OPTIMIZING ROUTING OF RESIDENTIAL SOLID WASTE COLLECTION: CASE STUDY OF CHIKOVA RESIDENTIAL AREA IN ZIMBABWE.

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ABSTRACT

During the last decade, metaheuristics have become increasingly popular for effectively confronting difficult combinatorial optimization problems. In the present paper, a metaheuristic algorithmic solution, the ArcGIS Network Analyst is introduced, implemented and discussed for the identification of optimal route in the case of Municipal Solid Waste (MSW) collection. ArcGIS Network Analyst applications are based on a geo-referenced spatial database supported by a Geographic Information System (GIS). GIS are increasingly becoming a central element for coordinating, planning and managing transportation systems, and so in collaboration with combinatorial optimization techniques they can be used to improve aspects of transit planning in urban regions. Here, the GIS takes into account all the required parameters for the MSW collection (i.e. positions of waste bins, road network and the related traffic, truck capacities, etc) and its desktop users are able to model realistic network conditions and scenarios. In this case, the simulation consists of scenarios of visiting varied waste collection spots in a part of the residential area of Chikova. Finally, the optimal solution is estimated by the routing optimization algorithm called Adapted Dijkstra's Algorithm and the optimized route was compared with the current route.

***Keywords**:* Solid waste collection, optimization, shortest route, optimum route.

Introduction

Sustainable solid waste management has evolved as a complex discipline and is one of the most difficult operational problems faced by most cities in Zimbabwe. Solid waste collections in most areas are being done in an adhoc manner as collection vehicles are assigned to areas without any serious demand analysis with route construction being left to the drivers thus increasing the solid waste management bill. One major subsystem of solid waste management is the problem of designing vehicle routes with all the realistic constraints (that are geographical, technological, financial and labour related). With this in mind, route optimization will be an important field of study regarding the improvement of municipal solid waste management processes. The basic problem in residential solid waste collection consists of collecting waste house by house as every street has to be traversed by the collection truck during the collection period hence an arc routing problem.

Solid waste have proved to be a nasty problem if left unattended and problems resulting from poor solid waste management have reached proportions requiring drastic measures in most countries. Indisposed waste increase the chances of disease transmission and possible infection, hence, removal and disposal of solid waste is a very important aspect of urban sanitation and a challenging problem to Public Health Engineers (Sincero 2004). This is so since random disposal of waste poses a serious health hazard to the community as well as affecting the tourism industry. However, over the past years, due to factors like solid waste collection and transportation costs, health, and environmental concerns, many municipalities have been forced to assess their solid waste management and examine its cost effectiveness and environmental impacts, in terms of designing collection routes (Johansson 2006). Solid waste management involves the control of generation, storage, collection, transportation and disposal according to the principles of public health, economic and other environmental considerations.

1.0 Background and Literature

Solid waste collection in Chikova residential area (area under study) is being done in an adhoc manner since a collection vehicle is assigned to the area without any serious demand analysis. Route construction decision is left to the driver, thus increasing the solid waste management bill. As a result of too much costs, solid wastes can go for weeks or months uncollected. Solid waste have proved to be a nasty problem if left unattended and problems resulting from poor solid waste management have reached proportions requiring drastic measures in the area. Indisposed waste increases the chances of disease transmission such as typhoid and cholera and possible infections.

Vehicle routing problems (VRP) are very difficult combinatorial problems which in practice often require modifications of the existing algorithms and development of new ideas in order to obtain solutions suitable within a given problem (Toth and Vigo, 2002). The problem with VRP problems is that most of practical VRP problems cannot be solved exactly within an acceptable period of time (Laporte, 1992). Thus, this is clear from literature, that there are many different modeling approaches that may be taken into consideration when faced with a solid waste collection problem, including the use of heuristic algorithms, genetic algorithms and fuzzy logic methods. Heuristic algorithms are adhoc; trial and error methods which do not guarantee to find the optimal solution but help find near-optimal solution in a fraction of the time required by traditional methods. Therefore, different and better heuristic and metaheuristic algorithms are being formulated and applied in vehicle routing problems due to its applications in logistic and supply-chains management. Routing problems are usually treated either as an arc-routing problem, where all of the arcs (lines or roads) of the network must be traversed or as node routing problem, where all nodes (points or places) on the network must be visited.

Arc routing problems (ARPs) are a special kind of vehicle routing in which the vehicles are constrained to traverse and service certain arcs within a graph, rather than visit certain nodes as in the standard vehicle routing problem. Two important ARPs can be derived from general routing problems, the Chinese Postman Problem (CPP) and the Rural Postman Problem (RPP). The CPP is commonly associated with mail delivery in urban settings that seeks the least cost traversal of all the streets. In turn, the RPP is associated with rural settings and can be described as follows: There are a number of villages whose set R of streets has to be serviced by a postman, and a set $A \forall R$ of links between the villages that do not have to be served, but may be used for travelling between villages. It seeks the least cost traversal of the subset $A \in R$ of required streets (Eiselt and Laporte, 1995).

