INTUITIONISTIC COFUZZY GRAPHS

R.Dhavaseelan,
Department of Mathematics,
Sona College of Technology,Salem - 5,Tamil Nadu ,India.

ABSTRACT

In this paper, we introduce the notion of an intuitionistic cofuzzy graphs and study some methods of construction of new intuitionistic cofuzzy graphs. We compute degree of a vertex, strong intuitionistic cofuzzy graphs and complete intuitionistic cofuzzy graphs. We also introduce and give properties of regular and totally regular intuitionistic cofuzzy graphs.

Keywords: Intuitionistic Cofuzzy Graphs; degree of a vertex; strong intuitionistic cofuzzy graphs; complete intuitionistic cofuzzy graphs; regular and totally regular intuitionistic cofuzzy graphs.

2010 Mathematics Subject Classification:05C07, 03E72.

1 Introduction and Preliminaries

The notion graph theory was first introduced by Euler in 1736. In the history of mathematics, the solution given by Euler of the well known Königsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. On the other hand, fuzzy graph theory as a generalization of Euler's graph theory was first introduced by Rosenfeld [7] in 1975. Later, Bhattacharya [2] gave some remarks on fuzzy graphs.The concept of cofuzzy Graphs by M.Akram [1]. For other notations, terminologies and applications not mentoned in this paper, the readers are referred to [3, 4, 5, 6, 8, 9]. Thus we introduce the notion of an intuitionistic cofuzzy graphs and study some methods of construction of new intuitionistic cofuzzy graphs. We compute degree of a vertex, strong intuitionistic cofuzzy graphs and complete intuitionistic cofuzzy graphs. We also introduce and give properties of regular and totally regular intuitionistic cofuzzy graphs.

Definition 1.1 [8, 9] Let μ be a fuzzy subset[8] on V, i.e., a map $\mu : V \to [0,1]$. A mapping $\nu : V \times V \to [0,1]$ is called a fuzzy relation on V if $\nu(x, y) \le (\mu(x), \mu(y))$ for all

Definition 1.2 [7] A fuzzy graph $G = (\mu, \nu)$ is a non empty set V together with a pair of functions $\mu : V \to [0,1]$ and $\nu : V \times V \to [0,1]$ such that $\nu(\{x, y\}) \le min(\mu(x), \mu(y))$ for all $x, y \in V$. Fuzzy graph is a graph consists pairs of vertex and edge that have degree of membership containing closed interval of real number [0,1] on each edge and vertex.

Definition 1.3 [1] A cofuzzy graph $G = (V, \mu, \nu)$ is a non-empty set V together with a pair of functions $\mu : V \to [0,1]$ and $\nu : V \times V \to [0,1]$ such that $\nu(\{x, y\}) \ge max(\mu(x), \mu(y))$ for all $x, y \in V$. We call μ the cofuzzy vertex set of G, ν the cofuzzy edge set of G, respectively. Note that ν is a symmetric confuzzy relation on μ. We use the notation xy for an element of E. Thus, $G = (\mu, \nu)$ is a cofuzzy graph of $G^* = (V, E)$ if $\nu(xy) \ge max(\mu(x), \mu(y))$ for all $xy \in E$.

2 Intuitionistic Cofuzzy Graphs

Definition 2.1 An intuitionistic cofuzzy graph is of the form $G = \langle V, E \rangle$ where

1. $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\gamma_1 : V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$ respectively, and

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1 \qquad (1)$$

2. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \ge max\{\mu_1(v_i), \mu_1(v_j)\} \qquad (2)$$

$$\gamma_2(v_i, v_j) \le min\{\gamma_1(v_i), \gamma_1(v_j)\} \qquad (3)$$

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$, $i, j = 1,2,...,n$.

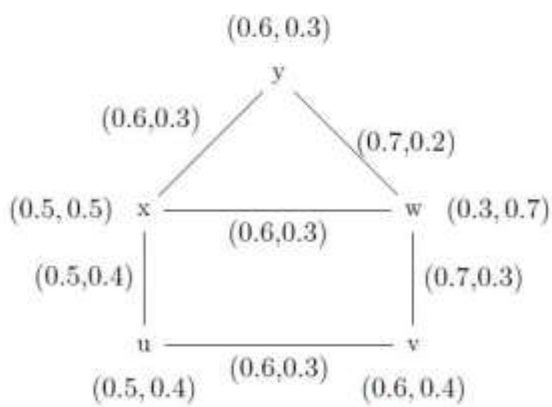


Figure 1: G : Intuitionistic Cofuzzy Graph

Notation 2.1 The triples $\langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ denotes the degree of membership and nonmembership of vertex v_i, The triples $\langle e_{ii}, \mu_{2ij}, \gamma_{2ij}\rangle$ denotes the degree of membership and nonmembership of edge relation $e_{ij} = (v_i, v_j)$ on V.

