))$

$= \beta_S(f(y) * f(z))$

$\leq \beta_S(f(x) * f(y)) \vee \beta_S(f(z) * f(x))$

$= \beta_S(f(x * y)) \vee \beta_S(f(z * x))$

$= \beta_S^f(x * y) \vee \beta_S^f(z * x)$

$$\Rightarrow \beta_S^f(y * z) \leq \beta_S^f(x * y) \vee \beta_S^f(z * x)$$

Thus, S^f is an intuitionistic fuzzy B-ideal of X.

Hence, the proof.

4. CARTESIAN PRODUCT OF INTUITIONISTIC B-IDEALS OF B-ALGEBRA

Definition 4.1: Let $S = \{\langle x, \alpha_S(x), \beta_S(x)\rangle / x \in X\}$ and $T = \{\langle y, \alpha_T(x), \beta_T(x)\rangle / x \in X\}$ be intuitionistic fuzzy sets of X. A Cartesian product of S and T defined by

$$S\ X\ T = \{\langle (x, y), \alpha_{SXT}(x, y), \beta_{SXT}(x, y)\rangle / x, y \in X\}$$

$$\alpha_{SXT} = \alpha_S(x) \wedge \alpha_T(x)$$

$$\beta_{SXT} = \beta_S(x) \vee \beta_T(x), \text{ where } x, y \in X$$

Definition 4.2: An intuitionistic fuzzy relation $R = \langle (x,y), \alpha_S(x,y), \beta_S(x,y)\rangle / x, y \in X\}$ on X is called an Intuitionistic fuzzy relation on S if

$$\alpha^R(x,y) \leq \alpha_S(x) \wedge \alpha_S(y)$$

$$\beta^R(x,y) \geq \beta_S(x) \vee \beta_S(y) \; for \; all \; x, y \in X$$

Definition 4.3: Let S be an intuitionistic fuzzy set in X. An intuitionistic fuzzy relation R on X is called a strongest intuitionistic fuzzy relation on S if

$$\alpha^R(x,y) = \alpha^R(x) \wedge \alpha^R(y)$$

$$\beta^R(x,y) = \beta^R(x) \vee \beta^R(y) \; for \; all \; x, y \in X$$

Theorem 4.4: For a subset S of a B-algebra X, let R be the strongest intuitionistic fuzzy relation on X. If S is an intuitionistic fuzzy B-ideal of XxX then

$$\alpha^R(x,x) \leq \alpha^R(0,0)$$

$$\beta^R(x,x) \geq \beta^R(0,0) \; for \; x \in X$$

Proof

Given: R is the strongest intuitionistic fuzzy relation of XxX, then

$$\alpha^R(x,x) = \alpha(x) \wedge \alpha(x)$$

$$\leq \alpha(0) \wedge \alpha(0)$$

$$= \alpha^R(0,0)$$

$$\Rightarrow \alpha^R(x,x) \leq \alpha^R(0,0)$$

$$\beta^R(x,x) = \beta(x) \vee \beta(x)$$

$$\geq \beta(0) \vee \beta(0)$$

$$= \beta^R(0,0)$$

$$\Rightarrow \beta^R(x,x) \geq \beta^R(0,0)$$

Hence the proof.

Theorem 4.5: Let S and T are intuitionistic fuzzy B-ideals in B-algebra X then SxT is an intuitionistic fuzzy B-ideal in XxX.

Proof

For any $(x, y) \in X \times X$,

$$(\alpha_S \times \alpha_T)(0,0) = \alpha_S(0) \wedge \alpha_T(0)$$

$$\geq \alpha_S(x) \wedge \alpha_T(x)$$

$$= (\alpha_S \times \alpha_T)(x,y)$$

$$\Rightarrow (\alpha_S \times \alpha_T)(0,0) \geq (\alpha_S \times \alpha_T)(x,y)$$

Similarly, we can also prove

$$(\beta_S \times \beta_T)(0,0) \leq (\beta_S \times \beta_T)(x,y)$$

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$

Now,

$$(\alpha_S \times \beta_S)\big((y_1, y_2) * (z_1, z_2)\big) = (\alpha_S \times \beta_S)(y_1 * z_1, y_2 * z_2)$$

$$= \alpha_S(y_1 * z_1) \wedge \beta_S(y_2 * z_2)$$

$$\geq \big(\alpha_S(x_1 * y_1) \wedge \alpha_S(z_1 * x_1)\big) \wedge \big(\beta_S(x_2 * y_2) \wedge \beta_S(z_2 * x_2)\big)$$

$$\geq \big(\alpha_S(x_1 * y_1) \wedge \beta_S(x_2 * y_2)\big) \wedge \big(\alpha_S(z_1 * x_1) \wedge \beta_S(z_2 * x_2)\big)$$

$$= \Big((\alpha_S \times \beta_S)\big((x_1 * y_1), (x_2 * y_2)\big)\Big) \wedge$$

$$\Big((\alpha_S \times \beta_S)\big((z_1 * x_1), (z_2 * x_2)\big)\Big)$$

$$\Rightarrow (\alpha_S \times \beta_S)((y_1, y_2) * (z_1, z_2))$$
$$\geq \Big((\alpha_S \times \beta_S)\big((x_1 * y_1), (x_2 * y_2)\big)\Big) \wedge \Big((\alpha_S \times \beta_S)\big((z_1 * x_1), (z_2 * x_2)\big)\Big)$$