Arc routing problems can be classified into different classes depending on the problem at hand some of which include the Capacitated Arc Routing Problem (CARP), Capacitated Arc Routing

Problems with Intermediate Facilities (CARPIF), Capacitated Arc Routing Problem with Turn Penalties (CARPTP), Stochastic Capacitated Arc Routing Problem (SCARP), Extended Capacitated Arc Routing Problem (E-CARP) and Periodic Capacitated Arc Routing Problem (PCARP). The Capacitated Arc Routing Problem (CARP) is an arc routing problem which involves determining a fleet of homogeneous size vehicles and designing the routes that minimizes the total cost. CARP with customers located along the edges of the network is an NP-hard problem and one such problem is in the designing of household vehicle collection tours as each vehicle is limited in its capacity. The Capacitated Arc Routing Problem (CARP) is an arc routing problem which involves determining a fleet of homogeneous size vehicles and designing the routes that minimizes the total cost. The CARPIF was first introduced by Ghiani et al. (2001) and it requires that a vehicle needs to unload or replenish at Intermediate Facilities (IFs). For the solid waste collection problem, a vehicle starts in the morning at an assigned depot. Waste is collected along the streets until the vehicle's capacity is reached, after which the vehicle needs to dump the waste at the nearest dumping site (IF), which may or may not include the depot. Ghiani et al. (2001) and Polacek et al. (2007) solved this problem by adding a subset I of intermediate facilities to the CARP. Constraints are added such that the waste collected between the depot and first IF, or between two IFs may never exceed the vehicle's capacity W. Some turns can be considered forbidden, others more time consuming or dangerous, especially for large waste collection vehicles. U-turns may be impossible to make due to narrow streets or forbid by traffic rules. Even right turns at robots or busy intersections may be more time consuming and should therefore be penalized. Belenguer et al. (2006) provides a method for including turn penalties in the CARP based on the work of Benavent and Solver (1999). This method allows the solving of the problem by adding a penalty cost associated with each turn to the objective function.

The SCARP can be defined as a problem having some element of uncertainty. Chu et al. (2006) mentioned that the stochastic component in solid waste collection refers to the demand of the customers and is treated as a decision variable in the problem. Heuristics and meta-heuristics have also become more popular in these modern days where technology rules the most. Heuristics are approximate techniques used to determine good feasible solutions for problems that are difficult or impossible to solve to optimality. Winston and Venkataramanan (2004) state that heuristic are characterized by using a greedy approach to obtain good solution in efficient time. Heuristics make incremental improvements to an existing solution by neighbourhood changes or local searches so they tend to get trapped in local optima and fail to find a global optimum resulting in meta-heuristics becoming more popular. Metaheuristics are a subset of heuristics that are based on intelligent search techniques which can overcome the problem of being trapped in a local optimum. This is achieved by accepting solutions that may not be an improvement or by considering several solutions at a time (Winston and Venkataramanan, 2004). Hirabayashi and Nishida (1992) proposed a Branch and Bound algorithm and Belenguer and Benavent (2003) proposed a Cutting Plane algorithm. Belenguer et al. (2006) extended the cutting plane algorithm for the mixed CARP. Eiselt et al. (1995) proposed an integer linear programming formulation for the undirected capacitated arc routing problem by replacing each edge with two arcs creating a directed formulation of the capacitated arc routing problem. Winston and Venkataramanan (2003) describe Tabu Search (TS) as making use of shortand long-term memory to emulate heuristic rules used by people when making day-to-day decisions. Hertz et al. (2000) used a TS algorithm called CARPET to solve the CARP; Greistorfer (2003) uses a Tabu Scatter Search metaheuristic; and Brandao and Eglese (2008) a deterministic TS algorithm. Ghiani et al. (2001) use the TS algorithm to solve the CARPIF and all of the authors were able to provide high quality solution for benchmark problems. Lacomme et al. (2006) used a Memetic Algorithm (MA) in solving the capacitated arc routing problem.

This memetic algorithm is a genetic algorithm hybridized with a local search which addresses different extensions of the CARP like mixed networks, parallel arcs and turn penalties. The memetic algorithm has provided good performance results on different bench mark problems. Belenguer et al. (2006) further extended this algorithm to also address problems with several dumping sites. Lacomme et al. (2006) use a GA with population management to solve the CARP. Results obtained by various authors suggest that the GA is highly successful for solving the CARP and its extensions. Polacek et al. (2007) use the Variable Neighbourhood Search (VNS) to solve the CARP and extended it to solve the CARPIF. Excellent results were obtained on four sets of benchmark problems of which two included the extension of IFs. Again the VNS showed slightly better performance results for the CARPIF compared to the results that Ghiani et al. (2001) obtained through the use of a TS algorithm. Another major advantage of VNS is that high quality final solutions can be achieved independently from the quality of the initial solution. Lacomme, Prins and Ramdane-Cherif (2001) formulated an integer linear model which generalizes a model proposed by Golden and Wong in 1981 for the basic undirected CARP. Despite the fact that this very first model had been developed long back, it has never been evaluated due to the lack of people powerful enough to tackle it.

