SPECIAL PYTHAGOREAN TRIANGLES AND 5-DIGIT DHURUVA NUMBERS

M. A. Gopalan,
Professor, Department of Mathematics, SIGC, Trichy, Tamilnadu,India.

S. Vidhyalakshmi,
Professor, Department of Mathematics, SIGC, Trichy, Tamilnadu,India.

E. Premalatha,
Assistant Professor, Department of Mathematics,National College, Trichy, Tamilnadu, India.

K. Presenna,
PG student, Department of Mathematics, SIGC, Trichy, Tamilnadu, India.

ABSTRACT

Pythagorean triangles, each with a leg represented by a 5-digit Dhuruva number are obtained. A few interesting results are given.

Keywords: Pythagorean triangles, 5-digit Dhuruva numbers

2010 Mathematics Subject Classification: 11D09, 11Y50, 11-04.

INTRODUCTION:

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. Thus the main objective of this paper is to find out the special Pythagorean triangles in connection with Dhuruva numbers.

In this communication, we have presented Pythagorean triangles each leg represented by 5-Digit Dhuruva numbers (53955, 59994) respectively. Also a few interesting results are obtained.

BASIC DEFINITONS:

Definition 2.1:

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2:

Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

METHOD OF ANALYSIS:

In this section, we exhibit Pythagorean triangles, each with a leg represented by the five digit Dhuruva number 53955 and denote this number by N.

To start with, it is noted that the leg y can not be represented by N as y is even and N is odd. Also z can not be written as sum of two squares. Since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P = p_1^{r_1} p_2^{r_2} \ldots\ldots\ldots p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 (\mathrm{mod}\ 4)$ then r_i is even. A prime $p \equiv 1 (\mathrm{mod}\ 4)$ can be written as $p = a^2 + b^2$

Now, consider x=N

$$\Rightarrow m^2 - n^2 = 53955$$

which is a binary quadratic Diophantine equation. Solving the above equation for m,n, we get 12 integer solutions and thus, we have 12 pythagorean triangles, each having the leg x to be represented by the five digit Dhuruva number N=53955 as shown in the table below:

S.No	m	n	x	y	z
1	246	81	53955	39852	67077
2	302	193	53955	116572	128453
3	518	463	53955	479669	482693
4	322	223	53955	143612	153413
5	622	577	53955	717188	719813
6	834	801	53955	1336069	1337157
7	1806	1791	53955	6469092	6469317
8	3002	2993	53955	17969972	17970053
9	5398	5393	53955	58222828	58222853
10	8994	8991	53955	161730108	161730117
11	2458	2447	53955	12029452	12029573
12	26978	26977	53955	1455571012	1455571013

Note that there are 8 primitive and 4 non-primitive triangles.

In a similar manner, it seen that there are 8 Pythagorean triangles, each having the leg y to be represented by the five digit Dhuruva number M =59994 as shown in the table below:

S.No	m	n	x	y	z
1	9999	3	9997992	59994	99980010
2	3333	9	11108808	59994	11108970
3	2727	11	7436408	59994	7436650
4	297	101	78008	59994	98410
5	1111	27	1233592	59994	1235050
6	909	33	825192	59994	827370
7	303	99	82008	59994	1011610
8	29997	1	899820008	59994	899820010

Note that there are 4 primitive and 4 non-primitive triangles.

From the above tables it is observe that

- $z - y = (m - n)^2$
- $z + y = (m + n)^2$

CONCLUSION:

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

ACKNOWLEDGEMENT:

*The finicial support from the UGC, New Delhi (F-MRP-5122/14(SERO/UGC) dated march 2014) for a part of this work is gratefully acknowledged.

REFERENCES:

[1]. W.Sierpinski, Pythagorean triangles, Dover publications, INC, Newyork, 2003.

[2]. M.A.Gopalan and G.Janaki, "Pythagorean triangle with area/perimeter as a special polygonal number", Bulletin of Pure and Applied Science, Vol.27E (No.2), 393-402, 2008.

[3]. M.A.Gopalan and A.Vijayasankar, "Observations on a Pythagorean problem", Acta Ciencia Indica, Vol. XXXVI M, No 4, 517-520, 2010.