Similarly, we can also prove

$$(\alpha_T \times \beta_T)((y_1, y_2) * (z_1, z_2))$$
$$\geq \Big((\alpha_T \times \beta_T)\big((x_1 * y_1), (x_2 * y_2)\big)\Big) \wedge \Big((\alpha_T \times \beta_T)\big((z_1 * x_1), (z_2 * x_2)\big)\Big)$$

Thus, $S \times T$ is an intuitionistic fuzzy B-ideal in $X \times X$.

Hence, the proof.

Result 4.6: Let S and T be intuitionistic fuzzy sets of a B-algebra X such that SxT is an intuitionistic fuzzy B-algebra of X. Then,

(i) Either $\alpha_S(x) \leq \alpha_S(0)$ or $\alpha_T(x) \leq \alpha_T(0)$ for all $x \in X$
(ii) Either $\beta_S(x) \geq \beta_S(0)$ or $\beta_T(x) \geq \beta_T(0)$ for all $x \in X$
(iii) If $\alpha_S(x) \leq \alpha_S(0)$ for all $x \in X$, then $\alpha_S(x) \leq \alpha_T(0)$ or $\alpha_T(x) \leq \alpha_T(0)$
(iv) If $\beta_S(x) \geq \beta_S(0)$ for all $x \in X$, then $\beta_S(x) \geq \beta_T(0)$ or $\beta_T(x) \geq \beta_T(0)$
(v) If $\alpha_T(x) \leq \alpha_T(0)$ for all $x \in X$, then $\alpha_S(x) \leq \alpha_S(0)$ or $\alpha_T(x) \leq \alpha_S(0)$
(vi) If $\beta_T(x) \geq \beta_T(0)$ for all $x \in X$, then $\beta_S(x) \geq \beta_S(0)$ or $\beta_T(x) \geq \beta_S(0)$

Theorem 4.7: If S and T are the intuitionistic fuzzy sets in B-algebra X such that SxT is an intuitionistic fuzzy B-ideal of XxX then either S or T is an intuitionistic fuzzy B-ideal of X.

Proof

Since, $\alpha_S(x) \leq \alpha_S(0)$ or $\alpha_T(x) \leq \alpha_T(0)$

Let $\alpha_S(x) \leq \alpha_S(0)$

And also, If $\alpha_S(x) \leq \alpha_S(0)$ for all $x \in X$, then $\alpha_S(x) \leq \alpha_T(0)$ or $\alpha_T(x) \leq \alpha_T(0)$

Let's take, $\alpha_S(x) \leq \alpha_T(0)$

$$\alpha_S(x) = \alpha_S(x) \wedge \alpha_T(0)$$

$= (\alpha_S \times \alpha_T)(x, 0)$

$\alpha_S(y * z) = \alpha_S(y * z) \wedge \alpha_T(0)$

$= (\alpha_S \times \alpha_T)\big((y * z), 0\big)$

$= (\alpha_S \times \alpha_T)\big((y, 0) * (z, 0)\big)$

$\geq \Big((\alpha_S \times \alpha_T)\big((x, 0) * (y, 0)\big)\Big) \wedge ((\alpha_S \times \alpha_T)\big((z, 0) * (x, 0)\big)$

$\geq \big((\alpha_S \times \alpha_T)(x * y, 0 * 0)\big) \wedge \big((\alpha_S \times \alpha_T)(z * x, 0 * 0)\big)$

$= \alpha_S(x * y) \wedge \alpha_S(z * x)$

$\Rightarrow \alpha_S(y * z) \geq \alpha_S(x * y) \wedge \alpha_S(z * x)$

Similarly,

We can prove $\beta_S(y * z) \leq \beta_S(x * y) \vee \beta_S(z * x)$

Hence, S is an intuitionistic fuzzy ideal of X.

Now let's prove T is an intuitionistic fuzzy ideal of X.