1.3 Main Objective

To determine an optimal route that minimizes total aggregate distance traversed and total cost in return.

1.4 Assumptions

- Solid waste is collected once per week.
- The collection vehicle is the same throughout the year.
- Fuel consumption ratio remains the same regardless of the truck being loaded or not.
- Fuel price at market rate remains unchanged at US1.35 per litre and only diesel is used.
- The route used will be the same all year through.
- No breakdown of vehicle during collection.
- Rate of truck depreciation is constant throughout the year estimated to be $20 for every 100 km.

3.0 Methodology

In this paper, ArcGIS Network Analyst was used to come up with geographical maps of the Chikova residential area indicating the residential streets and distances. Network Analyst uses the Dijkstra's Algorithm (Dijkstra 1959) in order to solve the Routing Problem and it can be generated based on two criterions:

(i) Distance criteria: The route is generated taking only into consideration the location of the refuse bins. The volume of traffic in the roads is not considered in this case.

(ii) Time criteria: The total travel time in each road segment should be considered as the: Total travel time in the route = runtime of the vehicle + solid waste collection time. The runtime of the vehicle is calculated by considering the length of the road and the speed of the vehicle in each road.

3.1 Model Algorithm

The objective is to find the shortest path (the path with the minimum total travel time) from the origin to the destination. The algorithm is as follows:

(i) Find the n^{th} nearest node to the origin to be repeated for $n = 1, 2, \ldots.44$ nearest node is the destination.

(ii) Input for the n^{th} iteration n_1 nearest nodes to the origin (solved for at the previous iterations), including their shortest path and distance from the origin. (These nodes, plus the origin, will be called solved nodes; the others are unsolved nodes.)

(iii) Candidates for the n^{th} nearest node, each solved node that is directly connected by a link to one or more unsolved nodes provides one candidate the unsolved node with the shortest connecting link. (Ties provide additional candidates.)

(iv) Calculation of the n^{th} nearest node, for each such solved node and its candidate, add the distance between them and the distance of the shortest path from the origin to this solved node. The candidate with the smallest such total distance is the n^{th} nearest node (ties provide additional solved nodes), and its shortest path is the one generating this distance.

3.2 Graph Transformation

To ease algorithmic design, the mixed multi graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{A}, \boldsymbol{R})$ is transformed into a fully directed multi graph $\boldsymbol{G}^* = (\boldsymbol{V}, \boldsymbol{A}^*, \boldsymbol{R}^*)$, by replacing each edge by two opposite arcs and by adding one dummy loop for the depot and one for each of the dumping sites. We define $\boldsymbol{V}$ as the set of n nodes which includes the depot node $\boldsymbol{V_0}$, as well as all the dumping nodes. The transformed $\boldsymbol{A}^*$ is defined as a set of m arcs, identified by indexes from 1 to m, instead of pairs of nodes to avoid ambiguities for parallel arcs. Each arc $u \epsilon \boldsymbol{A}^*$ begins at node $b(u)$, ends at node $e(u)$ and has a service cost $c(u)$. The $\boldsymbol{R}$ required tasks in $\boldsymbol{G}$, comprising of $\|\boldsymbol{E}\|$ required edges and $\|\boldsymbol{A}\|$ required arcs, given by $\boldsymbol{R}^* \subseteq \boldsymbol{A}^*$ with $\boldsymbol{R}^* = 2||\boldsymbol{E}|| + ||\boldsymbol{A}||$ tasks. Only one of the two arcs that represent an edge has to be collected in any feasible solution. To ensure this, both arcs u and v are linked by a pointer variable, $\mathbf{inv}(u)$ and $\mathbf{inv}(v)$, which ensures that when the algorithm selects one direction, both arcs can be marked "collected". This is done by coding an arc task in $\boldsymbol{G}$ as one arc u with $\mathbf{inv}(u) = 0$ in $\boldsymbol{G}^*$ while each edge task in $\boldsymbol{G}$ gives two opposite arcs, u and v, such that $\mathbf{inv}(u) = v$, $\mathbf{inv}(v) = u$, $q(u) = q(v)$ and $c(u) = c(v)$, possibly with distinct cost if the edge is windy $(c(u) \neq c(v))$. Each arc $u \epsilon \boldsymbol{R}^*$ has a demand $q(u)$ and a pointer $\mathbf{inv}(v)$. From now on all algorithms will work according to the directed encoding of the mixed multi graph.

