CONSTRUCTION OF $D(5)$ – DIO QUADRUPLES

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ABSTRACT

This paper concerns with the study of constructing special $D(5)$*- Dio Quadruples* (a, b, c, d) *such that the product of any two elements of the set increased by their sum and five is a perfect square.*

KEY WORDS

DIOPHANTINE QUADRUPLES, PELL EQUATION.

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INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus [5]. A set of m positive integers $\{a_1, a_2 ......a_m\}$ is said to have the property D(n), $n \epsilon z - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n. In this context, one may refer [1-4, 6-14,17,21,22] for an extensive review of various problems on Diophantine quadruples. This paper aims at constructing special Dio – quadruple where the special mention is provided because it differs from the earlier one and the special Dio – quadruple is constructed where the product of any two members of the quadruple with the addition of the same members and the addition of five satisfies the required property. In this context, one may refer [15,16,18-21].

METHOD OF ANALYSIS

Let $a = 3^n$ and $b = 3^n + 4$ be two integers such that $ab + a + b + 5$ is a perfect square.

Let $C_N(n)$ be any non-zero integer such that

$$(3^n+1)C_N(n)+3^n+5 = P_N^2 \tag{1}$$

$$C_N(n)(3^n+5)+3^n+9 = q_N^2 \tag{2}$$

Eliminating $C_N(n)$ from (1) and (2), we obtain

$$(3^n+5)P_N^2-(3^n+1)q_N^2 = 16 \tag{3}$$

Setting
$$P_N = 2X_N - 2(3^n+1)T_N \tag{4}$$

$$q_N = 2\,X_N - 2(\,3^n+5)T_N \tag{5}$$

in (3), we get

$$X_N^2 = (3^{2n}+6*3^n+5)T_N^2+1 \tag{6}$$

whose general solution is

$$\left.\begin{aligned} X_N(n) &= \frac{1}{2}\left\{\begin{aligned}&\left[\left(\frac{3^{2n}+6*3^n+7}{2}\right)+\sqrt{3^{2n}+6*3^n+5}\left(\frac{3+3^n}{2}\right)\right]^{N+1}+\\ &\left[\left(\frac{3^{2n}+6*3^n+7}{2}\right)-\sqrt{3^{2n}+6*3^n+5}\left(\frac{3+3^n}{2}\right)\right]^{N+1}\end{aligned}\right\}\\ T_N(n) &= \frac{1}{2\sqrt{3^{2n}+6*3^n+5}}\left\{\begin{aligned}&\left[\left(\frac{3^{2n}+6*3^n+7}{2}\right)+\sqrt{3^{2n}+6*3^n+5}\left(\frac{3+3^n}{2}\right)\right]^{N+1}-\\ &\left[\left(\frac{3^{2n}+6*3^n+7}{2}\right)-\sqrt{3^{2n}+6*3^n+5}\left(\frac{3+3^n}{2}\right)\right]^{N+1}\end{aligned}\right\}\end{aligned}\right\} \tag{7}$$

Substituting N=0 in (7) and using (4) and (1), we get,

$$C_0(n) = 2(2*3^n+6)-1$$

Now, consider the transformations

$$\left.\begin{aligned} P_N &= 2X_n + 2(3^n+1)T_N \\ q_N &= 2X_N + 2(3^n+5)T_N \end{aligned}\right\} \quad (8)$$

Taking N=0 in (7) and employing (8) in (1), we have,

$$c_0(n) = 4*3^{3n} + 36*3^{2n} + 104*3^n + 95$$

Denoting this $C_0(n)$ by $E_0(n)$, it is noted that the quadruple $(a, b, C_0(n), E_0(n))$ represents a special Dio-quadruple with property D(5).

Substituting N=1 in (7) and using (4) and (8) along with (1) in turn we get,

$$C_1(n) = 4(3^{3n} + 8*3^{2n} + 19*3^n + 14)(3^{2n} + 7*3^n + 12) - 1$$

$$E_1(n) = (2*3^{4n} + 22*3^{3n} + 84*3^{2n} + 130*3^n + 70)(2*3^{3n} + 20*3^{2n} + 64*3^n + 66) - 1$$

Note that $(a, b, E_0(n), C_1(n))$ is a special D(5) - Dio-quadruple. The repetition of the above process leads to many D(5) - Dio-quadruples as mentioned below:

(i) $(a, b, C_N(n), E_N(n))$ and

(ii) $(a, b, E_N(n), C_{N+1}(n))$, for N= 0, 1, 2, 3, 4,…

Some Numerical examples are tabulated below:

n	$(a, b, C_0(n), E_0(n))$	$(a, b, E_0(n), C_1(n))$	$(a, b, C_1(n), E_1(n))$	$(a, b, E_1(n), C_2(n))$
1	(3,7,23,839)	(3,7,839,28559)	(3,7,28559,970223)	(3,7,970223,32959079)
2	(9,13,47,6863)	(9,13,6863,974687)	(9,13,974687, 138398879)	(9,13,138398879, 19651666319)
3	(27,31,119,107879)	(27,31,107879, 96876239)	(27,31,96876239, 86994755759)	(27,31,86994755759, 78121193796359)

CONCLUSION

This paper concerns with the construction of sequence of special D(5) - Dio - quadruples. One may search for special Dio-quadruples of special numbers with suitable property.

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