Assume, $\alpha_T(x) \leq \alpha_S(0)$

Then,

$\alpha_T(x) \quad = \alpha_S(0) \wedge \alpha_T(x)$

$= (\alpha_S \times \alpha_T)(0, x)$

$\alpha_T(y * z) = \alpha_S(0) \wedge \alpha_T(y * z)$

$= (\alpha_S \times \alpha_T)\big(0, (y * z)\big)$

$= (\alpha_S \times \alpha_T)\big((0, y) * (0, z)\big)$

$\geq \Big((\alpha_S \times \alpha_T)\big((0, x) * (0, y)\big)\Big) \wedge ((\alpha_S \times \alpha_T)\big((0, z) * (0, x)\big)$

$\geq \big((\alpha_S \times \alpha_T)(0 * 0, x * y)\big) \wedge \big((\alpha_S \times \alpha_T)(0 * 0, z * x)\big)$

$$\geq ((\alpha_S \times \alpha_T)(0, x * y)) \wedge ((\alpha_S \times \alpha_T)(0, z * x))$$

$$= \alpha_T(x * y) \wedge \alpha_T(z * x)$$

$$\Rightarrow \alpha_T(y * z) \geq \alpha_T(x * y) \wedge \alpha_T(z * x)$$

Similarly, we can also prove, $\beta_T(y * z) \leq \beta_T(x * y) \vee \beta_T(z * x)$

Thus, B is an intuitionistic fuzzy B-ideal of X.

Hence the proof.

Theorem 4.8: Let S be an Intuitionistic fuzzy set in a B-algebra X and S^R be the strongest fuzzy relation on X. Then S is an intuitionistic fuzzy B – ideal of X if and only if S^R is an intuitionistic fuzzy B-ideal of XxX.

Proof

Suppose that S is an intuitionistic fuzzy B-ideal of X.

Then

$$\alpha_S^R(0,0) = \alpha_S(0) \wedge \alpha_S(0)$$

$$\geq \alpha_S(x) \wedge \alpha_S(y)$$

$$= \alpha_S^R(x, y)$$

$$\Rightarrow \alpha_S^R(0,0) \geq \alpha_S^R(x, y), for\ all\ (x, y) \in X \times X$$

Now, for any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$

$$\alpha_S^R(y_1 * z_1, y_2 * z_2) = \alpha_S^R(y_1 * z_1) \wedge \alpha_S^R(y_2 * z_2)$$

$$\geq (\alpha_S(x_1 * y_1) \wedge \alpha_S(z_1 * x_1)) \wedge (\alpha_S(x_2 * y_2) \wedge \alpha_S(z_2 * x_2))$$

$$\geq (\alpha_S(x_1 * y_1) \wedge \alpha_S(x_2 * y_2)) \wedge (\alpha_S(z_1 * x_1) \wedge \alpha_S(z_2 * x_2))$$

$$= \alpha_S^R(x_1 * y_1, x_2 * y_2) \wedge \alpha_S^R(z_1 * x_1, z_2 * x_2)$$

$$\Rightarrow \alpha_S^R(y_1 * z_1, y_2 * z_2) \geq \alpha_S^R(x_1 * y_1, x_2 * y_2) \wedge \alpha_S^R(z_1 * x_1, z_2 * x_2)$$

Similarly, we can also prove

$$\beta_S^R(0,0) \leq \beta_S^R(x,y)$$

$$\beta_S^R(y_1 * z_1, y_2 * z_2) \leq \beta_S^R(x_1 * y_1, x_2 * y_2) \vee \beta_S^R(z_1 * x_1, z_2 * x_2)$$

Thus, S^R is an intuitionistic fuzzy B-ideal of XxX.

Conversely,

Suppose that S^R is an intuitionistic fuzzy B-ideal of XxX.

We know that,

$$\alpha_S(0) \geq \alpha_S(x)$$

$$\beta_S(0) \leq \beta_S(x), for\ all\ x \in X$$

Now, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$

Then,

$$\alpha_S(y_1 * z_1) \wedge \alpha_S(y_2 * z_2) = \alpha_S^R(y_1 * z_1, y_2 * z_2)$$

$$\geq \left(\alpha_S^R\big((x_1, x_2) * (y_1, y_2)\big)\right) \wedge \left(\alpha_S^R\big((z_1, z_2) * (x_1, x_2)\big)\right)$$

$$= \left(\alpha_S^R\big((x_1, y_1), (x_2, y_2)\big)\right) \wedge \left(\alpha_S^R\big((z_1, x_1), (z_2, x_2)\big)\right)$$

$$= \big(\alpha_S(x_1, y_1) \wedge \alpha_S(x_2, y_2)\big) \wedge \big(\alpha_S(z_1, y_1) \wedge \alpha_S(z_2, y_2)\big)$$

In particular, if we take, $x_2 = y_2 = z_2 = 0$

Then, $\alpha_S(y_1 * z_1) \geq \alpha_S(x_1, y_1) \wedge \alpha_S(z_1, x_1)$

Similarly, we can also prove

$$\beta_S(y_1 * z_1) \leq \beta_S(x_1, y_1) \vee \beta_S(z_1, x_1)$$

Thus, S is an intuitionistic fuzzy B-ideal of X.

Hence the proof.

5. CONCLUSION

In this paper we have discussed about Intuitionistic Fuzzy B-Ideals, Homomorphism, Anti Homomorphism and Cartesian product of Intuitionistic Fuzzy B-Ideals.

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