3.3 Forbidden Turns, Turn Penalties and Distance Matrix

Forbidden turns and turn penalties are made transparent by including a set of permitted turns $\mathbf{turn}(u,v)$, with associated turn penalties, $\mathbf{pen}(u;\ v)$. A set of allowed successor arcs for arc u, $\mathbf{suc}(u)$, is created, such that arc $v \in \mathbf{suc}(u)$ if $e(u) = b(v)$ and $\mathbf{turn}(u,v)$ is allowed. Following the work of Lacomme et al. (2006) for the Capacitated Arc Routing Problem (CARP) with turn penalties, a feasible path from arc u to arc v is defined as a sequence of arcs $(\mu = u_1, u_2 \ldots u_k = v)$, such that $u_{i+1} \in \mathbf{suc}(u_i)$ for $i = 1, \ldots, k-1$. The cost of μ is defined by equation (3.1). Note that the cost of u and v is not included.

$$c(\mu) = \mathbf{pen}(u_1, u_2) + \sum_{i=1}^{k-1} \left(c(ui) + \mathbf{pen}(u_i, u_{i+1}) \right) \qquad (3.1)$$

Using an adaptation of Dijksta's shortest path algorithm, forbidden turns and turn penalties are included by pre-computing two $m \times m$ arc-to-arc matrices $\boldsymbol{D}$ and $\boldsymbol{P}$. $\boldsymbol{D}(u,v)$ is the cost/distance of the shortest path found from arc u to arc v and $P(u,v)$ is the predecessor of v on this shortest path. The full structure of the adapted Dijkstra's shortest path algorithm will be presented. The adapted Dijkstra's algorithm computes arc u of $\boldsymbol{D}$ and $\boldsymbol{P}$. It is called m times with $u = 1, 2, \ldots, m$ to calculate the arc-to-arc matrix $\boldsymbol{D}$ and predecessor matrix $\boldsymbol{P}$. The algorithm starts off by setting all arcs 'fixed value' to false and 'distance value' to infinite. Each iteration of the algorithm determines the next arc v that is closest to arc u. Once a shortest path between arcs u and v is obtained, arc v is fixed by setting $\mathbf{Fix}(v) = true$. This ensures that arc v cannot be selected again and determines each successor - arc z of arc v to see if the current shortest distance from arc u to arc z can be improved. The output of the algorithm is matrices $\boldsymbol{D}$ and $\boldsymbol{P}$, which includes turn penalties and forbidden turns. Matrices $\boldsymbol{D}$ and $\boldsymbol{P}$ are used as input for the TS algorithm to solve the routing of solid waste collection problem.

Algorithm : **Adapted Dijkstra's Shortest Path Algorithm**

Input : Begin nodes $b(u)$

End nodes $e(u)$

Distance/Service Cost $c(u)$

$\mathbf{turn}(u,v)$ and $\mathbf{pen}(u,v)$

Output : Shortest Path Matrix $\boldsymbol{D}(u,v)$

Predecessor Matrix $\boldsymbol{P}(u,v)$

Generate $\mathbf{suc}(u)$, the set of allowed successor − arcs for arc; u

for $i \longleftarrow 1$ **to** m **do**

$\boldsymbol{D}(u, v) = 1;$

$\mathbf{Fix}(v) = False;$

end

Find all successor arcs of arc u;

for $i \longleftarrow 1$ **to** m **do**

Set for each v in (u) :

$\boldsymbol{D}(u, v) = \mathbf{pen}(u, v)$ and

$\boldsymbol{P}(u, v) = u;$

end

Update matrices $\boldsymbol{D}$ and $\boldsymbol{P}$ with shortest path;

for $i \longleftarrow 1$ **to** m **do**

$\mathbf{Fix}(v) = False$ **do**

Choose min v and set $\mathbf{Fix}(v) = True;$

for each z in $\mathbf{suc}(v)$ with

$\boldsymbol{D}(u, v) + c(v) + \mathbf{pen}(v, z) \leq \boldsymbol{D}(u, z)\boldsymbol{do}$

$\boldsymbol{D}(u, z) \leftarrow \boldsymbol{D}(u, v) + c(v) + \mathbf{pen}(v, z);$

$P(u, z) \leftarrow v;$

end

end

4.0 Results and Discussions

The map in Appendix 1 shows Chikova Residential area as was created by ArcGIS using the original map. The map also shows the assumed distances between the nodes or assumed distances of all edges participating in the network. The map in Appendix II shows the area under study with all the streets in it as they are pointed out on the map using ArcGIS application and collection of solid waste should be done on both sides of the road. The following map in Appendix III shows the assumed distances between the nodes or assumed distances in meters of all edges participating in the network. The map also includes the assumed distances from the depot to the start node and from the end node to the landfill.

The map in Appendix IV shows the optimum route as it covers all arcs in the designed road network. Though the researchers have considered the optimum route as the shortest route to be traversed by solid waste collection truck, the route leaves some edges that increase amount of deadheading and people from these streets have to take their bins to the nearest node so they can be collected since edges that have been not traversed twice have minimum distances from each other. Coming up with an optimum route or shortest path that covers all arcs or edges proved difficult due to complexities in the designed road network of the area under study. The map shows the direction that should be used during the collection process, this is indicated by arrows. Some arrows are two way since the road will be traversed twice during the collection period.