Definition 2.2 An intuitionistic cofuzzy graph $H = \langle V^{'}, E^{'}\rangle$ is said to be an intuitionistic cofuzzy subgraph of the intuitionistic cofuzzy graph $G = \langle V, E\rangle$ if $V^{'} \subseteq V$ and $E^{'} \subseteq E$. In other words if $\mu_{1i}^{'} \leq \mu_{1i}$; $\gamma_{1i}^{'} \geq \gamma_{1i}$ and $\mu_{2ij}^{'} \leq \mu_{2ij}$; $\gamma_{2ij}^{'} \geq \gamma_{2ij}$ for every $i, j = 1,2,...,n$.

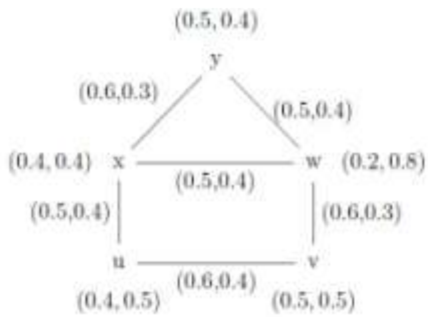


Figure 2: H : Intuitionistic Cofuzzy subgraph ($H \subseteq G$)

Definition 2.3 An intuitionistic cofuzzy graph $G = \langle V, E\rangle$ is said to be strong intuitionistic cofuzzy graph if $\mu_{2ij} = max(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = min(\gamma_{1i}, \gamma_{1j})$, for all $(v_i, v_j) \in E$.

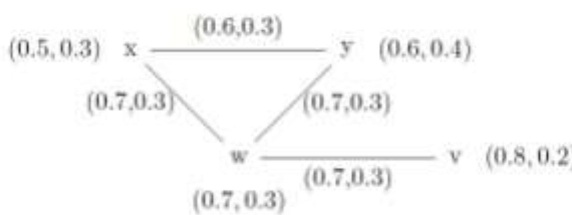


Figure 3: Strong Intuitionistic Cofuzzy graph

Definition 2.4 An intuitionistic cofuzzy graph $G = \langle V, E\rangle$ is said to be complete intuitionistic cofuzzy graph if $\mu_{2ij} = max(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = min(\gamma_{1i}, \gamma_{1j})$, for every $v_i, v_j \in V$.

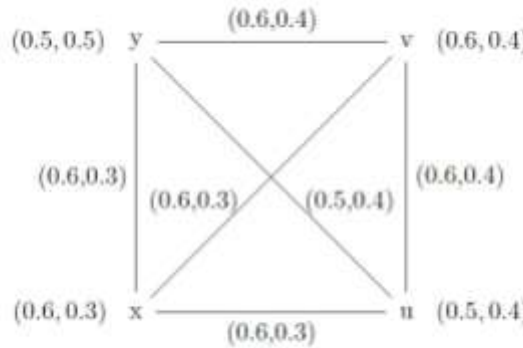


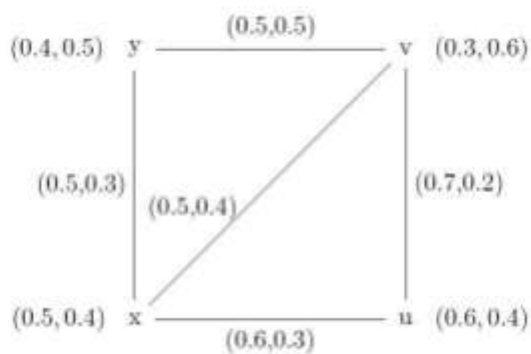
Figure 4: complete Intuitionistic Cofuzzy graph

Definition 2.5 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where $d_\mu(v) = \sum_{u \neq v} \mu_2(u,v)$ and $d_\gamma(v) = \sum_{u \neq v} \gamma_2(u,v)$

Definition 2.6 The minimum degree of G is $\delta(G) = (\delta_\mu(G), \delta_\gamma(G))$ where $\delta_\mu(G) = min\{d_\mu(v) \mid v \in V\}$ and $\delta_\gamma(G) = min\{d_\gamma(v) \mid v \in V\}$

Definition 2.7 The maximum degree of G is $\Delta(G) = (\Delta_\mu(G), \Delta_\gamma(G))$ where $\Delta_\mu(G) = min\{d_\mu(v) \mid v \in V\}$ and $\Delta_\gamma(G) = min\{d_\gamma(v) \mid v \in V\}$

Example 2.1 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. Draw as below



The degrees are $d_\mu(x) = 1.6, d_\gamma(x) = 1.0, d_\mu(u) = 1.3, d_\gamma(u) = 0.5,$
$d_\mu(v) = 1.7, d_\gamma(v) = 1.1, d_\mu(y) = 1.0, d_\gamma(y) = 0.8.$