[4]. M.A.Gopalan and S.Leelavathi, "Pythagorean triangle with area/perimeter as a square integer", International Journal of Mathematics, Computer sciences and information Technology, Vol.1, No.2, 199-204, 2008.

[5]. M.A.Gopalan and A.Gnanam, "Pairs of Pythagorean triangles with equal perimeters", Impact J.Sci.Tech., Vol 1(2), 67-70, 2007.

[6]. M.A.Gopalan and S.Leelavathi, "Pythagorean triangle with 2 area/perimeter as a cubic integer", Bulletin of Pure and Applied Science, Vol.26E (No.2), 197-200,2007.

[7]. M.A.Gopalan and A.Gnanam, "A special Pythagorean problem", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1435-1439,2007.

[8]. M.A.Gopalan, A.Gnanam and G.Janaki, "A Remarkable Pythagorean problem", Acta Ciencia Indica, Vol. XXXIII M, No 4, 1429-1434,2007.

[9]. M.A.Gopalan, and S.Devibala, "On a Pythagorean problem", Acta Ciencia Indica, Vol. XXXII M, No 4, 1451-1452,2006.

[10]. M.A.Gopalan and B.Sivakami, "Special Pythagorean triangles generated through the integral solutions of the equation $y^2 = (k^2 + 2k)x^2 + 1$ ",Diophantus J.Math., Vol 2(1), 25-30, 2013.

[11]. M.A.Gopalan and A.Gnanam, "Pythagorean triangles and Polygonal numbers", International Journal of Mathematical Sciences, Vol 9, No. 1-2, 211-215,2010.

[12]. K.Meena, S.Vidhyalakshmi, B.Geetha, A.Vijayasankar and M.A.Gopalan,"Relations between special polygonal numbers generated through the solutions of Pythagorean equation",IJISM, Vol. 2(2),257-258,2014.

[13]. M.A.Gopalan and G.Janaki, "Pythagorean triangle with perimeter as Pentagonal number", Antarctica J.Math., Vol 5(2), 15-18, 2008.

[14]. M.A.Gopalan and G.Sangeetha, "Pythagorean triangle with perimeter as triangular number",GJ-AMMS,Vol. 3, No 1-2,93-97,2010.

[15]. M.A.Gopalan , Manjusomanath and K.Geetha," Pythagorean triangle with area/perimeter as a Special polygonal number", IOSR-JM, Vol. 7(3),52-62,2013.

[16]. M.A.Gopalan and V.Geetha," Pythagorean triangle with area/perimeter as a Special polygonal number", IRJES, Vol.2(7),28-34,2013.

[17]. M.A.Gopalan and B.Sivakami, "Pythagorean triangle with hypotenuse minus 2(area/ perimeter) as a square integer", Archimedes J.Math., Vol 2(2), 153-166, 2012.

[18]. J.N.Kapur, Dhuruva numbers, Fasinating world of Mathematics and Mathematical sciences, Trust society, Vol 17, 1997.

[19]. Bert Miller,Nasty numbers, The mathematics teacher, No.9,Vol 73,649,1980.

[20]. Charles Bown.K, Nasties are primitives, The mathematics teacher, No.9, Vol 74,502-504, 1981.

[21]. P.S.N.Sastry, Jarasandha numbers, The mathematics teacher, No.9,Vol 37,issues 3 & 4 , 2001.

[22]. M.A.Gopalan V.Sangeetha and Manjusomanath, "Pythagorean triangle and Polygonal number",Cayley J.Math., Vol 2(2),151-156, 2013.

[23]. M.A.Gopalan and G.Janaki, " Pythagorean triangle with nasty number as a leg",Journal of applied Mathematical Analysis and Applications, Vol 4,No 1-2,13-17,2008.

[24]. M.A.Gopalan and S.Devibala, "Pythagorean triangle with Triangular number as a leg", Impact J.Sci.Tech., Vol 2(4), 195-199, 2008.

[25]. Dr.Mita Darbari, A connection between Hardy-Ramanujan number and special Pythagorean triangle," Bulletin of society for Mathematical services & Standards, Vol 3, No.2, 71-73, 2014.