4.4. The Optimum Route

Table 4.1 Comparison of the current route and the optimized route

Route	**Distance (km/year)**	**Fuel used (litres/year)**	**Cost of fuel (USD/year)**	**Depreciation Cost per year (USD)**
Current Route	90.12 per trip * 52 weeks = 4686.24	24.032 per trip * 52 weeks = 1249.66	1249 litres per year * $1.35 per litre = $1687.02	4686.24 /100 km *$20 per 100 km =937.25
Optimized Route	80.7 per trip * 52 weeks = 4196.4	21.52 per trip * 52 weeks = 1119.04	1119.04 litres per year * $1.35 per litre = $1510.70	4196.4/100 km *$20 per 100 km =839.28
Difference	489.84	130.62	176.32	97.97

Table 4.1 shows the total distance traversed by a truck using the current route and the optimized route. It also shows the total fuel used and the cost of fuel in both scenarios. The total distance of the current route is 4686 km per year and the total distance of the optimized route is 4196.4 km per year. The total distance, quantity of fuel and total cost of fuel of the current route are each minimized by 489.84 km saving a cost of $97.97 against depreciation. On the other hand fuel used in the current route is minimized by 130.62 litres saving $176.32 which could have been used on fuel.

Table 4.2: Table showing the path to be traversed during collection

From node	**To node**	**Distance (m)**	**From node**	**To node**	**Distance (m)**
Depot	1	1200	38	37	480
1	2	600	37	36	600

2	3	600	36	35	600
3	8	1200	35	33	960
8	7	600	33	34	600
7	4	1200	34	36	960
4	12	600	36	37	600
5	13	1200	37	31	2880
6	14	600	31	32	600
12	15	600	32	28	1200
13	16	1200	28	27	600
14	17	600	27	25	1500
15	18	600	25	21	720
16	23	3600	21	25	720
17	18	480	25	26	1800
18	19	1260	26	11	600
23	15	1260	11	26	600
18	19	2400	26	27	600
19	20	1260	27	28	600
15	13	1260	28	29	600
19	20	600	29	10	1800
20	13	840	10	11	600
13	20	840	11	10	600
20	21	780	10	9	600
21	22	600	9	1	2400
22	24	1500	1	9	2400
24	23	480	9	30	1800
23	24	480	30	42	2100
24	43	780	42	30	2100
43	44	3300	30	Landfill	3000
44	42	600	Landfill	30	3000
42	41	480	30	9	1800
41	40	1200	9	1	2400
40	39	480	1	Deport	1200
39	38	1200			**80.7 km**

5.0 Conclusion

Technological advancement and the development of routing softwares have proved to be the best way to go about when dealing with routing optimization problems for real life situations. ArcGIS network analyst has proved to be the best software to use since most combinatorial and heuristic methods can only solve problems to optimality which is not large as the one under study for instance Tabu Search and Ant Colony system. After implementation using ArcGIS network analyst and Dijkstra's Algorithm, the total distance travelled was reduced by 489.84 km per year that is a percentage reduction of 10.45% in the total distance traveled, total amount of fuel used and the total amount of money used to purchase fuel.