Minimum degree of a graph is $\delta_\mu(G) = 1.0, \delta_\gamma(G) = 0.5.$

Maximum degree of a graph is $\Delta_\mu(G) = 1.7, \Delta_\gamma(G) = 1.1.$

Definition 2.8 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. The total degree of a vertex $v \in V$ is defined as :

$$Td(v) = Td_\mu(v) + Td_\gamma(v) \text{ where}$$

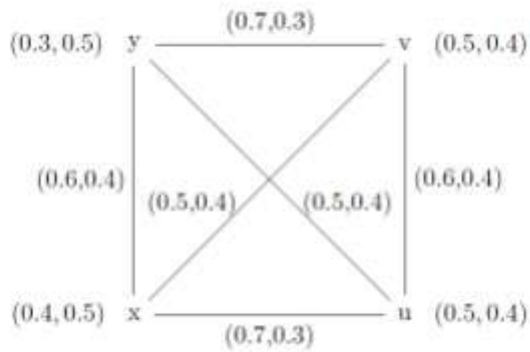
$$Td_\mu(v) = \sum_{(u,v)\in E} \mu_2(u,v) + \mu_1(v) \text{ and}$$

$$Td_\gamma(v) = \sum_{(u,v)\in E} \gamma_2(u,v) + \gamma_1(v).$$

If each vertex of G has the same total degree (r_1, r_2) then G is said to be an (r_1, r_2) totally regular intuitionistic fuzzy cofuzzy graph.

Definition 2.9 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. If each vertex has same degree (r, s) then G is called (r, s) regular intuitionistic cofuzzy graph. Thus $r = d_\mu(v), s = d_\gamma(v)$; for $v \in V$.

Example 2.2 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. Draw as below



$d(y) = (1.8,1.1), d(v) = (1.8,1.1),$ $d(u) = (1.8,1.1), d(x) = (1.8,1.1).$ So, G is a regular intuitionistic fuzzy cofuzzy graph. But G is not totally regular intuitionistic cofuzzy graph. Since $Td(y) = 3.7 \neq 3.8 = Td(v)$.

Proposition 2.1 If an intuitionistic cofuzzy graph is both regular and totally regular then (μ_1, γ_1) is constant.

Proof. Let G be a (r, s) regular and (k_1, k_2) totally regular intuitionistic cofuzzy graphs. So, $d_\mu(v_1) = r, d_\gamma(v_1) = s$ for $v_1 \in V$ and $Td_\mu(v_1) = k_1, Td_\gamma(v_1) = k_2$ for all $v_1 \in V$. Now,

$$Td_\mu(v_1) = k_1 \text{ for all } v_1 \in V,$$
$$d_\mu(v_1) + \mu_1 = k_1, \text{for all } v_1 \in V,$$
$$r + \mu_1(v_1) = k_1, \text{for all } v_1 \in V, .$$
$$\mu_1(v_1) = k_1 - r, \text{for all } v_1 \in V.$$

Hence $\mu_1(v_1)$ is a constant function.

Similarly, $\gamma_1(v_1) = k_2 - s$ for all $v_1 \in V$. Hence (μ_1, γ_1) is a constant.

Proposition 2.2 Let $G = \langle V, E\rangle$ be an intuitionistic cofuzzy graph. Then (μ_1, γ_1) is a constant function iff following are equivalent.

1) G is a regular intuitionistic cofuzzy graph,
2) G is a totally regular intuitionistic cofuzzy graph.

References

[1] M.Akram,Cofuzzy Graphs, The Journal of Fuzzy Mathematics,3(4)(2011),877-888.
[2] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letter, 6 (1987), 297-302.
[3] K. R. Bhutani, On automorphism of fuzzy graphs, Pattern Recognition Letter, 9 (1989), 159-162.
[4] F. Harary, Graph theory, 3rd edition, Addison-Wesley, Reading, MA, October, (1972).
[5] J. N. Mordeson and C. S. Peng, Operations on fuzzy graphs, Information Sciences, 78 (1994), 159-170.
[6] A. Nagoorgani and K. Radha, The degree of a vertex in some fuzzy graphs, International journal of algorithm, Computing and Mathematics, 2 (3) (2009), 107-116.
[7] A. Rosenfeld, Fuzzy graphs, Fuzzy sets and their applications (L. A. Zadeh, K.S. Fu, M. Shimura, Eds.), Academic Press, New York, (1975), 77-95.
[8] L. A. Zadeh, Fuzzy sets, Information and control, 8 (1965), 338-353.
[9] L. A. Zadeh, Similarity relations and fuzzy orderings, Information Sciences, 3 (2) (1971), 177-200.