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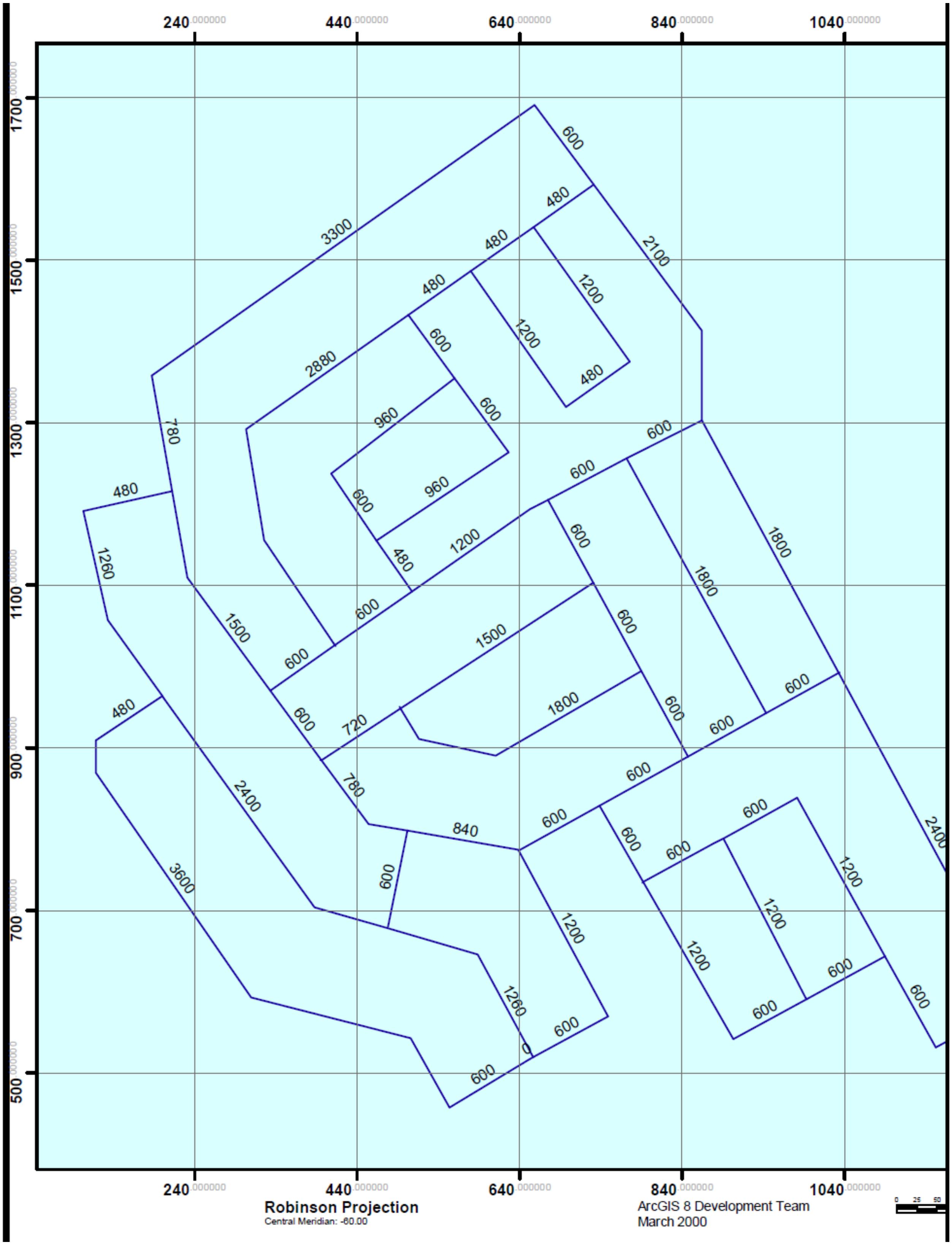
Table 4.3 Glossary of mathematical symbols.

Term	Description
$\boldsymbol{G}$	Connected mix graph with $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E}, \boldsymbol{A}, \boldsymbol{R})$
$\boldsymbol{G}^*$	Transformed fully directed graph with $\boldsymbol{G}^* = (\boldsymbol{V}, \boldsymbol{A}^*, \boldsymbol{R}^*)$
$\boldsymbol{V}$	Vertex set representing a streets intersection or dead ends
$\boldsymbol{A}^*$	Set of m arcs.
$\boldsymbol{R}^*$	Set of required arcs.
n	number of nodes in V, calculated as $n = \|\|V\|\|$.
m	number of arcs in $\boldsymbol{A}$,calculated as $m = \boldsymbol{A}^*$
$b(u)$	begin node of arc u.
$e(u)$	end node of arc u.
$q(u)$	demand of arc u.
$inv(u)$	pointer to indicate whether arc u is an edge or arc task in $\boldsymbol{G}$.

$suc(u)$	set of possible successor arcs for arc u.
V_0	depot node.
I	dumping node
k	vehicle
W	vehicle's capacity.

Appendix I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1040
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900
700
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3300
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Robinson Projection
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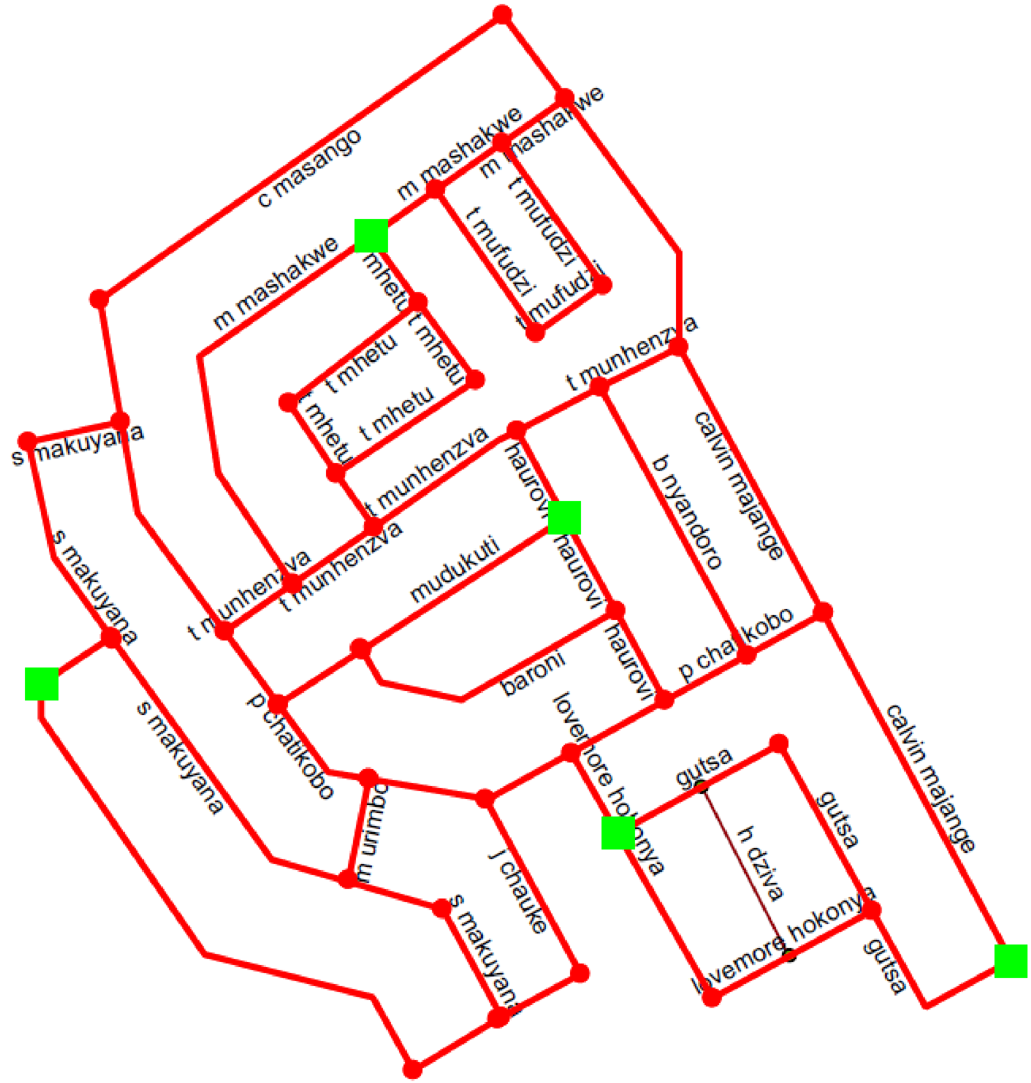
ArcGIS 8 Development Team
March 2000

Appendix II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haurovi
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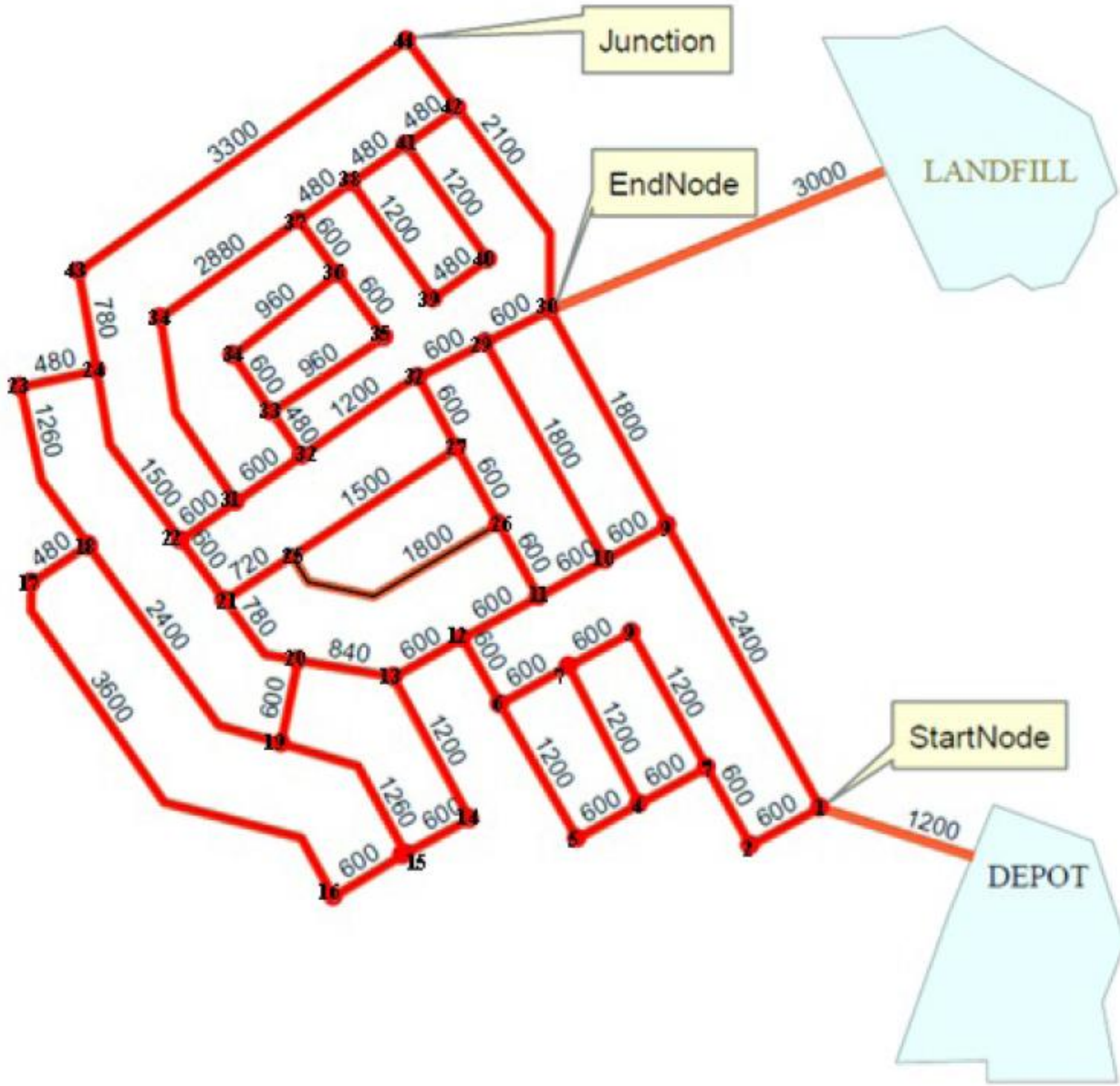
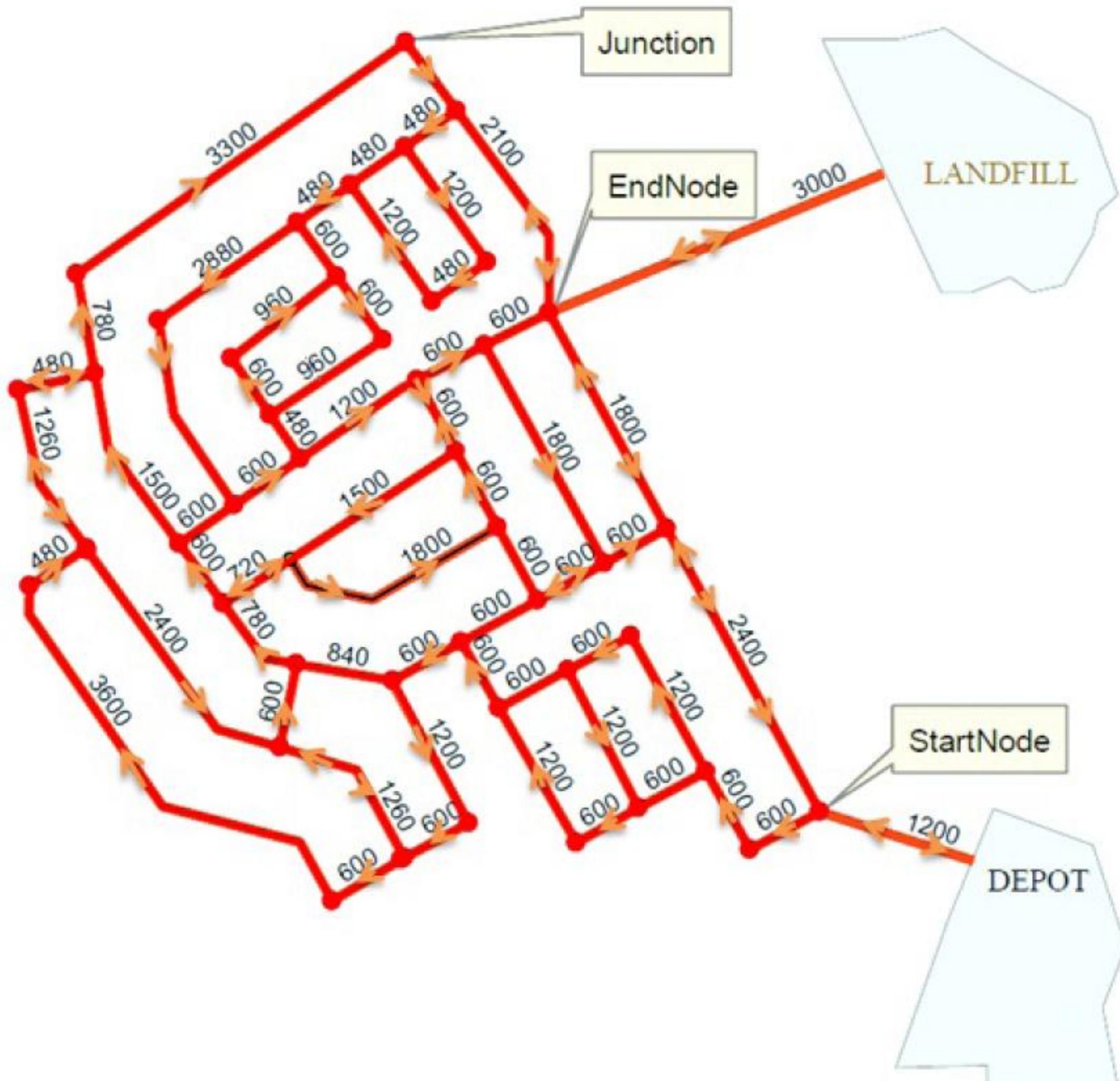
Appendix III



Appendix IV


Junction
EndNode
LANDFILL
3000
3300
2100
StartNode
DEPOT
1200